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Mathematical Elements

OF

NATURAL PHILOSOPHY,

CONFIRM'D BY

EXPERIMENTS:

Or, an INTRODUCTION to

Sir *ISAAC NEWTON*'s Philosophy.

Written in *Latin* by the late

W. JAMES s'GRAVESANDE, LL.D.

Professor of Mathematicks at *Leyden*, and F. R. S.

Translated into *English* by the late

J. T. DESAGULIERS, LL.D. F.R.S.

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PREFACE

TO THE

FIRST EDITION

OF

DR. s'GRAVESANDE's *Introduction to the* NEWTONIAN *Philosophy.*

IF we compare the Writings of different Philosophers concerning PHYSICS, we may easily see that they call different Sciences by the same Name, tho' they all profess to explain the true Cause of natural * *Phænomena*. And no wonder if they disagree among themselves, since even Mathematicians, who deal in Certainties, can hardly be kept from wrangling.

But that Diversity of Opinions should not deter us from searching after Truth; since Labour and Study will find it out; and the more we are in love with it, the less we are liable to Errors, excepting such as human Frailty renders unavoidable. We must proceed cautiously in *Physics*, since that Science considers the Works of supreme Wisdom, and sets forth,

† *What Laws* JEHOVAH *to himself prescrib'd,*
And of his Work the firm Foundation made,
When he of Things the first Design survey'd.

How the whole Universe is governed by those Laws, and how the same Laws run thro' all the Works of Nature, and are constantly observed with a wonderful Regularity.

We must take care not to admit Fiction for Truth, for by that means we shut out all further Examination. No true Explanation

 of

* *Appearances.*

† ——— *quas dum primordia rerum*
Pangeret, omniparens leges violare Creator
Noluit, æternique operis fundamina fixit.

of Phænomena can ſpring out of a falſe Principle: And what a vaſt difference there is betwixt learning the Fictions of whimſical Men, and examining the Works of the moſt wiſe God! Since an Enquiry into divine Wiſdom, and the Veneration inſeparable from it, is to be the Scope of a Philoſopher; we need not enlarge upon the Vanity of reaſoning upon fictitious Hypotheſes.

Nature herſelf is therefore attentively and inceſſantly to be examined with indefatigable Pains. That way indeed our Progreſs will be but ſlow, but then our Diſcoveries will be certain; and oftentimes we ſhall even be able to determine the Limits of human Underſtanding.

What has led moſt People into Errors, is an immoderate Deſire of Knowledge, and the Shame of confeſſing our Ignorance; but Reaſon ſhou'd get the better of that ill-grounded Shame, ſince there is a learned Ignorance that is the Fruit of Knowledge, and which is much preferable to an ignorant Learning.

Natural Philoſophy is placed among thoſe Parts of Mathematics, whoſe Object is Quantity in general. Mathematics are divided into pure and mixed. Pure Mathematics enquire into the general Properties of Figures and abſtracted Ideas. Mixed Mathematics examine Things themſelves, and will have our Notions and Deductions to agree both with Reaſon and Experience.

Phyſics belong to mix'd Mathematics. The Properties of Bodies, and the Laws of Nature, are the Foundations of mathematical Reaſoning, as all that have examined the Scope of the Science will freely confeſs. But Philoſophers do not equally agree upon what is to paſs for a Law of Nature, and what Method is to be followed in queſt of thoſe Laws. I have therefore thought fit in this Preface to make good the *Newtonian* Method, which I have followed in this Work. What that Method is, I have briefly ſet down in the firſt Chapter.

Phyſics do not meddle with the firſt Foundation of Things. That the World was created by GOD, is a Poſition wherein Reaſon ſo perfectly agrees with Scripture, that the leaſt Examination of Nature will ſhew plain Footſteps of ſupreme Wiſdom. It is confounding and overſetting all our cleareſt Notions, to aſſert that the World may have taken its Riſe from ſome general Laws of Motion, and that *it imports not* what is imagined concerning the firſt Diviſion of Matter. *And that there can hardly be any thing ſuppoſed, from which the ſame Effect may not be deduced by the ſame Laws of Nature*: and that for this Reaſon; That *ſince Matter ſucceſſively*

ſucceſſively aſſumes all the Forms it is capable of by means of thoſe Laws, if we conſider all thoſe Forms in order, we muſt at laſt come to that Form wherein this preſent World was framed; ſo that we have no Reaſon in this Caſe to fear any Error from a wrong Suppoſition. This Aſſertion, I ſay, overthrows all our cleareſt Notions, as has been fully proved by many learned Men; and is indeed ſo unreaſonable, and ſo injurious to the Deity, that it will ſeem unworthy of an Anſwer to any one that does not know that it has been maintain'd by any antient and modern Philoſophers, and ſome of them of the firſt Rank, and far removed from any Suſpicion of Atheiſm.

Then firſt laying it down as an undoubted Truth, that God has created all Things, we muſt afterwards explain by what Laws every thing is governed, and to mention only the Moon, we muſt explain, why

* *The Silver Moon runs with unequal Pace,*
Which yet Aſtronomers could never trace,
Or fix in Number her uncertain Place.
What Force her Apſides *has forward driven,*
And make her Nodes *recede i'th' Starry Heaven.*
What is her Pow'r to agitate the Sea,
Whoſe various Tides her Preſence ſtill obey;
When th' Ocean *ſwells its topmoſt Banks to lave,*
Or ebbs from weedy Shores with broken Waves,
Leaving the Sands, the Sailor's Terror bare——

In order to explain more fully which way we trace out the Laws of Nature, we muſt begin by ſome previous and preparatory Reflexions:

What Subſtances are, is one of the Things hidden from us. We know, for inſtance, ſome of the Properties of Matter, but we are abſolutely ignorant in what Subject they are inherent.

Who dares affirm that there are not in *Body* many other Properties, which we have no Notions of? And whoever could certainly know, that beſides the Properties of *Body* which flow from the

——— * *Qua cauſa argentea Phœbe*
Paſſibus haud æquis graditur; cur ſubdita nulli
Hactenus Aſtronomo numerorum fræna recuſet.
Cur remeant Nodi, curque Auges progrediuntur.
——— *Quantis refluum vaga Cynthia pontum*
Viribus impellit, dum fractis fluctibus ulvam
Deſe it, ac nautis ſuſpectas nudat arenas;
Æternis vicibus ſuprema ad littora pulſans.

Eſſence of Matter, there are not others depending upon the free Power of GOD, and that extended and ſolid Subſtance (for thus we define Body) is endowed with ſome Properties without which it could exiſt? We are not, I own, to affirm or deny any thing concerning what we do not know. But this Rule is not followed by thoſe, who reaſon in phyſical Matters, as if they had a complete Knowledge of whatever belongs to Body, and who do not ſcruple to affirm, that the few Properties of Body which they are acquainted with, conſtitute the very Eſſence of Body.

What do they mean by ſaying, that the Properties of Subſtance conſtitute the very Subſtance? Can thoſe Things ſubſiſt when joined together, that cannot ſubſiſt ſeparately? Can Extenſion, Impenetrability, Motion, *&c.* be conceived without a Subject to which they belong? And have we any Notion of that Subject?

We muſt give up as uncertain what we find to be ſo, and not be aſhamed to confeſs our Ignorance. Tho' we need not fear being too bold in affirming, that a Subject altogether unknown to us may perhaps be endowed with ſome unknown Properties. And thoſe Men, who, at the ſame time that they ſay, conformably to this Axiom, that we muſt not reaſon about Things unknown, lay it down as a Foundation of their Reaſoning, that nothing relating to Body is unknown to us, are beholden to meer Chance, if they are not miſtaken.

The Properties of Body cannot be known *à priori*; we muſt therefore examine Body itſelf, and nicely conſider all its Properties, that we may be able to determine what natural Effects do flow from thoſe Properties.

Upon a further Examination of Body, we find there are ſome general Laws, according to which Bodies are moved. It is paſt doubt, for inſtance, *that a Body once moved continues in motion: That Reaction is always equal and contrary to Action.* And ſeveral other ſuch Laws concerning Body have been diſcovered; which can no way be deduced from thoſe Properties that are ſaid to conſtitute Body; and ſince thoſe Laws always hold good, and upon all Occaſions, they are to be looked upon as the general Laws of Nature. But then we are are at a loſs to know, whether they flow from the Eſſence of Matter, or whether they are deducible from Properties, given by GOD to the Bodies which the World conſiſts of, but no way eſſential to Body; or whether finally thoſe Effects, which paſs for Laws of Nature, do not depend upon external Cauſes, which even our Ideas cannot attain to.

Who

Who dares affirm any thing upon this Point concerning all, or any Laws of Nature, without incurring the Guilt of Rashness? Besides, whoever examines the Phænomena of Nature will be fully persuaded, that many of its Laws are not yet discovered, and that many Particulars are wanting towards the compleat Knowledge of others.

The Study of Natural Philosophy is not however to be contemned, as built upon an unknown Foundation. The Sphere of human Knowledge is bounded within a narrow Compass; and he that denies his Assent to every thing but Evidence, wavers in Doubt every Minute; looks upon many Things as unknown, which the generality of People never so much as call in question. But rightly to distinguish Things known, from Things unknown, is a Perfection above the Level of the human Mind. Though many Things in Nature are hidden from us, yet what is set down in Physics, as a Science, is certain. From a few general Principles numberless particular Phænomena or Effects are explain'd and deduced by mathematical Demonstration. For the comparing of Motion, or, in other Words, of Quantities, is the continual Theme; and whoever will go about that Work any other way, than by mathematical Demonstrations, will be sure to fall into Uncertainties at least, if not into Errors.

How much soever then may be unknown in Natural Philosophy, it still remains a vast, certain, and very useful Science: It corrects an infinite Number of Prejudices concerning natural Things, and divine Wisdom; and as we examine the Works of GOD continually, sets that Wisdom before our Eyes; and there is a wide Difference betwixt knowing the divine Power and Wisdom by a metaphysical Argument, and beholding them with our Eyes every Minute in their Effects. It appears then sufficiently, what is the End of Physics, from what Laws of Nature the Phænomena are to be deduced, and wherefore, when we are once come to the general Laws, we cannot penetrate any farther into the Knowledge of Causes. There remains only to discourse of the Method of searching after those Laws, and to prove that the three *Newtonian* Laws, delivered in the first Chapter of this Work, ought to be followed.

The first is, *that we ought not to admit any more Causes of natural Things, than what are true and sufficient to explain their Phænomena.* The first Part of this Rule plainly follows from what has been said above. The other cannot be called in question by any that owns the Wisdom of the Creater. If one Cause suffices, it is needless needlefs to fuperadd another, efpecially if it be confidered, that an Effect from a double Caufe is never exactly the fame with an Effect from a fingle one. Therefore we are not to multiply Caufes, till it appears one fingle Caufe will not do the Bufinefs. In order to prove the following Rules, we muft premife fome general Reflections.

We have already faid, that mathematical Demonftrations have no Standard to be judged by but their Conformity with our Ideas, and when the Queftion is about natural Things, the firft Requifite is, that our Ideas agree with thofe Things, which cannot be proved by any mathematical Demonftration. And yet as we have occafion to reafon of Things themfelves every Moment, and of thefe Things nothing can be prefent in our Minds befides our Ideas, upon which our Reafonings immediately turn; it follows, that GOD has eftablifhed fome Rules, by which we may judge of the Agreement of our Ideas with the Things themfelves. All mathematical Reafonings turn upon the Comparifon of Ideas, and their Truth is evidenced by implying a Contradiction in a contrary Propofition. A rectilineal Triangle, for inftance, whofe three Angles are not equal to two right ones, is a thing impoffible, When the Queftion is not about the Comparifon of Quantities, a contrary Propofition is not always impoffible. It is certain, for inftance, that *Peter* is living, though it is as certain that he might have died Yefterday: Now there being numberlefs Cafes of that kind, where one may affirm or deny with equal certainty; it follows that there are many Reafonings very certain, tho' altogether different from the mathematical ones. And they evidently follow from the Eftablifhment of Things, and therefore from the pre-determined Will of GOD. For by forcing Men upon the Neceffity of pronouncing concerning the Truth or Falfhood of a Propofition, he plainly fhews they muft affent to Arguments, which their Judgments neceffarily acquiefce in; and whoever reafons otherwife, does not think worthily of GOD.

To return to *Phyfics*: We are in this Science to judge by our Senfes, of the Agreement that there is betwixt Things and our Ideas. The Extenfion and Solidity of Matter, for inftance, afferted upon that Ground, are paft all Doubt. Here we examine the thing in general, without taking notice of the Fallacy of our Senfes upon fome Occafions; and which way Error is to be avoided upon thofe Occafions.

We

We cannot immediately judge of all phyſical Matters by our Senſes. We have then recourſe to another juſt way of reaſoning, tho' not mathematical. It depends upon this Axiom, *(viz.) We muſt look upon as true, whatever being deny'd would deſtroy civil Society, and deprive us of the Means of living.* From which Propoſition the ſecond and third Rules of the *Newtonian* Method moſt evidently follow.

For who could live a Minute's time in Tranquillity, if a Man was to doubt the Truth of what paſſes for certain, whatever Experiments have been made about it; and if he did not depend upon ſeeing the like Effects produced by the ſame Cauſe?

The following Reaſonings, for example, are daily taken for granted as undoubtedly true, without any previous Examination; becauſe every body ſees that they cannot be called in queſtion without deſtroying the preſent Oeconomy of Nature.

A Building, this Day firm in all its Parts, will not of itſelf run to Ruin To-morrow. That is, the Coheſion and Gravity of the Parts of Bodies, which I never ſaw altered, nor heard of having been altered without ſome intervening external Cauſe, will not be altered To-night, becauſe the Cauſe of Coheſion and Gravity will be the ſame To-morrow as it is To-day. Who does not ſee that the Certainty of this Reaſoning depends only upon the Truth of the forementioned Principle?

The Timber and Stones of any Country, which are fit for a Building, if brought over here, will ſerve in this Place, except what Changes may ariſe from an external Cauſe; and I ſhall no more fear the Fall of my Building, than the Inhabitants of the Country, from whence thoſe Materials were brought, would do, if they had built a Houſe with them. Thus the Power which cauſes the Coheſion of Parts, and that which gives Weight to Bodies, is the ſame in all Countries.

I have uſed ſuch kind of Food for ſo many Years, therefore I will uſe it again To-day without fear.

When I ſee Hemlock, I conclude it to be poiſonous; tho' I never made an Experiment of that very Hemlock I ſee before my Eyes.

All theſe Reaſonings are grounded upon Analogy; and there is no doubt but our Creator has, in many Caſes, left us no other way of Reaſoning, and therefore it is a right Way. But the Foundation of Analogy is this, That *the Univerſe is govern'd by unchangeable Laws.*

Which being once admitted, we may afterwards make uſe of the ſame Method in other Matters, where no abſolute Neceſſity forces us to reaſon at all. When an Argument is good in one Caſe, there

is

is no reaſon why we ſhould refuſe our Aſſent to it in another. For who can conceive that *Things proved the ſame way are* not *equally certain?* Beſides, tho' we conclude in general, that this Method of Reaſoning is right from the Neceſſity of uſing it, yet it does not follow that particular Reaſonings depend upon that Neceſſity. I conclude from Analogy, that Food is not poiſonous; but is that Argument only good when I am hungry?

In Phyſics then we are to diſcover the Laws of Nature by the Phænomena, then by Induction prove them to be general Laws; all the reſt is to be handled mathematically. Whoever will ſeriouſly examine what Foundation this Method of treating Phyſics is built upon, will eaſily diſcover this to be the only true one, and that all Hypotheſes are to be laid aſide.

So much for the Method of Philoſophizing. I have now a Word to ſay of the Work itſelf.

The whole Work is divided into four Books. The firſt treats of Body in general, and the Motion of ſolid Bodies. The ſecond of Fluids. What belongs to Light is handled in the third. The fourth explains the Motion of celeſtial Bodies, and what has a Relation to them on Earth. The two firſt Books are contain'd in this Tome.

In order to render the Study of Natural Philoſphy as eaſy and as agreeable as poſſible, I have thought fit to illuſtrate every thing by Experiments, and to ſet the very mathematical Concluſions before the Reader's Eyes by this Method.

He that ſets forth the Elements of a Science, does not promiſe the learned World any thing new in the main; therefore I thought it needleſs to point out where what is here contained is to be found. I have made my Property of whatever ſerved my Purpoſe; and I thought giving Notice of it, once for all, was ſufficient to avoid ſuſpicion of Theft. I had rather loſe the Honour of a few Diſcoveries, diſperſed here and there in this Treatiſe, than rob any one of theirs. Let who will then take to himſelf what he thinks his own, I lay claim to nothing.

As to the Machines which ſerve for making the Experiments, I have taken care to imitate ſeveral from other Authors, have altered and improved others, and added many new ones of my own Invention. And no wonder I ſhould be forced to that Neceſſity, having made Experiments upon many Things never tried perhaps by any one before. For Mathematicians think Experiments ſuperfluous, where mathematical Demonſtrations will take place: But

as

as all Mathematical Demonſtrations are abſtracted, I do not queſtion their becoming eaſier, when Experiments ſet forth the Concluſions before our Eyes; following therein the Example of the *Engliſh*, whoſe Way of teaching Natural Philoſophy, gave me occaſion to think of the Method I have followed in this Work. I ſhall always glory in treading in their Footſteeps, who, with the Prince of Philoſophers for their Guide, have firſt opened the Way to the Diſcovery of Truth in philoſophical Matters, diſmiſſing all feign'd Hypotheſes out of Philoſophy.

As to the Machines, I will ſay thus much more by way of Advertiſement: That moſt of them have been made by a very ingenious Artiſt of this Town, and no unſkilful Philoſopher, whoſe Name is *John Van Muſſchenbroek*, and who has a perfect Knowledge of every thing that is here explain'd. Which Advertiſement, I ſuppoſe, will not be diſpleaſing to thoſe who may have a fancy to get ſome of the ſame Machines made for themſelves.

TO THE

READER,

CONCERNING THE

SECOND EDITION.

WHEN I first intended to write these Elements, my Design was that my Auditors shou'd be able to recollect, with ease, such Things as they had heard more largely explain'd and demonstrated; and that I might give an Idea of Natural Philosophy, treated of in a mathematical Manner, to such of my Readers, who were acquainted only with the first Elements of Geometry. Moreover, that this Book might be useful to Beginners, I pass'd over the more difficult Things, and often mention'd Propositions, which I said were prov'd by Geometricians.

But that the second Edition might be likewise of service to such of my Readers as were better acquainted with Mathematics, I annex'd the mathematical Demonstrations of all such Propositions, in the Scholiums to those Chapters, in which they are mention'd. And lest this shou'd confound other Readers, I took care to have them printed in a smaller Character. Yet I so dispos'd of the whole, that what is printed in a greater Character makes a kind of separate Treatise.

Likewise in the Scholiums I deliver'd some other Things, which cou'd not well be treated of in the Body of the Work, tho' they relate to what is explain'd, or serve by way of Illustration.

The second Edition is, also in other respects, more full and more accurate.

Many new Machines, and old ones improv'd, are exhibited in the Plates; and the Experiments, and their Success, are set forth in this Edition with greater Care.

For we have here likewise more fully explain'd, and have deliver'd, supported, and illustrated, with several new Experiments, our New Theory of Percussion, *which is founded upon* Leibnitz's *or rather* Huygens's *Doctrine of innate Forces.*

It was never my Way, nor is it now, however I may be provok'd, to quarrel about the Truth. That which appears true to me, I defend according

according to my best Ability, when there is occasion; and to remove all Appearance of Contention, as much as I am able, I have so endeavour'd to propose the Arguments which seem to me to be the Foundation of the fore-mention'd Theorys, that Answers to Difficulties may from thence be easily drawn; and I have undertaken to answer but a few directly: And I leave the Reader to judge, whether the Theories of Forces, *and* Percussions, *as well as of* Resistances *and* Retardations, *of Bodies mov'd in Fluids, don't exactly agree with the Phænomena, and with one another.*

As for my Work, every one may make use of it as he pleaseth, so that he does not think that I am bound to answer whatsoever may be objected. As long as I look upon those Things to be true which I have written, I think I may very justly be silent.

Altho' in many Things relating to the fore-mention'd Theories, I differ in my Opinion from Sir Isaac Newton, *yet I made no scruple to keep the Title of an* Introduction to the Newtonian Philosophy, *and to prefix it to the second Edition. For we illustrate many Things in this Book by what is deliver'd by that excellent Philosopher; and many Things, here explain'd, have a Tendency to make* Sir Isaac Newton's *philosophical Works, which deserve to be for ever celebrated by the greatest Philosophers, and read by all with Admiration, the more easily understood.*

He only, who in Physics reasons from Phænomena, rejecting all feign'd Hypotheses, and pursues this Method inviolably to the best of his Power, endeavours to follow the Steps of Sir Isaac Newton, *and very justly declares that he is a* Newtonian *Philosopher; and not he, who implicitly follows the Opinion of any particular Person.*

But that the Additions, and Emendations, of this Edition, might be of service to those who have the first Edition by them, I have taken care to publish a separate Supplement: In which, that I might be useful to those who have the first Edition, I have done all that I cou'd, but not what I wou'd have done. In the Supplement I have given Descriptions of all the new Machines, the Additions, and Propositions changed. But I cou'd not put into the Supplement, either the Improvements of the Machines, or those Things, by which, what is contain'd in the first Edition, is either illustrated, or more clearly and accurately expres'd: A compleat Supplement wou'd have given the Reader too much Trouble, and been too expensive.

THE

PREFACE

TO THIS

THIRD EDITION.

MY Intention, in writing this Book, was to give the Mathematical Elements of Natural Philoſophy. For this Reaſon I have choſen to treat of Things, in which what was certain might, in my Opinion, be ſeparated from what was doubtful; and I thought I might fairly omit what was deduc'd from feign'd Hypotheſes.

I don't deny but that Hypotheſes may open the way to Truth; but when that is prov'd to be true, which before was only ſuppos'd, there is no longer any room for Hypotheſes.

It is commonly ſaid, at this time, *if the Cauſe, which I imagine, ſhou'd not be true, there wou'd be no Cauſe.*

But this remains to be prov'd; for tho' we are unable to find out another Cauſe, it does not thence follow that there is no other Cauſe; becauſe that kind of Proof, which excludes all poſſible Cauſes, in treating of Nature, our Knowledge of which is very much limited, is very difficult.

Some endeavour to defend Hypotheſes upon different Principles. They argue, that all our Knowledge of natural Things is imperfect; that our firſt Reaſoning about them is built on Hypotheſes; and that that Analogy, without which we can diſcover nothing in Phyſics, is to be referr'd to Hypotheſes.

To theſe I have given an Anſwer in the preceeding Preface of the Year 1719. Afterwards, when I was oblig'd to ſpeak publickly, in the Univerſity, I again conſider'd this very Subject, and treated more diſtinctly of the Motives of Perſuaſion, in that Place where I ſpeak of Bodies; this Diſcourſe, tho' made publick, is annex'd to this Preface, to remove all Doubt that may yet remain.

The Defenders of Hypotheſes often likewiſe make uſe of Arguments, which are call'd *Argumenta ad hominem*, but this I have nothing to do with; for if any one ſhall make it appear that I have made uſe of one or two Hypotheſes, it will not from thence follow

that

that Hypothefes are to be admitted; at leaft I wou'd not allow this Conclufion myfelf, but wou'd reject them.

Befides the general Defign juft mention'd, I intended moreover this particularly, *viz.* to join Mathematical Demonftrations with Experiments; and fo difpofe the whole, that it might make a Syftem, and contain an Introduction to a deeper Knowledge in Phyfics.

There was no want of Authors, who had made Experiments, but moft of them were not to my purpofe; and fuch as wou'd have been of fervice to me, I cou'd no where find.

Moft of thofe, who have treated of Phyfics, which may be treated of in a mathematical way, have not concern'd themfelves about Experiments, which might have been of ufe to illuftrate their Demonftrations. And thofe, who have turn'd their Minds to Experiments, have bufied themfelves chiefly about thofe Experiments to which Mathematics can't lead one, and have not an intimate Connexion with them.

Thofe Courfes of Experimental Philofophy were much more to our purpofe, which were given about that time at *London* by the learned Dr. *Joh. Theoph. Defaguliers*, and *Francis Haukfbee* jun. the Experiments of the latter being explain'd by the learned Mr. *William Whifton*.

They were all explain'd in good Order, and together made a kind of a Body of Philofophy; but had refpect to the firft Principles only, and the more obvious Matters: and the firft of thefe Courfes was contain'd in thirty two Lectures, the laft in twenty fix.

But for myfelf, my Defign was more extenfive, and I propos'd to confider many Things about which Experiments had never before been made, as I mention'd in the foregoing Preface, where I have given an Account of what belongs to the firft Edition.

I fpoke of the fecond Edition in the Advertifement to the Reader, prefix'd to that Edition, which is likewife found before this Preface.

I wou'd willingly have comprehended in a feparate Supplement what has been added or alter'd in this Edition, which might have been of fervice to thofe who have the fecond Edition, or the firft with its Supplement; but fuch a Supplement wou'd have been too bulky, fince this Book is fo alter'd and inlarg'd, that it may well be look'd upon as a new Work.

But I have kept the fame Order, except that I have here divided the whole Work into fix Books, which before was only divided into

into four; because this Division was, I thought, more commodious for the present Disposition of Things.

There is also the same Matter, to which the late Additions are referr'd; and tho' I have treated of but few Things, which are not explain'd by others, yet had I no Intention to take off my Readers from the Study of other Authors: every one has a Method of his own, and different People different ones.

No one can get a thorough Knowledge of a Science, unless he compares different Treatises upon it; which I wou'd have so understood, as that a Person shou'd make choice of some Author, and by taking him for a Guide, first gain a general Knowledge of the Science, then a more particular one. Afterwards let him read other Authors, and several Treatises with the same Care, passing over such Things only, as he meets with elsewhere; and at last come to peculiar Treatises.

I have made it my Business to be serviceable to those chiefly, who take this Method in their Studies: Therefore I have endeavour'd to illustrate (at least with new Demonstrations) what may be met with elsewhere, and have omitted, as often as I well cou'd, what has been deliver'd by others, that the Authors themselves might be consulted about it: For it was my Intention to give an Introduction to the understanding of those Things, which have been deliver'd by others, especially such as require a deeper Research, as are many new Things daily made publick. For many excellent Philosophers have lately taken great pains in the Study of mathematical and experimental Philosophy, and do make continual Improvements in it.

This appears from the yearly Transactions of the many Academies which have been erected in several Parts of *Europe*, in the foregoing and present Ages, to the Improvement of these Sciences.

Besides those, whose Writings are found in these Transactions, we have daily Testimonies and Proofs of the Advantage of joining Mathematics and Experiments together from those celebrated Men, *Polenus*, *Desaguliers*, *Bernoulli*, *Wolfius*, *Musschenbroek*, and so many more, that it wou'd be tedious to mention them. To the Mathematico-physical Writings of these we may add what has been left about these Things by *Galileus*, *Torricelli*, *Gulielmini*, *Mariotte*, *Huygens* and many others, who have wrote about the particular Parts of Mathematics, belonging to Physics, and some of which I shall refer to in what follows.

But

But among thoſe, who have illuſtrated Phyſics by mathematical Demonſtrations and Experiments, Sir *Iſaac Newton* is to be reckon'd the Chief, who has demonſtrated, in his mathematical Principles of Natural Philoſophy, the great Uſe of Mathematics in Phyſics, inaſmuch as no one before him ever penetrated ſo deeply into the Secrets of Nature.

In Optics he has compos'd a new phyſical Syſtem, and altho' what he has deliver'd is very ſurprizing, yet the Strength of his Genius chiefly appears in that Art, by which he open'd himſelf the way, which he conſtantly follow'd, as if he had been led by *Ariadne*'s Clue of Thread, until he put in execution what he had propos'd.

His Experiments have a kind of Connexion one with another; and from one Experiment he has often, with great Subtilty, deduc'd what was to be try'd next, ſo as to enable him to come nearer to the Mark.

It is plain that what I have already ſaid about reading ſuch Things as are found elſewhere omitted by me, is to be underſtood under a Limitation; for moſt of what I deliver, is explain'd in other Authors; but in this Caſe, I ſaid I wou'd bring a new Demonſtration, *viz.* when it beſt ſuited my Deſign: for where another's Demonſtration explains a thing more clearly, without doubt it is fit to make uſe of it, and it wou'd be abſurd to do otherwiſe; but then this is not to be done frequently, for this wou'd tire the greater part of my Readers.

In this Edition there are many Machines added, and others ſo improv'd, that they may be almoſt all look'd upon as new; but as many of them are often made for the Uſe of the others, I have explain'd theſe and their Uſes with greater Care; and I thought I was bound to do this upon their account, who may make uſe of them now or hereafter.

In the former Editions I have not mention'd where that is to be found which I have taken from others, which I perceiv'd diſpleas'd many; but I will do what they require, if they think it neceſſary, and briefly run over the whole Work, and endeavour to call to mind, where thoſe Things, which are not mine, are to be found; and I will likewiſe refer the Machines to their Inventers; tho' there are few in this Edition taken from others.

All I require is, that if I ſhou'd have omitted any thing, the Reader wou'd think 'twas without deſign.

The

The firſt Book contains three Parts. In the firſt I examine into the general Properties of Bodies, which are commonly known.

In Schol. I. Chap. IV. I endeavour to illuſtrate the Diviſibility of Matter, by the Conſideration of the logarithmick Spiral; the Properties of which were firſt demonſtrated by *Wallis* *, *Barrow* †, and *James Bernoulli* ||.

I ſhew by the bye, in a particular Caſe, what Angle the Tangent makes with the Radius; but there is an Error in the ſecond Edition, which is corrected in this.

This Determination depends upon the Solution of this Problem. *The Center being given, and two Points taken at pleaſure, in the ſaid Spiral, together with the Number of Revolutions between the Points given, whether this Number be an Integer or a Fraction, to find out the Angle, which the Tangent makes with the Radius.*

The Solution is very eaſy, tho' at firſt View it appears difficult. For if we ſuppoſe the Curve in queſtion, keeping one of the given Points, with the Tangent to that Point, and alſo the Ordinates, to be chang'd into the common Logiſtick, whoſe Aſymptote ſhall paſs thro' the Center, and be perpendicular to the Radius paſſing thro' the Point that is kept, it will immediately appear without any Calculation, how by the Rule of Proportion, making uſe of the Tables of Logarithms and Tangents, we may have the Tangent of the Angle ſought.

What we have explain'd in the third Scholium of the ſame Chapter, concerning the Claſſes of Infinites, is from Sir *Iſaac Newton* ‡, but I have added the Demonſtration in my Edition of the Year 1725, which is here repeated.

The 5, 6, and 7th Experiments of the Vth Chap. are *Haukſbee's* **, the 11, 12, 13th are deſcrib'd by *Mariotte* ††; the reſt are commonly known. Many have wrote concerning the Cauſes of theſe Phænomena; but I have endeavour'd to illuſtrate them in the Scholiums, from other Principles; wherefore the Reader muſt not be ſatisfy'd with what others have deliver'd about this matter.

What is mention'd in Chap. VI. is commonly known.

In the ſecond Part of Book I. I treat of the Actions of Powers, but of ſuch as are deſtroy'd by the contrary Actions of other Powers; *i. e.* this ſecond Part treats of Equilibrium only.

There

* Tractatus de Cycloide; operum Tom. 1. pag. 560.
† Lectio 12. Geom Prob. 4.
|| Acta Lipſ. 1691. pag. 282: ſed præcipuè 1692. pag. 210.
‡ Schol. Lemm 10. Libri I. princ.
** Philoſoph. Tranſact. N. 305. p. 2223. N. 336. p. 539. N. 332. p. 395.
†† Mouvement des Eaux; part. 2. Diſc. I.

There are different Demonſtrations of Mathematics about Equilibrium; but I am pretty certain, that there are but few, in which he, who ſhall examine them with Attention, will not find, that that is taken for granted which wants proof. *Wallis* * and ſome others have given the true Foundation.

I treat of this Affair in Chap. VII. and conſider it abſtractly.

In Chap. VIII. I deliver ſome general Things concerning Gravity, the chief of what is there ſaid being this, That all Bodies deſcend with equal Gravity, when they are not hinder'd; the reſt is commonly known.

This equal Velocity, which has been a Matter of Contention among Philoſophers, was firſt demonſtrated by *Galileo* †, by Experiments with Lead and Cork. Afterwards Sir *Iſaac Newton* illuſtrated it more clearly with Gold, Silver, Lead, Glaſs, Sand, common Salt, Wood, Water, Wheat ||. And this has likewiſe been confirm'd by Experiments made with a very light Body and Gold, in glaſs Receivers, from which the Air has been exhauſted. And by ſuch an Experiment I have alſo prov'd what I have aſſerted about Gravity.

Moſt of the Experiments concerning the Balance and Center of Gravity, which are explain'd in Chap. X. are to be found in the Courſes of *Haukſbee* or *Deſaguliers* lately mention'd. Experiment 11. is deſcrib'd by *Caſſatus* in his Mechanics ‡.

Concerning the Center of Gravity *Wallis* firſt obſerv'd, that this Propoſition ſhou'd not be admitted without a Demonſtration, *viz.* That there is given in every Body a certain Point, about which it will be in Equilibrio in every Situation; therefore he demonſtrated that every Body has a Center of Gravity **. The Demonſtration which I have given of this Propoſition is to be found in Scholium I. in which I follow *Wallis* with reſpect to the Determination of the Center of Gravity ††. In the ſecond Scholium of this Chapter I have given a mechanic Arithmetic. This I did upon account of *Caſſini*, who demonſtrated that ſome arithmetical Operations might be perform'd by a Balance, whoſe Arms were divided into equal Parts ||||.

* Mechan. cap. 2. prop. 5.
† Mech. Dialog. 1.
|| Princip. Lib. 3. prop. 6.
‡ Lib. 1. Cap. 7.
** Mechan. cap. 4. prop. 15.
†† Ibid. prop. 24.
|||| Journ. des Scavans 27. Decemb. 1676.

As for the ſimple and compound Machines, of which none is made uſe of, but what is common, I need ſay nothing here about any of them, except the Wedge.

There is a ſtrange Difference of Opinions about this Machine. Thoſe who have examin'd into it with the greateſt care are *de la Hire* * and *Varignon* †.

But the Solution of this laſt Author, becauſe he has not taken notice of the Angle of the Wedge, can only be applied to thoſe Caſes, in which the Wedge fills the Angle, which is made by the ſeparated Parts of the Wood. My Solution is to be found in the Edition of the Year 1725.

In the firſt Edition of the Year 1719, I propos'd a Machine, to ſhew the Force of the Wedge, which was another's, and but a little alter'd; afterwards I rejected it becauſe it had too much Friction, and chiefly becauſe it did not ſhew the Action of the Wedge.

Therefore I gave a new one in the following Edition, which is more diſtinctly exhibited and explain'd in this.

After the Machines I treat of oblique Powers. In the foregoing Editions I have immediately conſider'd the Point, which is drawn by three Powers, and is at reſt; and I gave *Varignon*'s Demonſtration ||; afterwards I deduc'd the Reduction of an oblique Power to a direct one out of it.

I have now chang'd the Order, becauſe the ſecond Propoſition is more ſimple, and very eaſily demonſtrated; if we apply two direct Powers to a bended Lever, one of which is always oblique in reſpect of the other. From this Reduction I afterwards eaſily deduce what has relation to a Point drawn by three Powers, and reduce the Propoſition to *Varignon*'s Triangle.

I can't call to mind whoſe Demonſtration that by the bended Lever is, which deſerves to be eſteem'd on account of its Simplicity; but it is not mine.

Merſennus has likewſe treated of a Point, drawn by three Powers, and demonſtrated the Proportion between theſe to be the ſame as that between the Sides of a Triangle, whoſe Conſtruction he ſhews ‡. This is like the Triangle of *Varignon*, which I uſe, becauſe the Conſtruction of this is eaſier.

In the foregoing Editions I have given a Machine to demonſtrate the Powers, when more than three draw the ſame Point, which

* Mechan. Chap du Coin.
† Mechan. Sect. 8.
|| Projet d'une nouve Mech. Le même. 3. & prob. pag. 23.
‡ Phænom. baliſtica, prop. 6.

which I have alſo given here, becauſe it is very ſimple. But I have added a new one, which is more complex, but its Uſe is very extenſive, tho' in theſe Elements I do not treat of the intricate Caſes of oblique Forces, to which this Machine may be applied.

The third Part of Book I. treats of the Actions of Powers on Bodies, which are not retain'd.

Galileo's Doctrine of the Deſcent of heavy Bodies is explain'd in Chap. XVIII. and XIX.

In Chap. XX. I treat of Pendulums. Many Things, and indeed the chief Things, which are found in this Chapter, or its Scholia, are from *Huygens* *, but otherwiſe demonſtrated. Many Things are to be found here about the Cycloid, yet *Huygens* is not the firſt that found out this Curve; it was known to *Galileo*, and there has been much Diſpute about thoſe who found out its chief Properties †. But *Huygens* diſcover'd this Property, *viz.* That the Deſcent in a Cycloid is always made in the ſame time. He likewiſe was the firſt who gave the *Evolute* of this Curve, by help of which he ſhew'd the Way in which the Pendulum mov'd. Before his Time this kind of Curve was unknown to Mathematicians. *Huygens* was alſo the firſt who treated of the Center of Oſcillation. In the ſecond Edition I gave a Demonſtration of this Center, deduc'd from the general Theory of Compreſſions, which may be found in this alſo; and this was then ſufficient, becauſe I only conſider'd Bodies applied to the ſame Line. But now many Things are added both in the Subject and Scholia, which are of uſe in what follows; wherefore the Center of Oſcillation ſhou'd be determin'd likewiſe in other Caſes, which when I cou'd not do from the Theory of Compreſſions alone, without a more intricate Demonſtration, in N°. 476. I have called to my Aſſiſtance *Huygens*'s Principle, from which he has deduc'd all the Demonſtrations of the Center of Oſcillation.

In the laſt Scholium I demonſtrate a Cycloid to be a Line of the ſwifteſt Deſcent; which Property of it was firſt diſcover'd half an Age ago, by *John Bernoulli*, a famous Mathematician, who ſtill follows mathematical Studies with the Vigour of a young Man ‖.

Chap. XXI. is entirely new, and relates to Subjects as yet untouch'd by mechanical Writers, tho' they are uſeful.

* Vide Horol. Oſcill.
† Vide Groningii Hiſt. Cycloeidis.
‖ Acta Lipſ. An. 1697. pag. 206.

In Chap. XXII. of the Projection of heavy Bodies after *Galileo*, I demonſtrate, that the way of the Body projected is a conical Parabola. In this Chapter I ſolve two Problems, the firſt is, *The Velocity being given from a given Point, to project a Body to a given Point.* The ſecond is, *From a given Point, to project a Body to a given Point, thro' a given Point.*

The firſt of theſe is common, and many Solutions of different Mathematicians may be ſeen in *Blondel* *. The Solution which I give is *Cotes*'s †; but how eaſily I fell into this Solution, before I had ſeen the Solution of the famous Men mention'd, I have explain'd in my *Principles of univerſal Mathematics.*

Chap. XXIII. which is the laſt of Book I. treats of central Forces. I demonſtrate the chief *Huygenian* Theorems concerning this Motion ‖; theſe Theorems were deliver'd by this famous Man in the Year 1673, at the End of his Book *De Horologio Oſcillatorio.* I likewiſe illuſtrate thoſe Things, which are to be found concerning this Motion in Sir *Iſaac Newton*, and many of my Demonſtrations have the ſame Foundation with Sir *Iſaac*'s.

In the firſt Edition I had explain'd a Machine for comparing of central Forces; but the firſt Trial was not very perfect: afterwards I often mended it, and at length rejected it; but I have now given an Explanation of a new one, by which the Experiments are very accurately demonſtrated. Experiments indeed have been made about theſe Forces; *Deſaguliers* and others made many; but no one, as far as I know, has attempted any thing with regard to the comparing the Forces of Bodies ſeparately agitated, before the firſt Machine that was propos'd by me.

In Book II. I trreat of innate Forces, and the Colliſion or Congreſs of Bodies.

In Chap. I. I treat of the Nature of Forces; what I have firſt demonſtrated of theſe is, that the innate Force infinitely exceeds the Compreſſion. This Opinion is not new, altho' many who treat of Mechanics, ſpeak of the comparing of theſe Quantities, as if it was plain that they might be compar'd.

Ariſtotle firſt ſhew'd the Difference mention'd; for he aſks, why an Ax, when ſtriking, divides, but does not when it only preſſes ‡?

This very Queſtion takes it for granted that the Effect of Compreſſion is nothing, at leaſt infinitely ſmall in reſpect of the Effect of

* Art de jetter les Bombes. part. 3. Liv. 5. Chap. 5. 6. 7.

† Opera miſcellanea p. 87.

‖ Opera varia p. 188. Opera reliqua Vol. 11. Tom 2. p. 107.

‡ Mechanica, Quæſt. 20.

of Percuſſion; in the Anſwer alſo nothing elſe is contain'd. For this Anſwer, if expreſs'd in other Words, ſignifies, that in one Caſe there is only Compreſſion given, in the other Compreſſion with Percuſſion. *Galileo* deduc'd the ſame Concluſion from ſome Experiments *. But *Borelli* firſt clearly demonſtrated, that theſe Quantities are incommenſurable, and that Percuſſion is infinitely great, if it be compar'd with any Compreſſion whatſoever †.

In the II and III Chap. I treat of the Meaſure of Forces; many Authors have wrote of theſe, and two Opinions about them do now divide Philoſophers. All grant that the Force follows the Proportion of the Maſs; but many, where the Velocity is different, contend that the Force follows the Ratio of this laſt; while others endeavour to maintain that the duplicate Ratio of the Velocity obtains here.

The firſt Controverſy about this Meaſure, tho' indirect, was between *Huygens* and the Abbot *Catalan*, on occaſion of determining the Center of Oſcillation.

Whoever will examine *Huygens*'s Demonſtration in the fourth Part of his Book *de Horologio Oſcillatorio*, and compare them with *Catalan*'s Objections, will evidently ſee that in that Controverſy the Meaſure of the Force is treated of.

Theſe two famous Men conſider the Caſe in which ſeveral Bodies join'd together, deſcending with the Force of Gravity alone, and then looſed from one another, are carried upwards by the Velocities which they have acquir'd; or ſuch Bodies as deſcend ſeparated, and aſcend together. *Huygens*, in conſidering this Caſe, reaſons from this Axiom, *that Bodies cannot aſcend by the Action of Gravity*; and demonſtrates, that the Sum of the Products of the Weights of the ſeveral Bodies, multiplied by the Heights from which they fall, or to which they riſe, is the ſame both before and after they are looſed; that is, when the Weights are ſeparate, he ſeeks the Sum of the Products of the Square of the Velocities by the Maſſes. *Catalan* on the contrary reaſons thus: "If two equal "Weights ſuſpended ſeparately from the ſame Point, [at unequal "Diſtances] and rais'd to the ſame horizontal Plane, which paſſes "thro' the Point of Suſpenſion, be ſo let fall as to deſcribe ſimilar "Arcs — they will require ſuch Velocities, that their Squares will "be one to another, as the Heights whence thoſe Weights deſcend "perpendicular to the Horizon."

"Then

* Mechan. Dial. 4. in fine. Merſennus Cogit. Phyſ. Mat. Tom. III. Reflex. cap. 23.
† De vi percuſſionis, prop. 90.

"Then, if these two Weights be joined by a Line, or an inflexi-
"ble Rod, the which we suppose without Weight, and being
"suspended from the same Point at the same Distances, they be
"let fall from the same Height as before; a Pendulum com-
"pounded of these will acquire as much Velocity as the Sum of
"two simple Pendulums would do." And then immediately adds this Reason—"Because the Separation of the Weights does not "change the Quantity of Motion."

Huygens easily demonstrated, that these Principles would lead to absurd Consequences. For *Catalan* laid down with the rest, that the Quantity of Motion was proportional to the Product of the Velocity by the Mass, (which Proportion the Translation follows;) but *Huygens* easily demonstrated, that *Catalan*'s Opinion that this Quantity did not change, was absurd. As to the Quantity of Motion, he had already said before, what he demonstrated afterwards, in the Congress of perfectly elastic Bodies, (which he refer'd to Bodies perfectly hard) that the same Quantity of Motion was not preserv'd; but that the Sum of the Products of the Squares of the Velocities by the Masses, was not chang'd by the Collision, that Sum being the same before and after the Stroke: But afterwards he spoke more generally; when he said in his last Answer to *Catalan*, *That we must by no means take for a Law of Nature, that the same Quantity of Motion was preserv'd, unless what was spent and consum'd in acting on any thing; but that this was a constant Law of Nature, that Bodies kept their ascending Force*, (Force ascensionelle) *and therefore the Sum of the Squares of their Velocities always remain the same.* We are to observe, that this is understood of equal Masses, for *Catalan* had reduc'd the Question to this Case, as we have seen.

Before this Controversy was ended, another arose between *Leibnitz* and the same Abbot *Catalan*. *Huygens*'s last Dissertation but one was publish'd in the Year 1684, and the last, from which our Quotation is taken, was publish'd in 1690. But in the Year 1686, *Leibnitz* inserted a Paper in the *Leipzic* Acts of *March*, in which he subjoin'd these Words to the Demonstration, which he had given, and in which the Heights from which Bodies descend, and to which they ascend, are treated of. "*Hence it appears how "the Force is to be estimated from the Quantity of the Effect, which "it is able to produce; for example, from the Height to which it "can raise a heavy Body of given Magnitude and Kind, and not "from the Velocity which it can give to a Body.... Therefore we*

"*must*

" *muſt ſay, that the Forces are in a compound Ratio of the Bodies* " *and the Heights, from which falling, they could acquire ſuch Velocities. . . . . . Whence have riſen ſeveral Errors. . . . . Whence* " *alſo, I imagine, that it happens, that* HUYGENS'*s Rule concerning* " *the Center of Oſcillation of Pendulums, which is very true, has of* " *late been call'd in queſtion by ſome learned Men.*"

This agrees well enough with the Words of *Huygens* which we have quoted, and which, tho' they were publiſh'd after what we have quoted from *Leibnitz*, only explain thoſe Things, which are really contain'd in *Huygens*'s former Writings. Something like this often happens; thoſe that go before explain Things ſo, that nothing remains for the Author of a new Invention to do, but to explain more diſtinctly, and declare in more expreſs Words, what another has only hinted at more obſcurely. I do not deny but that *Leibnitz* is to be eſteem'd the Author of this Meaſure of Forces, which he explains in the Words which we have quoted from him; but I dare ſay, that *Huygens* led him to it.

Catalan, and afterwards *Papin*, anſwer'd *Leibnitz*: *Leibnitz* replied again, and ſeveral Papers were written upon the Subject. Then many others conſidered the ſame Queſtion.

This is what is treated of in the 2d and 3d Chapters of the ſecond Book of this Work; I have added many new Experiments to thoſe which I had given in the foregoing Edition. I do not contend about Words; but I have undertaken to prove and make out, by direct Experiments, theſe two Things, *viz. That Velocity is not communicated to a Body at reſt, but by an Action which muſt be as the Product of the Maſs into the Square of the Velocity. And that the Body mov'd never loſes its whole Velocity, unleſs it overcomes the Reſiſtance; that is, unleſs it produces an Effect, which follows the ſaid Ratio.* We mean here the entire Action, and that alone, which is ſpent in moving the Body; and the entire Effect, and that only, which the Body produces whilſt it loſes its Motion. He that denies theſe Propoſitions, denies what is evident to Sight: And he that agrees to them, and yet affirms that they follow from the Meaſure of Forces before receiv'd; I do not diſpute with him: I call *Force* that Power of acting in a Body, which muſt be meaſur'd by its whole Effect. Yet I deſire thoſe who make uſe of another Meaſure, to conſider whether they can explain not only ſome of, but all our Experiments concerning Force and Colliſion. I deſire likewiſe, when they ſhall conſider the Times, in which the

Effects

Effects are perform'd, that they may not uſe Fictions inſtead of the true Meaſure of the Times; I have deliver'd many Things about this Meaſure of the Time in the Scholia.

As for the Experiments, which relate to the Meaſures of the Forces, I ſhou'd obſerve that the noble and learned *Jo. Poleni* firſt immediately demonſtrated by Experiments, that Forces are ſpent in producing equal Effects when the Maſſes are to one another inverſely as the Squares of the Velocities *.

In N. 834. I have given this very Experiment of *Poleni*, making uſe of a Machine, that it may be more accurately perform'd.

In the ſecond Part of Book II. I treat of the Percuſſion of Bodies; about which Philoſophers have for a long time made many Miſtakes. At length in the Year 1669, *Wallis* deliver'd the true Laws, which take place in ſoft Bodies, tho' he applies them to thoſe that are perfectly hard; about the ſame time *Huygens* and *Wren* did the ſame, with regard to elaſtic Bodies †, tho' they do not ſpeak of Elaſticity.

Of theſe two laſt we have this Account in the Tranſactions of the *Engliſh* Royal Society. Without doubt, neither of theſe learnt any thing of the other about that Theory, before they compar'd together their Writings; but it was the beautiful Production of the Sagacity of each.

Huygens did indeed ſolve, ſome Years before, thoſe Caſes of Motion which were then propos'd to him, when he was at *London*; and plainly ſhew'd that he had then already diſcover'd Rules that enabled him plainly to do it. But he will not affirm that he diſcover'd any thing of his Theory to any of the *Engliſh* ‖.

Concerning this Theory I demonſtrate in Chap. V. Book II. that it is the ſame with *Wallis*'s, which is to be found in Chap. IV. except that it differs in this: *viz.* That when the Bodies are elaſtick, the change of Velocity from the Stroke is double of that which takes place, when they are not elaſtick, and by this means I have reduc'd this Theory to a great ſimplicity. But when I treat of demonſtrating *Wallis*'s Rules, I follow my own Theory.

As for the Experiments, many were made about Colliſion with pendulous Bodies, before the true Rules were known, but theſe were of no uſe; they had only relation to ſome Things in general, and are of no conſequence.

Wren

* De Caſtellis §. 118.
† Philoſ. Tranſactions, N. 43. and 46.
‖ Ibid. N. 46. pag. 927.

Wren and *Roofcius* firft gave more accurate Demonftrations, for they confirm'd the Rules given by fuch like Experiments before the Royal Society *. Afterwards *Mariotte* gave an entire Treatife about Collifion, with an accurate explanation of the Experiments.

I likewife make ufe of fufpended Bodies; but that Machine which I gave in the firft Edition, fufficiently perfect, agreeing very much with *Marriotte*'s, which was improv'd in the fecond Edition, is made ufe of here, being yet made more perfect; and I illuftrate Collifion itfelf by many new Experiments.

After that the true Rules of direct Collifion were difcover'd by the famous Men juft mention'd, there remain'd no Difficulty in explaining the oblique Collifion of two Bodies; which is perform'd by many learned Men, by applying *Kepler*'s Refolution of Motion to thefe Cafes; *viz.* by confidering feparately the Change of the direct Motion, and the lateral Motion remaining †.

Concerning the compound Collifion, whether direct, or oblique, of three Bodies meeting each other, a few Things, and fuch as belong to fimple Cafes, are to be found in Authors; yet did I not think that this Subject was to be entirely neglected, upon which Account I have added many Things in this Edition.

The laft Part of Book II. treats of the Laws of Elafticity. What I fay of the Motion of Fibres, is taken from *Merfennus*. *Huygens* difcover'd that the Vibrations of an elaftick Plate were made in equal Times. The firft Experiment, Chap. XIV. is alfo not new, but I have added Demonftrations to all thefe, and illuftrated the true Law of Elafticity by Experiments.

Book III. treats of Fluids, and Part I. of the Preffure of Fluids. Many Things of thofe which I explain are taken out of *Archimedes* ‖; almoft all the reft is explain'd by many Authors; among the chief I reckon *Simon Stevinus* ‡, Monf. *Pafcal* **, and Mr. *Boyle* ††. In thefe Things I can challenge nothing to myfelf except the Me-

* Ibid. No. 46. & Newtonus Princip. Schol. Corr. 6. Legis. Motûs 3tiæ.

† Paralipom. in Vitellionem Cap. 1. prop. 19. "Cùm quid obliquè movetur verfus fuperficiem, motus is componitur ex perpendiculari & parallelo fuperficiei. At fuperficies tantum ei parti objicitur, quæ eft in fe perpendicularis, non ei quæ eft fibi parallelos. Quare nec impedit partem fibi parallelon, fed patitur mobile refiliendo pergere ad partem alteram, fic ut advenerat.

‖ De infidentibus Humido.

‡ De la Statique. liv. 4. & 5.

** De l'Equilibre des Liqueurs.

†† Paradoxa Hydroftatica.

thod of explaining them, and many Things that relate to the Experiments; in which neverthelefs I have the fame Authors, efpecially Mr. *Boyle*, who have gone before me.

That learned Man *Jo. Georg. Leutmannus*, in the Acts of the Academy at *Peterfburgh*, has fhewn a Method of determining with accuracy the fmall Differences of Weights by a Balance, whofe Brachia are unequal *. Upon account of this Contrivance I examin'd into the Affair, and found that it might yet be more accurately perform'd hydroftatically; this Method is to be found in Chap. IV. Book III.

In the fecond Part of this third Book the Motions of Fluids, and their Efflux out of Veffels, are confider'd. Many Things are explain'd from Sir *Ifaac Newton*. What I have borrow'd from *Marriotte* and *Poleni* likewife gives great Light to this matter. In the remaining Controverfies, which have been manag'd by learned Men, I propos'd what feem'd to me to be true, giving my Reafons why I did fo.

Experiment 2. Chap. IX. is given by *Marriotte* †, the fourth and fifth I have already given in the foregoing Edition of the Year 1725. But I ought to take notice here that an ingenious and learned Man has explain'd one of thefe, *viz.* the fifth, in the Memoirs of the Royal Academy of Sciences of *Paris* for the Year 1736 ||: this Circumftance being different, he makes ufe of Tubes, having their Sides of Glafs; which Method I recommend to thofe to follow, who may have an Intention to make the Experiment. In the reft it might eafily be prov'd that my Method is to be preferr'd.

Chap. X. treats of the Motion of Rivers. What is to be found about determining the Velocity of the Water, in my Book, is taken from *Gulielmini* ‡. From which Author I have alfo taken fome other Things **.

What I demonftrate about the Motion of a Pendulum compar'd with the Motion of a Wave, in the following Chapter about Waves, is Sir *Ifaac Newton*'s ††.

In the third Part of Book III. the Actions of Fluids in motion are confider'd, and three Things examin'd: the Force of fpouting Fluids; the lateral Preffure of Fluids moving thro' Tubes; and, laftly,

* Tom. III. pag. 138.
† Mouv. des Eaux, 3 part. 2 difcours, fur la fin.
|| Memoirs pag. 191.
‡ Menfura Aquarum fluentium.
** De Fluminum Natura.
†† Princip. Lib. 2. prop. 44.

laſtly, the Reſiſtance, which muſt be overcome, that a Fluid, by help of a Machine, may be rais'd to any high Place.

As for the firſt, I don't deny that the Experiment, which I have given concerning the Force of Fluids, does not agree with the Experiments of learned Men; but I don't doubt but that the Event will always be ſuch, as I mention'd, if all the Cautions which I have deliver'd, be obſerv'd.

Concerning the lateral Preſſure in Tubes I ought to take notice, that ſome time after I had made the Machine, uſed in the Experiments of Chap. XIII. I ſaw one like it, which perhaps was made before mine, deſcrib'd in the Tranſactions of the Academy of *Peterſburgh*, by that celebrated Man, and Mathematician of the firſt Rank, *Daniel Bernoulli* *, who has given the ſame again in his elaborate Work, to which he has given the Name of *Hydrodynamice*.

As to the *Hydraulic* Machines, I demonſtrate in the following Chapter many Things hitherto neglected, which may be of great uſe; I conſider the thing only in a general way, an Application to peculiar Machines being not to my purpoſe.

In the laſt Part of Book III. I treat of Bodies mov'd in Fluids; and I have added nothing in this Edition, but have illuſtrated ſome Things more. Many of the Propoſitions demonſtrated in the Scholia of Chap XVI. are Sir *Iſaac*'s; but this is the chief Difference in the Demonſtrations. Where that great Man makes uſe of the Quadrature of the Hyperbola, I have ſubſtituted the logiſtic Line, which it is known may be eaſily done, by which nevertheleſs the Demonſtrations become leſs abſtract; but Sir *Iſaac* was unwilling to depart from his general Method of determining the Magnitudes by the Quadratures of Curves, in this particular Caſe, which he has happily made uſe of upon many Occaſions.

Huygens was the firſt who ſhew'd the Properties of this logiſtic Line at the End of his Treatiſe of Gravity; and that famous Mathematician *Guido Grandi* has demonſtrated theſe Propoſitions of *Huygens* in a peculiar Treatiſe †.

My fourth Book treats of Air and Fire. In the firſt Part concerning Air, I conſider its Weight and Elaſticity, and explain the chief Phænomena depending upon theſe.

The Weight of the Air, which was known to the Antients, and prov'd by Experiments ||, was not uſed in explaining Phænomena.

c 2 *Galileo*

* Tom. IV. pag. 194.

† Vide Hugenii Opera reliqua Tom. I.

|| Ariſtoteles de cœlo lib. 4. cap. 4.

Galileo himſelf, who, taking *Ariſtotle* for his Guide, had weigh'd Air condens'd in a Bottle *, never ſuſpected that the Water in Pumps was ſuſtain'd by the Weight of Air, although he knew that the Water cou'd not be rais'd in ſucking Pumps beyond a certain Height †.

Torricelli was the firſt, who, when in the Year 1643 he had diſcover'd the like Effect with Mercury, and ſeen that it was not ſuſtain'd in a Tube, which had the Air taken out of it, beyond a certain Height, ſuſpected that this Effect was to be attributed to the Preſſure of the Air riſing from its Weight. That this Conjecture was true, Monſ. *Paſcal*, in the Year 1648, prov'd by a remarkable Experiment, made at his Deſire, by which it appear'd that the Height of the Mercury in the Torricellian Tube was leſs by three Inches at the Top of the Mountain *le Puits de Domme*, in the County of *Auvergne*, than at the Bottom of that Mountain; becauſe he demonſtrated that the Preſſure was diminiſh'd with the Quantity of Air preſſing downwards ||.

A little while after theſe Things were diſcover'd, many new Experiments were made about the Air, the chief of which were thoſe of *Ottho de Guericke*, *Robert Boyle*, and thoſe of the *Italian* Academy *del Cimento* ‡: which laſt, tho' very commendable, are leſs remarkable, at leaſt as to what relates to the Air; for they have extracted it only from ſmaller Balls with *Torricellian* Tubes inſerted in their upper Part. But *De Guericke* and *Boyle* have made uſe of greater Veſſels, and by help of Pumps have taken out the Air; and that which is put to this uſe, is call'd an *Air-Pump* to this day.

Concerning the Inventor of this Pump *Boyle* informs us, that *Ottho de Guericke* was the firſt who extracted Air out of a Veſſel, but himſelf the firſt that had a Machine made fit for this purpoſe **. But *Papin* firſt join'd two Pumps together, which were carried with contrary Motions, that the deſir'd Effect might be obtain'd in a ſhorter time, and more commodiouſly; for the having two Pumps join'd together very much diminiſhes the Labour ††.

After *Boyle* many Pumps differently conſtructed were made uſe of, all of which have the ſame Foundation with *Boyle*'s and *Guericke*'s Method.

I have

* Dialog 1. Mechan. † Ibid.

|| Paſcal Recit de la grande Experience de l'Equilibre des liqueurs.

‡ Vir Clariſſ. P. van Muſſchenbroek anno 1731. Latino idiomate editionem dedit horum Exp. cum commentario pulcherrimo & multis experimentis novis.

** Prooem. Experim. Phyſico-Mechan.

†† Boylei Exp. Phyſ. Mech. continuatio 2da in præfatione.

I have used different Methods propos'd by others, and endeavour'd to amend what Inconveniency I found. But after I had try'd different Methods, and improv'd them, I came at length to that Construction of a double Pump, explain'd in Chap. IV. of Book IV. which Construction I have afterwards apply'd to a smaller single Pump, which is also mention'd in the same Chapter. I think the double Pump is preferable to all others that I know; yet I do not contend that this can't be made more perfect: but I ought to inform those who shall attempt this, that it often happens, that, when we make any Alteration, to avoid one Inconveniency, there arises another and greater, which is only discover'd by use. Which also may relate to other Machines; an unexperienc'd Person may easily make a Machine worse, by endeavouring to mend it.

The Foundation of the Construction of any kind of Pump whatever, by which the Air is exhausted, is the Elasticity of the Air; wherefore de *Guericke* cou'd scarce be ignorant of this Property. *Galileo* who, as may be seen above, compress'd the Air in a Bottle, when he weigh'd it, had his Eye upon the Effect of the Elasticity. *Aristotle* himself, when he found blown Bladders to be heavier than when they were empty *, made use of compress'd Air, otherwise the Weight wou'd not have been encreas'd. Nevertheless no one before *Boyle* treated of this very thing; this great Man was the first, who distinctly explain'd, and illustrated by Experiments this Property †.

In Chap. V. and VI. I explain many Experiments now commonly known, concerning the Weight and Elasticity of the Air, which yet cannot be said of them all, *viz.* the seventh and eighth of Chap. V. and some others. In all I shew how we shou'd proceed in them, and in many I do not depart from the commonly receiv'd Method. In treating of these I have given only two new Machines, one for the Compression of Air, and the other, by help of which the Experiment of Bodies falling *in vacuo* may be frequently repeated.

As I have occasion to speak of the Air, I observe some Things in general of other elastick Fluids. Concerning these Fluids many Things are to be found in *Boyle*, who gives them the Name of fictitious Air ||; and concerning these, the very diligent Dr. *Stephen Hales* has deliver'd many Things well worth regard, who refers all these Fluids to Air ‡.

Sound

* Loco supra indicato.
† Exp. Phys. Mechan. & alibi.
|| Exp. Phys. Mech. contin. 2da.
‡ Vegetable Staticks. Vol. 1. Chap. 6.

Sound is propagated by Air; I treat of this alſo in this Part of Book IV. explaining the *Newtonian* Theory; and the Demonſtration is Sir *Iſaac*'s, which I give of the Agitation of the Particles of Air in Scholium 1. Chap. VII. but I have propos'd it in a leſs abſtracted manner. Concerning this, great Men have obſerv'd that there is a Fault in this Demonſtration of Sir *Iſaac Newton*'s; but the learned *Gabr. Cramer*, Profeſſor of Philoſophy and Mathematics at *Geneva*, has demonſtrated the Propoſition to be true *.

Of the Velocity of Sound many have made Experiments, but the chief are thoſe which are made by Dr. *William Derham*, celebrated for many Works †.

In Part II. of Book IV. I treat of Fire. From known Obſervations and Experiments, I have deduc'd ſome Properties of Fire, which have furniſh'd me with Explanations of many Phænomena.

In Phyſics when we cannot reaſon from the ſimple Laws of Nature, no other way lies open, and what we are ignorant of, is to be left untouch'd. I have acted thus not only in many Things, which relate to Fire; but have omitted many other Things, which are explain'd by Writers upon Phyſics; becauſe I was unwilling to admit Hypotheſes.

In Chap. IX. I have made mention of lucid Stones: What I have deliver'd is to be found in the Tranſactions of the Academy of Sciences of *France* ‖.

Where I treat in Chap. X. of the Expanſion of Bodies by Heat, I demonſtrate by a ſingle Experiment only, that this obtains in all the Parts. But this Property of Bodies has been explor'd with care by others, chiefly by the illuſtrious *Herman Boerhaave*, lately the Ornament of our Academy ‡. But the firſt who brought this Dilatation to an accurate Meaſure is the famous *Pet. Van Muſſchenbroek*, my friendly Collegue, who makes happy Diſcoveries in experimental Philoſophy daily **. *J. Ellicot* has given a Machine, different from *Muſſchenbroek*'s, which is a very ingenious one, and deſcrib'd in the Tranſactions of the Royal Society at *London* ††.

This is made for this End, *viz.* that the Dilatations of different Bodies, when the ſame Degree of Heat is given, may be compar'd one with another. But although I often attempted this with great Care,

* Newt. Princ. editio Genev. Tom. II. pag. 364.
† Philoſ. Tranſact. N°. 313. pag. 2.
‖ Memoir. de l'an. 1730. p. 524. & 1735. p. 347.
‡ Vide Elementa Chemiæ. Tom. I. p. 138.
** Tentamina Exper. Acad. del Cim. part. 2. pag. 12.
†† Philoſ. Tranſactions. N°. 443.

Care, making uſe of ſuch a Machine, and another very accurate one ſent me from *England*, yet I cou'd never attain to it; but I learn'd this from my Experiments, that it was vain to attempt ſuch a Compariſon, unleſs the Bodies upon which the Experiments were made, were immers'd in a Fluid, that the Heat might be communicated to them from a circum-ambient Fluid.

Speaking of Fire, I have occaſionally deliver'd a few Things about Electricity, that it might appear, that there is a Connexion between ſome Phænomena of Fire and Electricity. Theſe Experiments are from *Haukſbee*; from whence I have alſo taken the Experiments, which I have given of Attrition *in vacuo:* but the Machine, which I make uſe of in theſe Experiments, is different from that, which that indefatigable Inveſtigator of new Experiments made uſe of. There are a great many Things belonging to Electricity; he who wou'd have a more diſtinct Knowledge of what relates to this Subject, may conſult the Places which are ſet down at the Bottom of this Page *. I have many Experiments of lucid Mercury in Chap. XI. *Picard* firſt diſcover'd this Property in the Barometer †. Afterwards *John Bernoulli*, commended above, demonſtrated this Property alſo in greater Veſſels from which the Air was exhauſted, who likewiſe diſcover'd that Mercury ſhines when the Air is preſent ‖.

The Vth Book treats of Light. In the ſecond Edition the Guides that I follow'd were Dr. *Barrow* in his Optical Prelections, *Huygens* in Dioptrics, Sir *Iſaac Newton* in Optics, of whom I have now ſome Optical Prelections which were not publiſh'd at that time. Before this Vth Book was ſent to the Preſs, there came to my hands an ingenious Work concerning Optics of Dr. *Robert Smith*'s, *Cambridge* Profeſſor. The Deſign of this learned Man was to give an entire Treatiſe of Optics; mine to give the Elements only: Therefore it is no wonder if many Things, that are to be found in my Book, are likewiſe conſider'd by this Author. But the Method which I take, and the Demonſtrations which I uſe, do ſufficiently differ from the beautiful Demonſtrations of this Author, ſo that I can't be thought to have ſtole them. Beſides, this learned Man treats of ſuch Things, as don't belong to our Deſign. But I have borrow'd from him what I have ſaid about the

* Philoſ. Tranſactions N. 417. 423. 426. 431. Mem. de l'Ac. des Sci, années 1733. pag. 23. 73. 233 457. — 1734. pag. 341. 503. — 1737. pag. 86. 307.
† Acad. des Sci. Tom. X. an. 1675. pag 566.
‖ Acad. des Sciences. M. 1701. pag. 1. & 14.

the Reaſon of an Object's appearing ſingle that is ſeen by both Eyes. It immediately appears that this Cauſe agrees entirely with the Principles from which I have drawn Concluſions in many Places. I have explain'd theſe Principles more largely alſo in my Treatiſe of Logick, *viz.* that the Senſes teach nothing of themſelves, but that we owe all the Uſe of them to Experience.

I had explain'd in the firſt Edition an Apparatus of Machines for making Experiments of Light; the ſame improv'd and enlarg'd is given here, that more Experiments may be made, and all of them with greater Eaſe and Accuracy, and that new ones alſo might be made.

The firſt of all theſe Machines is of uſe to keep the Sun's Rays in the ſame Line; *Farenheit* firſt made uſe of ſuch a one, who by moving a Glaſs, with a Handle, eaſily reduc'd the Ray to its firſt Poſition; he might have made uſe of a Clock to direct this Glaſs. The Foundation of the Conſtruction of this Machine was plain and ſimple; he made uſe of two Glaſſes; the firſt, which was continually mov'd by the Handle, reflected the Sun's Rays along the Axis of the Earth, and by the ſecond he directed them as he pleas'd. But the Rays are too much weaken'd by this double Reflection.

I make uſe of one Glaſs only, which I direct by a Clock; but if any one wou'd do this continually with his Hand, and beſides, after a Quarter or half an Hour, alter the Situation of the Machine a little, the Conſtruction wou'd be very ſimple.

In the ſixth or laſt Book I have added a few Things. In the firſt Part, I explain what relates to the Motions of heavenly Bodies and their Appearances; in the ſecond, I deliver the phyſical Cauſes of thoſe Motions, according to the Opinion of Sir *Iſaac Newton*. What is to be found in my Book about the Figure of the Earth, I have deduc'd from later Meaſures, as is ſhewn in its Place.

The BOOKSELLER to the READER.

*WHAT was printed in a * ſmaller Character at the end of this Preface, the famous Author had a Deſign to alter; but about that time he died. It would be a kind of Crime to add or to take from the Writings of ſo great a Man; therefore I deliver this latter Part as it was found in his Papers. What Emendations and Additions*

* The latter Part of the Preface from the Words [The 5th Book treats of Light, &c.] is, in the Latin, printed in a ſmaller Character than the reſt of the Preface.

ditions he had intended to make, may be ſeen from the two following Fragments, which he left imperfect, and which I have here annex'd, that no Part of the Writings of a great Philoſopher might be loſt.

OUR fifth Book treats of the Phænomena of Light; and therein a Science is explain'd, which is entirely owing to latter Philoſophers. For if we take a View of more ancient Writings, it immediately appears, that the true Cauſes of the Phænomena of Light were not known to thoſe Authors, and that they have deliver'd many things to us quite uſeleſs. Every one may ſee Proofs of this Aſſertion, who looks over the Treatiſes written upon Opticks by *Euclid*, *Heliodorus*, *Alhazenus* the *Arabian*, *Vitellio*; to which we may add what that famous Monk of the thirteenth Century, *Roger Bacon*, has left upon Opticks; and likewiſe *Jo. Bapt. Porta*, whoſe Book concerning Refraction was publiſh'd in the Year 1598.

This fifth Book contains four Parts, the firſt of which treats of the Motion of Light, and its Inflection or Bending. What belongs to the direct Motion and Velocity is explain'd, and the Authors mention'd, to whom we owe what relates to this Subject. The Inflection, or Bending of Light, which is treated of afterwards, was taken from the Obſervations of *Grimaldi* *, but chiefly from what Sir *Iſaac Newton* has diſcover'd about it †.

In the Part concerning Refraction I examine its Laws, and treat of Glaſs Lenſes and their Uſe, as of Microſcopes and Teleſcopes alſo.

Snellius firſt diſcover'd the true Law of Refraction, as *Huygens* tells us ‖. Glaſs Lenſes were diſcover'd about the end of the thirteenth Century; about which Invention *Will. Molyneux* may be ſeen ‡, the Father of him who is mention'd in Chap. I. of this V. Book . . . . . . But, in my opinion, it is not certain who was the Inventor of Lenſes; and I am perſuaded, againſt the Opinion of the Author juſt ſpoken of, that the above-mention'd *Roger Bacon* is not to be look'd upon as the Perſon, except with a Limitation. This learned Man mentions ſome Places of *Bacon*'s to confirm his Opinion, which, taken ſeparately, ſeem to evince it; but theſe Places ſo taken have a different Senſe, which he will give them, who ſhall examine the Words in Connexion with them, and at the ſame time conſider with

 Attention

* Acad. des Sciences. H. An. 1715. pag. 52.
† Optic. Lib. III.
‖ Hugenii Opera reliqua. Vol. II. pag. 2.
‡ A Treatiſe of Dioptricks. Pag. 257.

Attention other Places. When *Bacon* ſpeaks of Objects ſeen *thro' a Medium* *, he does not underſtand Objects placed out of the Medium, but within it, the Eye being placed in another Medium. He likewiſe ſuppos'd the Sun, Moon, and Stars to be in the Vapours which ariſe from the Earth, but the Spectator placed in a rarer Medium without the Vapours, and thus to contemplate theſe Bodies †.

Nevertheleſs, what *Bacon* mentions of Glaſſes, might naturally lead to the Conſtruction of Lenſes. According to his Principles (of the Truth or Falſity of which I ſhall ſay nothing here) he affirm'd, that an Object, in a Glaſs terminated by a convex Surface, when the Eye was placed in the Air, appeared enlarged near the Centre of the Sphere; from whence he inferred, that Letters would appear enlarged, by a ſmaller Segment of a Glaſs Sphere apply'd to them.

As for the ſingle Microſcopes, of which I have treated, I ſhall ſay nothing of them, they are only convex Lenſes. *Drebbelius* is the Inventor of compound Microſcopes ‖.

Jo. Bapt. Porta *is the firſt who made mention of Teleſcopes, affirming diſtant Objects to appear* . . . . .

The Author had an Intention to enlarge the Place which I have mark'd with Points in the former Fragment, by examining Bacon's *Words, as appears from the ſecond Fragment.*

But, as it appears to me, the Inventor of Lenſes is unknown, and that the above-mention'd *Roger Bacon* is not to be look'd on as ſuch; if we ſpeak ſtrictly, I am perſuaded againſt the Opinion of the Author ſpoken of; but I think there is no doubt but that he open'd the Way to this Invention.

This will appear, by quoting ſome Places out of *Bacon*: *But if they are not plain Bodies, thro' which the Sight ſees, but ſpherical, &c.* ‡. He does not here treat of Lenſes, nor of Objects ſeen beyond any Medium. The Author examines the Phænomena of convex and concave Mediums, as he examin'd the plain ones, and conſiders Objects immers'd in one Medium, while the Eye is placed without this Medium in another; and he no where conſiders double Refraction, which takes place when the Object is placed without the Medium.

Yet

* De Perſpectiva. Pars III. Diſtinct. 2. cap. 3.
† Ibid. cap. 4.
‖ . . . . .
‡ De Perſpectiva. Pars III. Diſtinct. 2. cap. 3.

Yet *Bacon* has hit the Nail on the Head; and, if he had join'd Experiments with Theory, he had discover'd Glass Lenses, and avoided many Errors in Refraction which he considers.

For when he had affirm'd that an Object appears enlarged, *if the Eye is in a more subtile Medium, and the Convexity of the Medium, where the Object is, be towards the Eye* *: He afterwards applies this to Glass; and, that he might consider an Object as if it was in a Glass, he supposes the Glass as having its opposite Side plain, and immediately applied to the Object. And tho' this Author speaks of a plano-convex Lens, he had no notion of a Glass, beyond which Objects would appear enlarged after a double Refraction. The Place is remarkable: *But if a Man looks at Letters, and other small Things, thro' a Medium of Crystal or Glass, or any other diaphanous Body, the Letters being placed beneath, and there be a smaller Portion of a Sphere, whose Convexity is towards the Eye, and the Eye in the Air, he will see the Letters much better, and they will appear larger to him*, &c. He immediately adds; *therefore this Instrument is useful for old Men, and those that have weak Eyes.*

* De Perspectiva, Pars III. Distinct. 2. cap. 3.

AN *ORATION CONCERNING EVIDENCE.

NO one, I will not ſay who is ſkill'd in mathematical Studies, but only a Beginner, and firſt entring upon them, can avoid perceiving, that theſe Sciences lay claim to a particular Way of proving any Truth; and that mathematical Demonſtrations are accompany'd with ſuch a kind of Evidence, as overcomes an Obſtinacy inſuperable by any other means.

Hence ſo many learned Men have labour'd to illuſtrate other Sciences with this ſort of Evidence: and I don't ſcruple to affirm, that the Study of Mathematicks has given Light to Sciences, very remote from them.

But what do Men not abuſe! This very Advantage in the Mathematicks in the diſcovering of Truth, has given occaſion to ſome to reject Truth itſelf, though ſupported by the moſt firm and evident Arguments.

While they contend that nothing is to be taken for Truth but what is prov'd by mathematical Demonſtration, in many things they take away all Criterium of Truth, while they boaſt that they defend the only Criterium of Truth.

But every one will eaſily perceive how inconſiſtent ſuch are with themſelves, if he ſhall aſk them, whether or no thoſe who live on the Earth don't ſtand in need of one another's Aſſiſtance, whether the Sun, which they obſerve riſing, will not ſet after a certain Time; whether they ever made any doubt of the *Romans* being formerly the moſt powerful People, and of *Rome*'s being the Metropolis of their Empire. Yet theſe things can't be prov'd by mathematical Demonſtrations.

Therefore

* Spoken at *Leyden* the eighth of *February*, in the Year 1724, when the Author went out of his Rector's Office.

Therefore there is an Evidence different from mathematical Evidence, to which we can't well deny our Aſſent, and which no one will deny in thoſe things in which nothing but the Love of Truth is the Occaſion of Doubt.

Latter Philoſophers have given the Name of moral Evidence to that which is different from mathematical, and call that Perſuaſion which follows moral Evidence, moral Certainty; making uſe of a Name unknown to the Ancients.

This alſo is kept within its own true Bounds by few, whilſt many contend, that Propoſitions are ſupported by moral Evidence which have ſcarce the leaſt Probability.

I thought, when I was to ſpeak before ſuch a famous Company of noble and learned Men, that I ſhould do nothing foreign from my Office, nor unacceptable to you, if I ſpoke at this time of both kinds of Evidence, mathematical and moral, and the Perſuaſion ariſing from thence.

I ſhall endeavour to expreſs my Sentiments, and deliver plainly what you may underſtand; this being all that *Cicero* requir'd of a Philoſopher, who was himſelf the moſt eloquent of Philoſophers, not much demanding Elcquence from a Philoſopher.

Being about to deliver the Nature of mathematical Evidence, and to tell why no one in his Senſes can deny his Aſſent to it, I ſhall conſider the Nature of our Mind.

Our Mind contains Ideas, and compares them one with another; in this all Knowledge is placed.

Our Mind perceives Ideas, is conſcious of this Perception; and no one can doubt, whether it truly perceives the Idea, which it perceives.

I don't here ſpeak of the Agreement between theſe Ideas and external Things, I only ſpeak of that Repreſentation which is preſent to the Mind. While the Idea of a Building is preſent to my Mind, I may make it a queſtion, whether there is really any particular Building without me anſwering to this Idea, or can be; but I ſhall not doubt of this, *viz.* whether I think of a Building which is repreſented to my Mind: for the Mind is conſcious of its own Perception.

The Mind is not always ſenſible of one only Idea at a time; but the greater its Capacity is, the more Ideas it takes in at a time.

But when many Ideas are together preſent to the Mind, the Mind neceſſarily perceives the Compariſon of them, and forms an Idea of this Compariſon to itſelf.

To

To deny, when two Ideas are preſent to me, that I perceive whether or no they differ from each other, and in what reſpect; would be denying that theſe Ideas were preſent to me, which really are preſent: I ſpeak only of the Ideas, of that which is preſent to the Mind.

Theſe things therefore are inconſiſtent, that the Mind ſhould perceive Ideas, without perceiving the true Compariſon between theſe Ideas; and upon that very account it will be conſcious of this Perception, and be perſuaded that there remains no doubt concerning this Compariſon, *i. e.* it will aſſent to this Propoſition, That there is really ſuch a Compariſon between theſe Ideas which it perceives. If I compare the Idea of the Number Seven with the Idea of the Sum of theſe Numbers Four and Three, I immediately perceive that there is little difference between them; and there can be no doubt, whether Three and Four, taken together, make Seven.

Behold, I have given you the Foundation of mathematical Evidence; you ſee why this does naturally force our Aſſent.

You have that, by which you may ſolve the Difficulties, in which Scepticks endeavour to involve the Truth. Let them ſay that Truth ſtands in need of a Criterium, that a true Criterium is to be diſtinguiſh'd from a falſe one, and that a new Criterium is wanting for this Purpoſe; which new Criterium will likewiſe want its Criterium, and ſo on *in infinitum*: therefore, that it is a Contradiction to ſuppoſe any Criterium.

I anſwer, that the Evidence itſelf is the deſir'd Criterium of Truth; *viz.* the very Perception of the Compariſon between two Ideas. That the Criterium of Evidence is Conſciouſneſs; but that this brings its own Criterium along with it: for I do not want a Criterium to make me certain, that I am conſcious of the Idea that is preſent to my Mind. Can I avoid perceiving the Idea which I do perceive? While I am conſcious, am I therefore not conſcious that I am conſcious? To demand another Criterium of Conſciouſneſs implies a Contradiction: but I have juſt prov'd, that this is the only Foundation of mathematical Evidence.

Others alledge, that we labour in vain when we endeavour to diſcover the Foundations of Evidence, ſince nothing can be known. For in order to know any Thing, we ought to underſtand thoroughly how it differs from another Thing, but we can't diſcover this Difference, unleſs we were well acquainted with Things: therefore Things themſelves, and their Differences will for ever be conceal'd from us.

I think

I think it proper to illuſtrate the Argument by an Example.

I can't know a Triangle unleſs it appears to me in what it differs from a Square, which will be unknown to me, as long as I am unacquainted with a Triangle and Square.

But who does not ſee that Things are here ſeparated, which can't be ſeparated? and who can make any doubt of my knowing what a Triangle is, and in what it differs from every thing elſe, at one View of the Mind?

But let us now leave Scepticks.

The Foundations of mathematical Evidence being laid down, we ſhall eaſily ſhew why Mathematicks lay claim to that invaluable Privilege of not erring: whoſe juſt Title that it may be made appear, I ſhall mention in a few words the Object and Method of Mathematicks.

Firſt, Mathematicks are employ'd about Ideas, and thoſe only; and a Mathematician, conſider'd as ſuch, is not at all concern'd whether or no the Ideas, about which he reaſons, agree with any thing being. When he proves, for example, that the Square of the Hypoteneuſe of a right-lin'd and right-angled Triangle is equal to the Squares of the remaining Sides taken together, he is not ſollicitous about the Triangle itſelf; neither does he care whether or no the Squares are made, he only attends to the Ideas of the Squares, and pronounces that the thing would be ſo, if they were made. A Mathematician reaſons always from this Suppoſition, if it is given.

By this Hypotheſis he takes care not to fall into Error; the natural Philoſopher acts contrarily, ſuppoſing the Thing to be true that he feigns, and ſeldom avoids Miſtakes.

In thoſe Parts of Mathematicks alſo in which Things themſelves are treated of, the ſame Hypotheſis is the Foundation of the Demonſtrations, *viz.* if Things are ſo. Theſe we call mix'd Mathematicks, to diſtinguiſh them from pure, *i. e.* ideal Mathematicks.

Whilſt an Aſtronomer views the Stars, and meaſures their Courſes, he does not act as a Mathematician. A Mathematician deduces his Concluſions from previous Obſervations, and conſiders the Ideas of Obſervations only, and affirms nothing of the heavenly Motions except upon Suppoſition, if the Obſervations are without Error.

Aſtronomers alſo often from Obſervations feign an Hypotheſis about Motion, in which Caſe the mathematical Concluſions, tho' true in themſelves, can't be apply'd to the Things themſelves, unleſs there is no Error in the Hypotheſis about Motion, and the Obſervations upon which it depends, or a Compenſation for that Error.

But this does not concern a Mathematician as ſuch; he attends only to Ideas, and that Evidence which we call mathematical, and which I have prov'd to bring its Conviction with it, takes place in Mathematicks, from whence it had its Name.

2dly, The Object of Mathematicks is Quantity. Thoſe who follow this Study, conſider this firſt in general, and examine particular Quantities in peculiar Parts of Mathematicks: A Geometrician meaſures Extenſion: He who treats of Mechanicks, compares Forces. Motion is conſider'd in ſeveral Parts of Mathematicks.

Quantities of the ſame kind only can be compar'd; and their Ideas, if ſimple, are very diſtinct, and may be compar'd without Danger of Miſtake. If more complex, they may be reſolved the moſt eaſily of all others into particular Ideas, that the Parts may be compar'd.

Laſtly, in comparing complex Ideas, Mathematicians make uſe of a Method, by which Error may be eaſily avoided. From what is more ſimple, they proceed to what is more compound; which, as I juſt obſerv'd, they compare by Parts: which if it can't be done, they call to their aſſiſtance intermediate Ideas, that there may be no Compariſon, except of Ideas whoſe Agreement or Diſagreement appears at firſt view.

Ideas alone are not conſider'd in mathematical Sciences only; in others alſo mathematical Evidence claims a Place: And in thoſe likewiſe where Things are treated of, the Reaſonings relate to the Ideas only; and mathematical Evidence will have a Place hypothetically, if the Ideas agree with the Things, as I have taken notice with regard to mix'd Mathematicks.

But the Method, which Mathematicians uſe, may be applied to all Sciences; and Mathematicks have nothing peculiar to them, except their Object, *viz.* Quantity; ſo that we may indeed avoid Error more eaſily in Mathematicks, and yet eſcape it in other Sciences by the ſame Art, if we compare Ideas in the ſame manner as Mathematicians do; which, nevertheleſs how difficult it is in many Occaſions, I will preſently ſhew.

Logick treats of the Method of Reaſoning, *viz.* of ſuch Rules by which Ideas ought to be compar'd one with another; *i. e.* the Ideas of the Compariſons of other Ideas are the Object of Logick; and this Science relates to Ideas entirely, and differs from Mathematicks in its Object only: which does not much alter the Nature of the Evidence, by which Logick may be illuſtrated. The Rules

alſo,

alſo, (to mention this one thing only) which are commonly deliver'd about Syllogiſms, yield to no mathematical Theorem in Certainty.

Ontology, a Science that conſiders the common Properties of all Things, is entirely converſant about Ideas. This general Idea *To be* is the Object of this Science; and altho this ſeems a ſimple Idea, yet this Science is not contain'd within ſo narrow Bounds, and it gives great Light to other Siences, if it is freed from thoſe Obſcurities in which Philoſophers have involv'd it.

In this no particular thing is conſider'd; but all kinds of Things are referr'd to Claſſes, that we may be able to determine their general Differences.

We likewiſe conſider in *Ontology* the more general Compariſons of thoſe Things, which either are or may be; and herein is the greateſt Advantage of this Science plac'd: for inſtance, he who ſhall examine what is demonſtrated concerning Cauſe and Effect, will eaſily ſee its Uſe in ſolving the moſt difficult Queſtions.

Pneumatology is converſant about the Properties of all *Intelligencies.* As all Knowledge depends upon Thoughts, our *Mind* has inherent in it the Idea of *itſelf* before all others; and we acquire Notions of the Properties of our Underſtanding without any outward Aſſiſtance. But as this Science treats of theſe alone, what is demonſtrated in it, is ſupported by mathematical Evidence, and is mathematically true.

If we turn ourſelves to that Part of Pneumatology which treats of GOD, we ſhall ſee that this likewiſe is wholly converſant about Ideas, and deduc'd from ſuch Notions, of which the Mind can in no wiſe doubt; which follows from the Nature of them: and that therefore even thoſe Things relating to the ſupreme and infinite Intelligence, concerning which People have diſputed, depend upon mathematical Evidence.

There exiſts ſomething now; therefore ſomething has exiſted from Eternity.

I think; *i. e.* there is ſomething Intelligent; from thence I infer that the firſt Cauſe of this is eternal, and infinitely exceeds in Intelligence that Intelligence which it has created; upon which account I am oblig'd to attribute a Power to it by which a Mind may be form'd, *i. e.* infinitely exceeding all that I can frame any Idea of to myſelf.

This appears at the firſt View; if I conſider the thing with Attention, I eaſily perceive, that there is an Intelligence without beginning, whoſe Being can be attributed to no external Cauſe; that it muſt therefore be *Self-exiſtent*, and that nothing can put an End to its Perfection, and that there is only *one* ſuch.

It is plain then that GOD is *one, eternal*; *of infinite Knowledge*; and that his Power is confin'd within no Bounds. Which Things being demonſtrated, hence other Things flow that are diſcover'd of GOD. For inſtance, *infinite Goodneſs* is deriv'd from *infinite Wiſdom*. For it is eaſy to prove that all that is oppos'd to it proceeds from a Defect of Underſtanding, and can be only in a limited Intelligence.

I affirm that that very Argument by which we prove the Being and Wiſdom of GOD, deduc'd from an Examination into Things, is accompanied with mathematical Evidence.

I confeſs that it is not mathematically certain, that the Planets move, that the Sun communicates Life to Plants by its Heat, that the Bodies of Animals are made in a wonderful manner, theſe Things belong to moral Evidence: but as for what relates to Ideas, and thoſe only, there is ſomething without me, whatſoever it is, whereby an Idea is rais'd in my Mind of this vaſt Aſſemblage of Things, diſpos'd in the wiſeſt manner, and govern'd by Laws to be admir'd by every Intelligence.

I don't enquire now whence the Ideas ariſe, which I acquire upon an Examination of the Univerſe; I affirm nothing of their Origin, but I can never imagine that I myſelf am the Cauſe of them. Whence it follows that there is an Intelligence without me, that has rais'd them in me, in what manner I don't determine: Of the Wiſdom of which Intelligence, if I judge from theſe Ideas, I cannot doubt of its infinitely exceeding all that I can frame any Idea of.

It is to my purpoſe to touch upon theſe Things, but not to examine them fully.

To the Sciences which are founded upon mathematical Evidence, *i. e.* whoſe Stability and Firmneſs depends ſolely upon the Examination of Ideas, I refer alſo the firſt Foundations of Ethicks; *i. e.* all Things which in general relate to the Foundations of the Duties of Intelligences one to another, and chiefly to the ſupreme Intelligence from which they had their Origin, and from which they ought to look for all their Happineſs.

I am perſuaded that it will ſeem a great Paradox to you, that I attribute to ſo many other Parts of Philoſophy that Evidence and Stability, which have put Mathematicks out of the Reach of Error and Contention; whilſt the innumerable Diſſenſions of Philoſophers

losophers in Mathematicks plainly ſhew that there is Error in them, at leaſt in ſome meaſure.

I confeſs that Philoſophers have often err'd, nay, I will add yet more Strength to the Objection, and freely own, that there is nothing ſo ſtrange, nothing that can be conceiv'd ſo diſtant from right Reaſon, which may not be equal'd by the metaphyſical Dreams of ſome Philoſophers; but few Miſtakes have been made in what relates to pure Mathematicks, and theſe have been very eaſily ſet right by others: Nevertheleſs the ſame Evidence, and Method of reaſoning, take place in Metaphyſicks, and Mathematicks. But whence it comes to paſs that more Errors are made in Philoſophy than Mathematicks, is eaſily ſhewn. I don't ſpeak of ſuch, who have the Aſſurance to proclaim themſelves Judges of the moſt difficult Things, and exceeding their Capacity, while they are ignorant of the firſt Rules of reaſoning, and are ſcarce acquainted with the firſt Elements of a Science.

Neither will I ſpeak here of the Paſſions of the Mind, affecting the Mind much more in Metaphyſicks than Mathematicks.

It will be ſufficient if I ſhall ſhew that, when the Paſſions are ſet aſide, and there is a ſincere Deſire of finding out the Truth, a limited Intelligence cannot avoid Error, and ſhall demonſtrate that it is not avoided in other Sciences ſo eaſily as in Mathematicks.

I have ſufficiently explain'd how Mathematicians avoid Error, but I muſt add ſomething.

They make uſe of no Word, without explaining with exactneſs what they mean by it, and accurately enumerating the particular Ideas contain'd in a compound Idea. By the ſame Word they always expreſs the ſame thing. They comprehend in the plaineſt Words the Truths demonſtrated, that they may make uſe of them like Axioms.

Whilſt Quantity is treated of, theſe Cautions may be ſtrictly obſerv'd, and it is not indeed impoſſible, to apply the ſame Rules to the other Parts of Philoſophy mention'd; but it is ſcarce in the Power of the human Underſtanding to make uſe of them in all Things with due Care.

When we reaſon about the Acts and Properties of our Mind, we have indeed Ideas of them, but we are ignorant of many Things concerning the Nature of the Mind, and it is often very difficult, to draw ſuch a Concluſion only, as may not be overthrown by the ignorant.

In the Ideas alſo, which we examine, there are often contain'd many more ſimple Ideas, all of which we don't always conſider, whence the Concluſion is not indeed uncertain, in that reſpect in which it was drawn; but if we afterwards apply it to the ſame Idea, conſider'd in another reſpect, it will be always uncertain, and very often falſe. But how difficult is it to avoid this in thoſe Sciences in which we expreſs by the ſame Word not only the ſame compound Idea conſider'd in various reſpects, but frequently Ideas entirely diſagreeing?

Men wou'd not fall into Miſtakes ariſing from thence, if, while they make uſe of a Propoſition before demonſtrated, they wou'd keep preſent in their Mind the Demonſtration. Then they wou'd not apply that which was determin'd of one Idea to another. But what Man cou'd ever perceive, at one View, the Connexion of a Propoſition, deduc'd from its firſt Principle, by the Aſſiſtance of ſeveral intermediate Propoſitions, with its firſt Principle?

It follows then from the Imbecillity of the Mind, that it is more difficult for it to avoid Error in Ontology and Pneumatology, than in Mathematicks; but that with Attention, that Caſe in which there is no room for ſuſpecting Error, may be ſeparated from others, ſufficiently appears from what has been ſaid.

I am not ignorant that ſuch abſtracted Things are not ſuited to this kind of Diſcourſe, except they are illuſtrated by Examples; but I deſire to be excuſed if I don't explain theſe Things further, and if I don't diſpleaſe by Examples, many Philoſophers who are of my Opinion in theſe Things.

Having ſhewn the Sciences which have mathematical Evidence, I obſerve of all others, that they may be put on a Level with mix'd Mathematicks. In theſe the Ideas of things without us are treated of. Whether or no theſe Ideas agree with the Things themſelves, does not concern mathematical Evidence, to which Perceptions and Reaſonings only ſhou'd be referr'd, and theſe can be converſant only about the Ideas of Things.

Therefore, Divinity, Ethicks, Phyſicks, and Hiſtory have not mathematical Evidence. All Sciences in general, relating to the Knowledge of Nature, are to be referr'd to Phyſicks. And to Hiſtory belongs in general all Relation of Things tranſacted.

In all theſe the Foundations of the Sciences, and their Firmneſs, does not depend upon our Ideas.

In Divinity we muſt firſt determine, whether the ſupreme and infinite Intelligence has made any particular Declaration of his

Will

Will to Mankind, and where it is to be found; this can never be determin'd from a ſimple Compariſon of Ideas: But when it appears that there is ſuch a Declaration, the Concluſions, to be drawn from what GOD has declar'd, will relate to Ideas; and the Stability of the Reaſonings will be mathematical, *viz.* hypothetical, as it always will when the Ideas of Things are treated of, if GOD has reveal'd this; but whether or no he has reveal'd it, belongs to that other kind of Evidence which I ſaid was call'd Moral.

I have lately ſhewn, that the Foundations of Ethicks, as far as they relate to the Duties of Intelligences in general, belong to Ideas. With regard to Men, it is requiſite that it be made appear to us that Men living together in a Society, ſtand in need of one another's Aſſiſtance; *i. e.* it is neceſſary that we acquire an Idea of Society between Men, having ſuch Affections of Mind, as we really obſerve in them. If we refer to this Civil Society alſo, as it ſhou'd be referr'd when we ſpeak of the Duties of Men; it is neceſſary, that it be known, in each particular Society, which is to be conſider'd, where the Power is to be found whence the Laws ſhou'd proceed, and what Laws it has promulg'd.

I cannot collect from the Obſervation of Ideas alone, the Knowledge of all theſe Things, *i. e.* a Perſuaſion of the Agreement of the Ideas, which I have in my Mind of theſe Things, and the Things themſelves.

But this Agreement being laid down, the juſt Reaſonings from them will appear to be mathematically true. But whether or no the Ideas anſwer to the Things themſelves, belongs to moral Evidence.

In Phyſicks alſo I can only have moral Evidence of the Motions, whereby the Bodies that compoſe the Univerſe are acted, and the Laws by which they are govern'd.

I don't refer to moral Evidence, that Queſtion which is ſtated by ſome, Whether there are Bodies? Thoſe, who deny this, yet agree, that inſtead of particular Bodies there is ſomething external to us, whereby an Idea of any particular Body is excited, and that the ſame thing excites an Idea of the ſame Body, in the Minds of different Men; ſo that this very thing, whatſoever it is, is look'd upon by all as the Body itſelf; and acts upon us in the ſame manner as the Body itſelf wou'd act, and with reſpect to us, it matters nothing whether there is any true Body external to us, or that *ſomething elſe*, which never differs from a true Body in reſpect to us. Whence it appears, that if there has been any thing vain and uſeleſs deliver'd

by

by Philoſophers, as there often has, we may juſtly refer to this Claſs the Diſputes of thoſe, who deny the Exiſtence of Bodies.

When in Phyſicks we get a good Knowledge of Phænomena by the Help of moral Evidence, *i. e.* when it appears that we have ſuch Ideas of theſe Phænomena, as agree with the Things themſelves; the Reaſonings about theſe Ideas will be mathematically true, and the Concluſions may be applied to the Things themſelves.

In Hiſtory alſo, to which I likewiſe refer the common Actions of Men, we have only moral Evidence. We don't diſcover by a Compariſon of our Ideas alone, what was done by this or that Man. But when we have the Ideas of Things tranſacted, by comparing theſe one with another, we draw Concluſions ſupported by mathematical Evidence.

You ſee that moral Evidence, and the Perſuaſion thence ariſing, relates to the Agreement between the Ideas in our Mind, and the Things themſelves external to us; whilſt mathematical Evidence is converſant about the Agreement which is between the Compariſon of Ideas and the Idea of this Compariſon. When we perceive this, it implies a Contradiction, as I have ſhewn, to ſuppoſe Error in theſe. We err in thoſe Things, which belong to mathematical Evidence, when having compar'd any two Ideas, we apply the perceiv'd Compariſon to other Ideas.

When we are converſant about Things external to us, we do not acquire an Idea of a thing by the Perception of it; Things themſelves don't act upon our Minds, we can't conceive how they ſhou'd. Therefore we can't deduce the Foundations of moral Evidence from a ſimple Examination of the Mind, and of Things conſider'd by themſelves. We have Aſſiſtances external to the Things themſelves, by which we acquire Ideas of Things external to us.

Theſe Aſſiſtances are the Senſes, Teſtimony, and Analogy.

Moral Evidence has theſe three Foundations, while Mathematicks have only one, in the Perception of Ideas.

Mathematical Reaſonings depend upon ſuch an Evidence as brings its own Conviction along with it.

But moral Evidence is the Foundation of Perſuaſion, not of itſelf, but by the Will of GOD.

If we conſider the thing in itſelf, it does not imply a Contradiction, to ſuppoſe that the Senſes, Teſtimony, and Analogy, may lead us into Error, whatſoever Cautions we uſe; but it implies a Contradiction,

Contradiction, to ſuppoſe that GOD intended theſe to be the Foundations of Perſuaſion, and that they ſhou'd not lead us to Truth, when we make uſe of them with due Care.

But it will not be difficult to demonſtrate, by Arguments mathematically certain, that GOD intended the Senſes, Teſtimony, and Analogy, to be ſuch Foundations, and that his Intention in this was not vain.

This appears from ſuch Arguments, as we make uſe of to prove the Being of GOD, and his infinite Goodneſs.

From hence I infer, that he deſign'd Men ſhou'd make uſe of ſuch Benefits as he has given them; but I ſhall ſhew that Men can't make uſe of thoſe Things which are neceſſary to ſupport their Life upon this Earth, where GOD has plac'd them, unleſs we allow thoſe to be the Criteria of Truth which I have mention'd, whence it will appear that they are ſuch. The ſupreme Wiſdom had been inconſiſtent with itſelf, if when it had given Things themſelves, it had denied the Faculty of judging of them. Which nevertheleſs does not exclude the making uſe of due Cautions.

Who can doubt of Men's continually ſtanding in Need of ſuch Things, of which they can judge only by their Senſes? Yet GOD has granted Men the Uſe of theſe Things; therefore he deſign'd that they ſhou'd enjoy them, *i. e.* he intended That, without which they cou'd not enjoy theſe Things; therefore he intended that they ſhou'd judge of them, and make uſe of their Senſes, which we ſee were given to Men by divine Providence to this end.

No Man can live alone, he ſtands in need of the Aſſiſtance of others; but that all may aſſiſt one another, a Communication of Ideas is neceſſary; which very thing, the Beneficence of the Deity, which can never be ſufficiently prais'd, has granted to Men, by giving them Speech.

By help of this, ſince we can't obſerve by our Senſes all Things neceſſary for us, we make uſe of the Obſervations of others; whereby the Neceſſity of Teſtimony is eſtabliſh'd; from whence I infer, that GOD intended we ſhou'd give Credit to Teſtimonies, when regulated by due Cautions.

Innumerable Particulars, which we can't be without, and which were not denied us by GOD, can't be examin'd by us one by one, ſo that we may be aſſur'd of their Uſe, neither is the Teſtimony of others ſufficient in theſe, which can't be given of all Particulars, many of which, when made trial of, prove uſeleſs.

How

How miſerable wou'd Men be in compariſon of what they are now, who cou'd not apply Concluſions drawn from Obſervations to what they had not obſerv'd, who cou'd never judge of what was to come by what was paſt!

Who wou'd plow the Land, ſow Seed, gather Fruit, or take any Care for the future, if every Event was uncertain? Who does not ſee the Inconveniences of theſe Things?

Miſerable Men, who wou'd daily be in doubt, whether the next Food that they took might not prove Poiſon! who upon the ſetting of the Sun wou'd be afraid of an eternal Night; and when it was ſhining, expect that it ſhou'd be extinguiſh'd every Inſtant.

I need not mention any thing more; the Goodneſs of GOD has deliver'd us from theſe; he has given us Power of applying our Obſervations to Things not obſerv'd, whereby we diſtinguiſh what is neceſſary for Life from what is hurtful, and often determine about Things to come.

I have reaſon to hope, that I ſhall again gather the Seed which is hid in the Earth, after it has ſprung up again, and produc'd perhaps an hundred fold, except ſomething extraordinary ſhall happen.

When I ſee the Sun ſet, I am perſuaded, upon good Grounds, that it will be conceal'd from my Sight but a few Hours.

I am not afraid of a ſtrong Building's falling of itſelf.

From Analogy therefore we muſt reaſon in Phyſicks, and who will doubt of the Almighty Creator of the Univerſe's deſigning this, who whilſt he knows the Creator to be good, conſiders the Things created?

But as GOD intended this, he intended thoſe Things alſo which are neceſſary to communicate Force to ſuch Reaſonings; *i. e.* he confin'd the Univerſe to fix'd and unchangeable Laws. For theſe being given, Analogy ſtands upon a good Foundation, and being taken away, all Things in Phyſicks are uncertain, and the whole human Race wou'd ſoon be extinct.

You ſee how different the Foundations of Aſſent are according to their different Circumſtances. But although theſe Foundations differ, tho' mathematical Evidence does not agree with moral, yet a different Perſuaſion does not proceed from thence. I can no more deny my Aſſent to ſuch Things, as are drawn from the Foundations of moral Evidence which I have explain'd, when due Care is taken, than to thoſe which are prov'd by a mathematical Demonſtration. I ſhall never doubt of *London*'s being a City of *England*,

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nor of the three Angles of a right-lined Triangle taken together, being equal to two right ones.

If moral Evidence, which we have from God, was not firm, and ſufficient to make us give our Aſſent to it, we cou'd, by a mathematical Demonſtration, prove God not to be good.

But that God is good, and that this appears alſo by mathematical Demonſtration, follows from what is ſhewn before. Neither does it belong to this Place to anſwer the Objections, which are brought by ſome againſt the divine Goodneſs, which we are fully aſſured of, from the Conſtitution of Things, which we are, in a great meaſure unacquainted with.

Neither will I detain you by refuting the Objections of Scepticks; it is ſufficient for me to have propos'd ſuch Arguments, whence Anſwers to their Objections naturally flow.

But ſomething muſt be ſaid of the Opinions of Scepticks, that you may ſee by what Anſwers they endeavour to overthrow the Arguments of Dogmatiſts.

Socrates was the firſt who gave occaſion to Scepticiſm. He, according to *Cicero, call'd back Philoſophy from occult Things, and Things cover'd with Darkneſs by Nature, in which the Philoſophers who went before him had been converſant, and brought it back to common Life, that he might enquire into Virtues and Vices, and Good and Evil.* Of other Things he affirm'd nothing, he confuted others, ſaying, that he knew nothing, and that upon this account he excell'd others, becauſe they thought they knew what they were ignorant of; but that he knew only this one thing, that he knew nothing: and all that he ſaid was about the Praiſe of Virtue, and perſuading all Men to the Study of it. He, or rather his Scholar *Plato*, laid the Foundation of that which is call'd the firſt Academy.

Arceſilas was the Founder of the ſecond Academy; he openly affirm'd that nothing at all cou'd be comprehended, and that he was not certain of this, *viz.* that he knew nothing, defending Scepticiſm entirely.

But the *Academicks* alone did not doubt of all Things, *Pyrrho* ſet up another School, and there was not one Art of doubting only amongſt the Philoſophers. *Aulus Gellius* has obſerv'd that the *Pyrrhonians* and *Academicks* differ'd in their Ways of doubting in general. Theſe did as it were comprehend, that nothing can be comprehended, and did as it were diſcern, that nothing can be diſcerned.

cerned. Thoſe affirm'd that Truth appears no manner of way, becauſe nothing appears to be true.

Sextus Empyricus, whoſe entire Work of Scepticiſm is ſtill extant, has deliver'd a like Difference between theſe Philoſophers, but in another way. *Arceſilas*, the Founder of the ſecond Academy, ſaid, *that a wrong Aſſent was a Good, and according to Nature*; *but* Pyrrho *thought that it was ſo, not according to Nature, but according to Appearance.*

But how this agrees with theſe Words of *Tully*, I don't well ſee: *Arceſilas denied that any thing could be known, even this very Thing.*

They all diſputed about all Things, and contended, that nothing could be affirmed, the contrary of which might not be prov'd by Arguments equally good: And oppoſed the moſt abſurd Trifles to the moſt ſolid Reaſons. Yet they took it very ill, if any one in the Affairs of Life made uſe of their own Arguments againſt them.

The Sophiſt *Diodorus* having put his Shoulder out, came to the Phyſician *Exophilus*, who made anſwer to him, that his Shoulder was not put out; making uſe of the Argument, by which the Sophiſts, and *Diodorus*, endeavoured to prove that all Motion was impoſſible. Then the Sophiſt entreated him that he would leave off ſuch Diſcourſe, and apply ſome Remedy that his Art ſupplied him with.

But let us hear *Lacydes*, who was a great Promoter of the School Subtilties. He was very penurious in his Houſe-keeping, and took care of his own Victuals himſelf, and ſealing up his Pantry, he put the Seal, thro' the Key-hole of the Door, into the Pantry; ſo that when the Door was unlock'd, he might take his Ring, and when he pleaſed firſt lock the Door, and then ſeal it, and afterwards throw the Ring again into the Pantry. But his Servants finding out this Contrivance, when their Maſter was out, unlock'd the Pantry, and took what Victuals and Drink they pleaſed, then ſhut it, ſealed it, and put the Seal thro' the Key-hole.

But *Lacydes* finding the Veſſels empty which he had left full, thought that this was a Confirmation of what he had heard taught by *Arceſilas*, *That nothing can be comprehended.* I ſhut the Door with my own Hands, ſaid he; I ſealed it, I put the Seal into it; but returning, and opening the Cellar, have found only the Seal, and not the other Things; was I not in the right to doubt of every thing? But having diſcovered the Theft, he took the Seal along with

with him: upon which his Servants opening the Pantry, and taking out what was in it, ſealed it with another Seal, and often with none at all; and ſtrenuouſly diſputed, making uſe of their Maſter's Argument, that the Cellar was either ſeal'd with the Seal with which he had ſeal'd it, or that it had not been ſeal'd at all: At laſt the Philoſopher opening his Mind without diſguiſe, ſaid to them, we diſpute one way, but live another.

All Scepticks have come to this, *viz.* to affirm that the Art of doubting is not to be applied to the Affairs of Life.

Carneades put away his Pupil and Friend *Mentor*, becauſe he caught him with a Harlot; he did not ſuſpect his Senſes to be deceitful in that Caſe.

" It is ſufficient, as I think, ſays *Sextus Empyricus*, to frame our " Lives by Experience acquir'd by Uſe, and according to common " Obſervations, and pre-conceiv'd Notions in the Minds of Men, " without concerning ourſelves about Opinions, not aſſenting to " thoſe Things which are affirm'd out of a dogmatical Curioſity; " and are of no ſervice to the common Affairs of Life."

But how inconſiſtent is this! If the Senſes are deceitful, if nothing can be comprehended, if Reaſon leads us into Error, why muſt we uſe them in the ordinary Affairs of Life? How ſhall we be aſſured of Experience, common Obſervations, and the Opinions of Mankind? Whence will it appear that theſe Things will be of ſervice to us? But it will be ſufficient to ſhew by a ſingle Example, by what Subtilties they maintain the Foundations of their Doctrine.

They ſay that there is nothing certain, and that this Propoſition includes itſelf. The Dogmatiſts make anſwer, that this Propoſition is either true or falſe; if it is true, this at leaſt will be certain, that there is nothing certain; if falſe, all Things are not uncertain. But don't imagine that the Scepticks are worſted by this Dilemma. They anſwer that it is neither true nor falſe. But let us leave theſe, ſeeking a Medium between Truth and Falſhood.

Let us return to moral Evidence. I have ſhewn the Foundations of it to be the Senſes, Teſtimony, and Analogy; but I hinted that due Cautions were to be obſerved. I don't maintain, that the Senſes never lead us into Error, that we are to give Credit to every Teſtimony, and that all kinds of Obſervations whatever give us room to reaſon analogically.

We muſt enquire into the Cautions by which Error may be avoided, and in order to the Diſcovery of theſe, the Foundations of moral Evidence, which have been mentioned, muſt be admitted, but it will appear that we have made no Error in the Diſcovery of the Cautions, if we find that we have admitted nothing againſt them, in the Diſcovery of them.

The Rules relating to the Senſes are to be taken from ſuch Things as Phyſics teach us about them.

The Rules concerning human Teſtimony have for their Foundation Obſervations made of Mankind.

And for Analogy, we ought to reaſon from this Propoſition lately demonſtrated, *That the Univerſe is governed by unchangeable Laws.*

It would be tedious, tho' not foreign from my Purpoſe, to explain theſe Things more fully, and I ſhould treſpaſs upon your Patience if I ſhould point out the Rules themſelves.

I will only ſay this, that their chief uſe is, that we may be able to avoid Error, for we do not always come to a Knowledge of the Truth by the Aſſiſtance of them.

But altho' we ſeek after Truth, altho' this is the End of our Studies, if we do not attain to it, it is ſomething to have eſcaped Error. When we are not able to reach the deſired Haven, it is better to ſail upon the Sea, than to put to Shore in an Enemy's Country. It is better to remain ſtill in doubt, than to be in danger of running into Error.

No State of the Mind, if we except Scepticiſm, is worſe in the Search after Truth than that which is for taking away every Queſtion, and denying Aſſent to either ſide of it. Neverthele ſs no State of the Mind is more common.

Every one without examining the Queſtion much, amongſt different Opinions, makes choice of one in particular, and takes no Notice of the Arguments which might be brought to overthrow it: He thinks he muſt contend for this as he wou'd for all that is dear and valuable to him; and ſeeks for Arguments, whereby he may defend his own Opinion, and anſwer others, without being at all concern'd about the Truth.

Hence innumerable Errors proceed, and the Reaſon why they are ſcarce ever ſet right: The Mind of Man always inclines to Extremes, either making a doubt of all Things, or giving its Aſſent to weak Arguments, ſcarce ever ſeparating what is certain from what is doubtful.

How much greater Improvement wou'd Men make in the ſearching after Truth, if they wou'd perſuade themſelves that it was very eaſy for them to err, and that they had not well examin'd every thing to which they aſſent, and wou'd uſe diligence in endeavouring to correct their Miſtakes?

But I have not ſaid all that is ſufficient about this Matter. I have paſs'd over, without mentioning, the Rules about avoiding Error, where I treat of moral Evidence; and I will omit alſo what relates to the Origin of Errors, being content to have ſhewn the general Foundations of Evidence.

Probability has great Affinity with what I have explain'd. This is very extenſive, and is by many confounded with moral Evidence; if I had time, I wou'd ſay ſomething about it. But I will now add only this, which I am apt to think will be more acceptable to you, namely, that *I have done.*

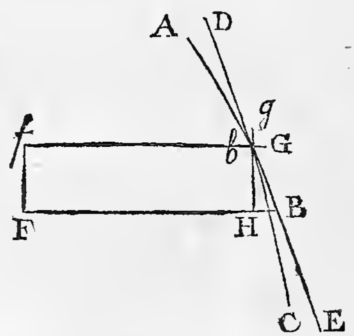
TO

TO THE

READER:

Concerning Demonſtrations which have Quantities infinitely ſmall for their Foundation.

IN many Demonſtrations, which I have given in the Scholia, I conſider infinitely ſmall Quantities, and ſo propoſe them, that they may be underſtood by ſuch Readers, as are unacquainted with what Geometricians have deliver'd about them. But that they might not in any wiſe doubt of the Truth of the Demonſtrations, and leſt they might not form an exact Idea of them, I thought it wou'd be of ſervice to premiſe ſomething.



Let there be any Curve A B C, which a Line D E touches in B; let there be any two right Lines F B, *f* G, parallel, join'd by the Line F f, of which *f G* cuts the Curve at *b*; let H *b* alſo be parallel to F *f*, cutting the Tangent D E at *g*. Now if we ſuppoſe F *f* to be diminiſh'd, *i. e.* the Line, *f* G to be carry'd with a parallel Motion, whilſt *g b* H is alſo carry'd with a parallel Motion, along the Interſection of this Line with the Curve, it is manifeſt that the Ratios between *g* B, *g* H, H B, are not chang'd, until all the Lines vaniſh together, *f* G, F B, coinciding.

In the ſame Motion of the Line *f* G, the Ratios between *b* B, *b* H, H B, are continually changing, till there are no Ratios remaining

maining upon their vaniſhing; but at the very Inſtant of their vaniſhing, the Ratios are different from all the former.

Thus a Body that is falling, and by falling freely, continually encreaſes its Velocity, when it comes to any Point, it has a Velocity greater than all the Velocity which it had before it came to that Point, but leſs than all thoſe, which it will have after it has paſs'd by the Point; and the particular Velocity by which it goes to that Point, is different from all others, whereby it goes to any other Points. In like manner I don't ſpeak here of the Ratios, which the Quantities have before they vaniſh, or after their diſappearance or vaniſhing, but of thoſe which they have while they are diſappearing or vaniſhing.

But in the very Inſtant of their vaniſhing, becauſe the Curve coincides with the Tangent in the Point of Contact, the Points G, *g*, *b*, are confounded, and the Ratios between *b* B, *b* H, H B, don't differ from the Ratio's *g* B, *g* H, H B.

When in the Demonſtrations I ſuppoſe B *b* infinitely ſmall, I conſider it as a right Line, and alſo ſuppoſe the Equality of the Ratios mention'd: yet theſe Things are not mathematically true, except in the Inſtant of the vaniſhing; therefore when I ſpeak of infinitely ſmall Quantities, you are to underſtand thoſe that are vaniſhing, and the Demonſtrations will yield to no mathematical Demonſtration.

For it is manifeſt, that at the Inſtant of the vaniſhing of *f b* and F B, they are confounded and truely equal; therefore in whatſoever Demonſtration I put an infinitely ſmall Portion of the Curve *b* B, as I conſider it vaniſhing, I may ſafely look upon ſuch Lines as *f b* and F B as equal.

Theſe Demonſtrations ſhould be diſtinguiſhed from thoſe in which there is any Error, tho' inſenſible; ſuch as the Demonſtration N. 2384. whence I collect that Sound, whether great or ſmall, is always mov'd with the ſame Velocity thro' the ſame Air; which is not mathematically true, but the Difference of the Velocities, when there is any, is ſo ſmall, that it can't be perceived by any means; wherefore we take no notice of this Difference in Phyſicks. In like manner as in practical Geometry, when we conſider the Height of a Mountain, we take no notice of the Alteration that may be made in it by the Addition of a Grain of Sand. But in ſuch Demonſtrations, infinitely ſmall Quantities are not treated of, but finite ones; for not only the Ratio between the Diameter of a Grain of Sand, and the Height of a Mountain, but likewiſe between the Diameter of that, and the Diameter of the Earth, or if you pleaſe the Diſtance of any fixed Star from the Earth, may be expreſſed by a finite Number.

In theſe Demonſtrations in which we conſider thoſe Quantities as equal, which differ ſo infinitely, an Error in a Demonſtration will not be ſenſible; and therefore, in treating of Things themſelves, of which we judge by our Senſes, theſe Demonſtrations are juſtly admitted by Mathematicians; they do not belong to pure Mathematicks, which yet admit, as I have ſhewn, Demonſtrations, founded upon infinitely ſmall, or evaneſcent Quantities.

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Mathematical Elements

OF

NATURAL PHILOSOPHY

CONFIRM'D BY

EXPERIMENTS.

BOOK I.

PART I. Concerning Body in general.

CHAP. I.

Concerning the End of Natural Philosophy, and the Rules of Philosophizing.

NAtural Philosophy is conversant about *natural Things* and their *Phænomena*.

DEFINITION 1, and 2.

Natural Things are all Bodies; and the Assemblage or System of them all is called the Universe. 1.

DEFINITION 3.

Natural Phænomena, are all Situations, and Motions, of natural Bodies, not immediately depending upon the Action of an intelligent Being, and which may be observed by our Senses. 2.

We don't exclude those Motions out of the Number of natural Phænomena, which are made in our Body by the Will: In these we must distinguish what depends upon the Will, from that which is to be attributed to another Cause. There is Motion made in a 3.

 deter-

determinate Manner, and at certain Times, which ought to be aſcribed to the Determination of the Will, and does not relate to Natural Philoſophy: But the Motions proceeding from the Action of the Muſcles, whoſe Action depends upon ſome other Motion, are theſe natural Phænomena; but the Motion ariſing from the immediate Action of the Mind, and which is entirely unknown to us, is not a natural Phænomenon.

All theſe Motions are governed by certain Rules.

The Sun riſes and ſets daily, and the time of its riſing and ſetting is always determin'd, according to the Time of the Year and the Place; Plants of the ſame kind, under the ſame Circumſtances, are produced and grow in the ſame manner: and the ſame may be ſaid of other things. It is manifeſt to one who conſiders it with Attention, that certain Rules are obſerv'd even in thoſe Things which appear to us entirely fortuitous and uncertain.

For an Examination into Things brings us to this Axiom, which is the Foundation of all Reaſonings in Natural Philoſophy; That
4. *the Creator of the Univerſe governs all Things, by Laws determin'd by his Wiſdom, or ſpontaneouſly flowing from the Nature of the Things.*

5. *Natural Philoſophy explains natural Phænomena*, i. e. *treats of their Cauſes.*

When we enquire into theſe Cauſes, we muſt conſider Body it ſelf in general; then we muſt diſcover by what Rules the Creator of all things pleas'd that all their Motions ſhould be perform'd. *Theſe Rules, are call'd the Laws of Nature.*

Definition 4.

6. *A Law of Nature then is, the Rule and Law according to which God thought fit that certain Motions ſhould always*, i. e. *in all Caſes, be perform'd.*

7. Therefore a Law of Nature is with reſpect to us, every ſimple Effect, which continues the ſame upon all Occaſions, whoſe Cauſe is unknown to us, and which we find cannot flow from any Law known to us, tho' perhaps it may from a more ſimple Law, unknown to us.

For with reſpect to us it matters not, whether any thing depends immediately upon the Will of God, or is produc'd by an intermediate Cauſe, of which we have no Idea.

8. The Laws of Nature can be found out only from an Examination of natural Phænomena.

Phænomena

Phænomena are to be explain'd by the help of Laws, discover'd in this manner.

But in the Investigation of them, the following Rules of Sir *Isaac Newton* are to be observ'd, which are founded upon the fore-mention'd * Axiom. * 4.

RULE 1.

That no more Causes of natural Things are to be admitted than such as are true, and sufficient to explain their Phænomena. 9.

RULE 2.

That there are the same Causes of natural Effects of the same kind. 10.

RULE 3.

That the Qualities of Bodies, whose Virtue cannot be increas'd and diminish'd, and which belong to all Bodies, upon which Experiments can be made, are to be look'd upon as Qualities of all Bodies. 11.

CHAP. II.

Concerning Body in general.

EXtension is the first Thing to be consider'd in Body. 12.

The Idea of Extension is almost always present to our Mind; its Idea is most simple; and therefore cannot be describ'd by any Words.

Every Body is extended, and if you take away the Extension of Body, you take away the Body itself. 13.

Yet every thing that is extended is not Body; but it can't be determin'd how Body differs from Space, without examining other Properties of Bodies. 14.

The next Thing to be consider'd in a Body is *Solidity*. A Body excludes every other Body from the Place occupied by itself, and fluid Bodies as well as those that are hardest have this Property also. 15.

The third Property of Body is *Divisibility*; which is a Consequence of Extension. For there may be always conceived an Extension less than any given Extension, whence we see that there are Parts in all Extension, which Parts in a Body may be mutually separated from each other: because 16.

17. Body is endued with a fourth Property, ſince it may be tranſferr'd from one Place to another, whence a Body is ſaid to be *moveable*.

18. But it continues in Motion by an *innate Force*.

When there is no Obſtacle, a Body yields to the leaſt Motion: but a greater Action is requir'd, if the Body is to be mov'd to the ſame Diſtance in a leſs time, or to a greater, in the ſame time: That likewiſe is a greater Action, whereby a greater Body is mov'd, than that by which a leſs is mov'd, if the Diſtance to which it is carry'd is the ſame. Therefore *a quieſcent Body reſiſts Motion, not*
19. *while it is at reſt, but while it is moving*. Upon this account a Body is call'd inactive, *and is ſaid to have Inertia or Inactivity. This in all Bodies is proportional to the Quantity of Matter*; for it is equally in every Particle of Matter.

20. Every Body has ſome Figure, and is figurable, becauſe it is terminated: but its Figure may be chang'd, becauſe a Body may be reſolv'd into Parts, which, as they are moveable, may be diſpos'd in different manners with reſpect to each other.

CHAP. III.

Concerning Extenſion, Solidity, and Vacuity.

21. WE muſt now conſider that Queſtion (ſo often handled by the Learned) *concerning a Vacuum*; *viz.* whether there is any Extenſion void of all Matter; for ſuch Extenſion is call'd a *Vacuum*, *Emptineſs*, or *mere Space*.

That there is really a Vacuum is prov'd from Phænomena, and therefore I ſhall conſider this Queſtion more particularly in Chap. XII. Book V. I will now ſpeak only of the Poſſibility of a Vacuum.

22. It may be inferr'd, that a Vacuum is poſſible from an Examination of our Ideas alone. For every thing is poſſible, which we can clearly conceive to exiſt. For if there is that in any thing, which hinders its Exiſtence, the Idea of the Impediment is contain'd in the Idea of the Thing, and is the Reaſon, why the Poſſibility of the Thing can't be ſo well conceiv'd.

Therefore the Queſtion comes to this, whether we can have an Idea of Extenſion that is not ſolid.

We

We acquire an Idea of Solidity by the Touch; we perceive certain Bodies to resist us, and indeed those resist us at all times, which hinder our descending to the lowest Places. From this Resistance we infer, that a Body excludes every other Body from the Place occupied by itself, *i. e.* that it has Solidity; which Idea of Solidity we transfer to more subtile Bodies, which don't come within the reach of our Senses, upon account of the Smallness of their Parts: And it appears by Experience, that these, as well as the hardest, resist other Bodies. 23.

EXPERIMENT 1.

The Air in which we live almost always escapes our Sight and Touch, yet in a well-clos'd Syringe it resists the Piston, so that it can't be thrust to the Bottom of the Pump by any Force. 24.

But the Idea of Solidity is not contain'd in the Idea of Extension; this is deduced from the Idea of Resistance, and we acquire it only by the Touch. Therefore if a Person had never touch'd a Body, he would be utterly unacquainted with Solidity, yet would he have an Idea of Extension. 25.

EXPERIMENT 2.

When a Body is set against a concave Mirror at a proper Distance, a Spectator sees the Image of it hanging in the Air before the Mirror, as I explain this in Book IV. This Appearance is like the true Body, having the most lively Colours; yet it does not resist. 26.

If any one had never seen any thing except such Images, and his own Body like such an Image, would he have any Idea of Solidity? It does not appear that he would; yet he will certainly have an Idea of Extension.

Here I don't enquire what such an Image is; I speak of Ideas.

Space does not differ from Body in a Privation of Solidity only.

Space is infinite, and it will appear to one who considers the thing with Attention, that it can be terminated by no Bounds. For no Space can be conceiv'd terminated, whose Bounds may not be surrounded by other Space; and an Idea of Extension confin'd by Bounds, and not included in other Extension, destroys itself. Wherefore to one who is attentive to the Matter, it is a Contradiction to suppose Bounds to Space. But Bodies are finite. 27.

We plainly see that *there are Parts in Space*, but they can't be separated from one another, *they are immoveable, as well as Space itself.* 28.

itself. But the Parts of Body undergo a Removal, and therefore a Separation.

29. The Idea of Space is very ſimple; that of Body is more complex.

Solidity is call'd by ſome Impenetrability, who endeavour to deduce it from the Nature of Extenſion: If to a cubick Foot of Extenſion, ſay they, another cubick Foot of Extenſion be added, we ſhall have two cubick Feet; for they have each all things requiſite to conſtitute that Magnitude; therefore one Part of Space excludes all others, and cannot admit them.

30. *I anſwer*, That it implies a Contradiction to ſuppoſe the Removal of a Part of Space from one Place to another; therefore it follows from the Immobility, and not from the Impenetrability, or Solidity of the Parts of Space, that two Parts of Space can't both occupy the ſame Place.

CHAP. IV.

Concerning the Diviſibility of Body in infinitum, and the Subtilty of its Particles.

31. THE Extenſion of a Body implies its Diviſibility, *i. e.* Parts may be conſider'd in it.

Yet the Diviſibility of Body differs from the Diviſibility of Extenſion, for its Parts may be ſeparated from each other (28.) But as Diviſibility depends upon Extenſion, it ought to be examin'd under the Conſideration of Extenſion.

32. *Body is diviſible in infinitum, i. e.* there is no Part that can be conceiv'd in its Extenſion, tho' ſmall, but that there will be a ſmaller.

33. Plate I. Fig. 1. Let there be a Line A E, perpendicular to B D; and let F G, be diſtant from A a little, and perpendicular to it; let there be Circles deſcrib'd from the Centers C, C, C, &c. and with the Radii C *a*, C *d*, &c. cutting the Line F G, in the Points *i*, *i*, &c. by how much the greater the Radius A C is, by ſo much the ſmaller is the Part *i* F: the Radius may be increas'd *in infinitum*, and the Part *i* F diminiſh'd; which nevertheleſs can never be reduced to nothing; becauſe the Circle can never coincide with the right Line B F.

Therefore the Parts of any Magnitude may be diminiſh'd *in infinitum*, and there is no End of the Diviſion.

But

But a greater Paradox is deduced from this Demonſtration. It is plain from this, that the mixt Angle, which the Circle makes with the Tangent, may be diminiſh'd *in infinitum*. But this Angle, tho' thus diviſible, is leſs than every right-lined Angle *; and a right-lin'd Angle, which is itſelf diviſible *in infinitum*, as all Quantity is, howſoever it is diminiſh'd, does yet exceed all the mixt Angles juſt mention'd. 34.

* 16 El. III.

Therefore *an infinitely ſmall Part of any Quantity divided in infinitum, is itſelf diviſible in infinitum.* 35.

And the ſame thing is proved in the following Scholia by other mathematical Demonſtrations, by which alſo it appears, that *there is an infinite number of Claſſes of Diviſions in infinitum.* 36.

From the Diviſibility of Body I deduce, *that any Particle of Matter being given, howſoever ſmall, and any finite Space being given, howſoever great, it is poſſible, that the Matter of that ſmall Particle may be diffus'd thro' that whole Space, and ſo fill it, that there will be no Pore in it, whoſe Diameter will exceed the ſmalleſt Line given.* Which that I may demonſtrate, I conceive the Space fill'd to be divided into cubick Cells, whoſe Sides are equal to, or leſs than this ſmalleſt Line given: the Number of the Cells will be finite, and the Particle may be divided into ſo many Parts, as there are Cells; ſo that we may conceive one Particle placed in each of the Cells; we may further conceive a concave Sphere to be form'd by each of theſe ſmall Particles. Upon account of the Diviſibility of Matter, any concave Sphere may be continually increas'd, by diminiſhing the Thickneſs of the Matter; but as there is ſuch a Sphere in each of the Cells, they may each be increas'd, 'till thoſe that are next to each other touch one another, and all together fill the Space. 37.

The chief Objections againſt the Diviſibility *in infinitum*, are, That an Infinite cannot be contain'd in a Finite; That it will follow from its Diviſibility *in infinitum*, that all Bodies are equal, or that there is one Infinite greater than another. 38.

But the Anſwer to theſe is eaſy; That the Properties of determin'd Quantity are not to be attributed to an Infinite, conſider'd generally. Who has ever prov'd that there cannot be an infinite Number of infinitely ſmall Parts in a finite Quantity; and that all Infinites are equal? I demonſtrate the contrary in the following Scholia.

If, after having examin'd the poſſible Diviſibility of Matter, we look into the Subtilty of the Parts in Bodies, it will appear that 39.
this

this vaſtly exceeds our Comprehenſion; and there are in Nature innumerable Examples of ſuch Particles ſeparated from one another.

Mr. *Boyle* proves this by divers Arguments.

40. He ſpeaks of a ſilken Thread 300 *Engliſh* Ells long, that weighed but two Grains and an half.

41. He meaſur'd Leaf-gold, and found by weighing it, that 50 ſquare Inches weigh'd but one Grain. If the Length of an Inch be divided into 200 Parts, the Eye may diſtinguiſh them all; therefore there are in one ſquare Inch 40000 viſible Parts, and in one Grain of Gold there are two Millions of ſuch Parts, which viſible Parts no one will deny to be farther diviſible.

42. A whole Ounce of Silver may be gilt with eight Grains of Gold, which is afterwards drawn into a Wire thirteen thouſand Foot long.

43. In odoriferous Bodies we can ſtill perceive a greater Subtilty of Parts, and which are ſeparated from one another; ſeveral Bodies ſcarce loſe any ſenſible Part of their Weight in a long Time, and yet continually fill a very large Space with odoriferous Particles. Whoever will be at the pains to make Calculations concerning thoſe ſubtile *Effluvia*, will find the Number of Parts to be amazing.

44. By help of Microſcopes, ſuch Objects as would otherwiſe eſcape our Sight, appear very large: There are ſome ſmall Animals ſcarce viſible with the beſt Microſcopes; and yet theſe have all the Parts neceſſary for Life, Blood, and other Liquors: Who does not ſee how great the Subtilty of thoſe Particles muſt be, which make up ſuch Fluids?

S C H O L I U M I.

That an Infinite is contain'd in a Finite.

THEY who deny that Matter is diviſible *in infinitum*, call that an Infinite, than which there is no greater; and I readily grant that Matter is not infinitely diviſible, according to this Definition of an Infinite. I maintain that a Body cannot be divided into ſuch a Number of Parts, as will be the greateſt of all, and that there is no End of the Diviſion.

But I call that an Infinite which exceeds a finite, *i. e.* every Thing whoſe
45. Magnitude, how great ſoever, can be determin'd; but it may be deduced from the Conſideration of a decreaſing geometrical Progreſſion, *that there is contain'd in a finite Quantity, a Number of Parts exceeding every finite Number.*

Who does not ſee that this Progreſſion, *viz.* $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, *&c.* may be continued *in infinitum*, and that there is no End of the Continuation? But I ſhall demonſtrate that the Sum of all the Terms never exceeds Unity, nay that

that it is exactly equal to Unity, if we conceive the Progression to be really continued *in infinitum*.

Let the Line A E be Unity; the half of this A B is the first Term $\frac{1}{2}$; BC the half of what remains is the second Term $\frac{1}{4}$; the third Term will be CD $\frac{1}{8}$; by dividing D E into two equal Parts, we have the next Term; and in the same Manner the Series may be continued *in infinitum*, and the Defect of the Sum of the Terms of the Series AB, BC, CD, &c. from the whole Line A E, will always be equal to the last Term of the Series, howsoever it is continued. But as long as the Number of Terms is finite, the Denominator of the Fraction expressing the last Term is a finite Number, and the last Term is a finite Part, whereby the Sum of the Series is deficient from an Unity. 46. Plate II. Fig. 1.

But if the Number of Terms exceeds every finite Number, the Denominator of the last Term will exceed every finite Number, and will express a Part of the Line A E less than every finite Part, and therefore the Difference, between the Sum of the Series and the Line A E, will vanish, *i. e.* these Quantities will be equal. *Q. E. D.*

We have no Idea of an Infinite; therefore what I demonstrate concerning an Infinite, has no relation to Ideas; but what immediately follows from undoubted Principles, is certain, and what is also deduced from them, cannot be questioned.

Very many Things exceeding our Comprehension are plainly demonstrated concerning the Divisibility of Matter, among these what relates to the Curve called the Logarithmic Spiral, is chiefly remarkable.

Concerning the Logarithmic Spiral, and its Measure.

The Property of this Curve is, that all the Angles which it makes with Lines drawn to its Center, are equal. 47.

Let C be the Center; the Angle of the Curve at A, *i. e.* which the Tangent of the Curve makes with the Radius AC, namely BAC, is equal to the Angle EDC, which the Tangent makes with the Line D C, in any other Point D. Plate II. Fig. 2.

If this Angle is a Right one, the Spiral will change itself into a Circle, but if an Acute one, it plainly appears that the Curve continually draws near to the Center; and yet it cannot come to it except after infinite Windings.

Having laid down the Beginning of the Curve A, let us suppose the first Revolution to be terminated at F, and the second at G; as these are made in the same Manner, they are similar Curves, differing only in Magnitude, and having their Distance from the Center diminished in the same Proportion; therefore AC is to FC, as this last is to GC, and all the Distances from the Center, terminated by successive Revolutions, make a decreasing geometrical Progression. If F falls upon a Point equally distant from A and the Center C; in which Case the Angle BAC will exceed 83 Degrees 42 Min. the second Revolution will be terminated in G, the middle Point between

F and C, which also should be applied to the following Gyrations: And
48. *any Point that is moved in the Curve* when it makes any entire Revolution, towards the Center, passes through the half of its Distance from the Center in the Beginning of its Revolution. Therefore, *although it will come to as small a Distance from the Center as you please*, it cannot reach it in one Revolution; and if the Number of Revolutions is increased, as much as you please, yet will it not come to the last, and *the Number of Revolutions will exceed every finite Number.*

Yet it is certain that *the Curve reaches to the Center and is there terminated.*
49. For the Parts of the Curve passed through by the Point describing the Curve, in every one of the successive Revolutions, make a geometrical Progression. For each of the Revolutions is form'd in the same manner; and are to one another as the Distances from the Center, which we see make a geometrical Progression. But the Sum of all the Terms of such a Progression decreasing and continued *in infinitum*, is finite, as I have demon-
* 46. strated it to be in a peculiar Case *, and which may be proved of all by a like way of reasoning. Therefore the Sum of all the Parts to be passed through by a Point in every one of the Revolutions, that it may come to the Center, is also finite: And *this Point will come to the Center with a*
50. *finite Velocity, and in a finite Time, after it has made an infinite Number of Gyrations.* I determine the Length of the Way passed through after the following Manner.

51. Let there be a Portion of a Curve ABEG; whose Center is C; with
Plate II. which Center, and the Radius CG, let there be described a Portion of a
Fig. 2. Circle GL, cutting the Line CA in L.

Let us suppose LA to be divided into the equal, but small Parts, AD, DI, IL, through whose Divisions let us conceive Portions of Circles describ'd with the Center C, cutting the Curve at B and E; and the Radii BC, EC being drawn, let the right-angled Triangles ADB, BFE, EHG be made, in which, upon account of the small Lines AD, DI, IL, the Hypotenuses, though Portions of a Curve, may be looked upon as right Lines; for the Number of the Parts in AL may be conceived to be increased *in infinitum*, those Things remaining which have been explained hitherto, as well as those that follow.

The Triangles mentioned are all similar to each other; because they are rectangular, and besides, they have the Angles BAD, EBF, GEH equal;
* 26. El. 1. from the Nature of the Curve. They are likewise equal *, because of the homologous equal Sides AD, BF, EH, which follows from the Equality of the Parts AD, DI, IL.

From A let the Line A c be drawn, making the Angle CA c with CA, equal to the Angle CAB; let C c, L g, I e, D b, be drawn perpendicular to AC in the Center C, and in the Points L, I, D, cutting A c in the Points c, g, e, b; and $b f$ and $e h$ being drawn parallel to AC, the Triangles AD b, $b f e$, $e h g$ are made, which are similar and equal to each other, and to the Triangles ABD, BFE, EHG, as appears from the Construction. Therefore

Therefore the Hypotenuses A b, b e, e g, are equal to the corresponding Hypotenuses AB, BE, EG, *i. e.* the Line A g is equal to the Portion of the Curve ABEG.

Hence it appears how any Portion of the Curve may be measured, and that the Curve is equal to the Line A c, if it be continued to the Center. 52.

SCHOLIUM II.

Concerning the Inequality of Infinites.

IT will plainly appear, *that all Infinites are not equal*, if we consider that a Line, which is extended one way, may be drawn out *in infinitum*, and that such a Line is infinite; yet will it be less than another Line, which we conceive to be produced *in infinitum* both ways, and the Sum of both will exceed this also. 53.

An infinite Line contains an infinite Number of Feet, and twelve times the Number of Inches.

We likewise discover the Inequality of Infinites, by comparing the different Logarithmical Spiral Curves, mentioned in *Scholium* I.

Besides the Curve just mentioned, and in part described here, let us conceive another Logarithmical Spiral also, proceeding from A, and tending towards the Center in such manner, as to come to F in two Revolutions; in two more it will come to G; because two Revolutions are required, that in its way to the Center it may pass through half the Distance from it: The Number of Revolutions in this is double the Number of Revolutions in the first Spiral, when it approaches equally to the Center with the first ADF; and it will come to the Center in twice the Number of Revolutions: yet neither Curve will reach the Center, without an infinite Number of Revolutions. 54. Plate I. Fig. 2.

SCHOLIUM III.

Concerning the Classes of Infinites.

WHAT relates to the various Classes of Infinites, is the greatest Paradox of all that is demonstrated concerning them, and immensely exceeds our Ideas.

Let there be a Parabolick Curve ABC, having any Abscis AD, and the Ordinate corresponding to it DC. 55. Plate II. Fig. 4.

It is a known Property of this Curve, that an Ordinate is a mean Proportional between the Abscis and a certain determined Line, called the Parameter: Wherefore, if any Abscis is called x, its corresponding Ordinate y, and the Parameter a, we have in all the Points of the Parobola $\div$ x, y, a; therefore $ax=yy$ *: which Equation therefore expresses the Nature of the Parabola. If x vanishes y vanishes also, and the Parabola agrees with AF, drawn

* La Hire Sect. Con. lib. 3. pro. 2.

drawn parallel to the Abſciſſes, through A, and the whole is beneath this Line, which is tangent to it, and with which it makes the mixt Angle FAC.

If a is increaſed, x remaining, y is increaſed, and the Parabola enlarges itſelf, or rather a new one is made, in which all the Ordinates exceed the correſponding Ordinates of the firſt Curve; ſo that the firſt Curve is included in the ſecond, which paſſes between the firſt and the Tangent AF, and makes a ſmaller mixt Angle with it. But the Parameter may be increaſed *in infinitum*, and upon that account the Angle which the Parabola makes with the Tangent, may be diminiſhed *in infinitum*; as I have lately demon-

* 33. ſtrated this concerning a Circle *.

56. Keeping the Axis AD and Vertex A, let there be given another Curve AEG, and let its Ordinates be called z, and their Relation to the correſponding Abſciſſes x, be expreſſed by this Equation, $bbx=z^3$: b denotes a conſtant Line.

By increaſing b all the z are increaſed, and the Curve is changed into one more open, and the Angle of Contact is diminiſhed, which by increaſing b may be diminiſhed *in infinitum*.

57. Therefore *we have two Claſſes of Angles decreaſing in infinitum: but the ſecond taken all together is infinitely ſmall with reſpect to the firſt*. For I demonſtrate that any Angle in the ſecond is exceeded by any Angle, *i.e.* howſoever ſmall, in the firſt.

Let c be a third proportional to a and b, howſoever taken; therefore $ac=bb$. By multiplying the Equation $ax=yy$ by c, we have $acx=yyc$, i. e. $bbx=yyc$. In the ſecond Curve bbx is equivalent to z^3; therefore $z^3=yyc$, if the Abſciſs x is the ſame in both Curves.

From this Equation we deduce z; $c::yy, z^2$: Whence it appears that yy is exceeded by zz, i.e. that y is ſmaller than z, as long as this is exceeded by c, whence it follows that the ſecond Curve is given between the Tangent and firſt Curve, whilſt it proceeds from A, before z is equivalent to c, which that it obtains univerſally appears from this Demonſtration.

58. Let us now ſuppoſe a third Curve AI to be given, whoſe Axis alſo is AD, and whoſe Equation is $d^3x=u^4$, the ſame Abſciſſes x remaining; u is any Ordinate; and d a determined Line; if we increaſe this, we alter the Curve, and diminiſh the Angle which the Curve makes with the Tangent AF; and by theſe Curves is formed the third Claſs of Angles, which may be diminiſhed *in infinitum*, and in which there is no Angle, which is not exceeded by any Angle in the ſecond Claſs.

b and d being given in any Proportion, let bb be to dd; as d to a fourth, which we will call e; therefore bbe will be $=d^3$; and the Equation of the Curve $bbx=z^3$, being multiplied by e, will be chang'd into this $bbex=d^3x=z^3e$; and therefore $z^3e=u^4$, if regard is had to the ſame Abſciſſes in both Curves; therefore $u:e::z^3:u^3$; therefore u exceeds z, as long as e exceeds u, and by proceeding from A, the Curve, whoſe Abſciſſes are u, paſſes between AF and the other Curve. *Q. E. D.*

The

The Curves, whose Equation is $f^4 x = t^5$, a determin'd Quantity f being laid down in each of the Curves, and any Ordinate t, will give a new Class of Angles less than all that have been mention'd, and in the same manner *Classes may be form'd in infinitum, and all the Angles in any Class are always exceeded by all the Angles in the foregoing Class, and do exceed all the Angles in the following Class.* 59.

Between any two Classes there is given an infinite Series of Classes; all of which have the same Property, that any Angle of one is infinitely small with respect to the Angles of the foregoing Class, i. e. is exceeded by them all, *and infinitely great with respect to the following Class,* all the Angles of which it exceeds. 60.

The Curves $ax = yy$ and $bbx = z^3$ form different Classes; because the Dimension of the Ordinates z^3 in the second exceeds by Unity the Dimension y^2 of the first Curve; but I will shew that the Classes differ, how little soever these Dimensions differ, whence will appear what was propos'd: because Numbers without end may be given between these Numbers 2 and 3, and any others, which differ from one another, none of which, how little soever different, can be given, between which other Numbers again without end cannot be given. 61.

Let ax be $= yy$ and $g\,1\frac{1}{10}\,x = s^{\frac{21}{10}}$; i. e. $g^{\frac{11}{10}}x = s^{\frac{21}{10}}$, s denotes the Ordinates; and g a constant Line, as long as the Curve is not chang'd. As a is to g, so let $g^{\frac{1}{10}}$ be to a fourth Quantity, which let be call'd $b^{\frac{1}{10}}$; therefore $g^{\frac{11}{10}} = ab^{\frac{1}{10}}$; by multiplying the Equation $ax = yy$ by $b^{\frac{1}{10}}$, there is given $ab^{\frac{1}{10}}x = g^{\frac{11}{10}}x = y^2 b^{\frac{1}{10}} = s^{\frac{21}{10}}$; whence I deduce $s^{\frac{1}{10}}. b^{\frac{1}{10}} :: yy . ss.$ Therefore near the Point A, where s is necessarily less than b which is determin'd, y will be also less than s; whence follows what was said of the Angles.

It is also deduc'd from the Consideration of mean Proportionals, that there are given intermediate Classes *in infinitum*, differing also *in infinitum*, between any two Classes of Quantities, which differ *in infinitum*. 62.

If A is infinitely great with respect to a, any mean Proportional b, between these Quantities, is less than A, and greater than a, yet has it not a finite Ratio to A or a; for the Ratio of A to a is compounded of the Ratios of A to b, and b to a, and a Ratio compounded of two finite Ratios is also finite; therefore as A and a differ *in infinitum*, the Ratio between A and b, or b and a, exceeds every finite Ratio; wherefore it is also infinite. Mean Proportionals between two Quantities may be given *in infinitum*.

SCHOLIUM IV.

Concerning the Subtilty of Parts.

THE Weight of the Gold for gilding the Silver, mentioned in N° 28. is $\frac{1}{60}$ of the Weight of the Silver. When the Weights are equal, a Piece of Gold is to a Piece of Silver, as 10 to 19, therefore the Bulk of the Gold with which the Silver is cover'd to the Bulk of the Silver, as 1 to 114. for 10 : 19 : : 60 : 114. 63.

A

A Cubic Foot of Water weighs $63\frac{1}{2}$ Pounds, Silver is ten times heavier; therefore a cubic Foot of Silver weighs 635 Pounds.

A Cube is to a Cylinder of the ſame Diameter and Height, nearly as 14 is to 11; therefore the Weight of a cylindric Foot of Silver is 499 Pounds, 7984 Ounces.

One Ounce is ſtretched out into a Wire of 14000 Feet, and in a cylindric Foot there is contain'd a Wire of 111776000 Feet in Length, *i. e.* there are ſo many Wires each one Foot long.

The Surfaces of Circles are as the Squares of their Diameters, therefore the Square of the Diameter of the Wire to the Square of one Foot is, as 1 to 111776000; the Roots of which Numbers are 1 and 10572, in which Ratio the Diameters mentioned are: therefore the Diameter of the Wire is $\frac{1}{10572}$ of a Foot, or $\frac{1}{881}$ of an Inch. Gold is put round it, and the Bulk is increas'd $\frac{1}{114}$, *i. e.* the circular Section of the Wire is ſo much increas'd, which will happen if a Plate is put round the Wire, whoſe Thickneſs is a fourth Part of the $\frac{1}{114}$ Part of the Diameter; for the Area of the Circle is found by multiplying the Circumference by the fourth Part of the Diameter.

Therefore the Thickneſs of the Gold is $\frac{1}{456}$ of the Diameter of the Wire, which is $\frac{1}{881}$ of an Inch, ſo that the Thickneſs of the Gold is $\frac{1}{401736}$ of an Inch.

Theſe ſmall gilt Wires are made flat, that they may be wound about ſilken Threads, whereby their Surface is at leaſt three times as great as before, and the Thickneſs of the Gold is diminiſh'd in the ſame Ratio, ſo that it is $\frac{1}{1205208}$.

The Wire is not equally gilt in all Parts, and the Thickneſs of the Gold is in ſome Places perhaps twice as ſmall as in others, wherefore I ſhall not at all exceed the Truth, if I lay down the Thickneſs, where it is leaſt, $\frac{1}{2000000}$ of an Inch; *i. e.* the thouſandth Part of an Inch is divided into two thouſand Parts.

Gold is thus actually divided; and therefore the Particles, which are ſeparated by Art, have no greater Diameter, and in a golden Sphere of one Inch Diameter there are 8.000.000.000.000.000.000. ſuch Parts; and in a very ſmall Grain of Sand, *viz.* whoſe Diameter is the hundredth Part of an Inch, there are 8.000.000.000.000. ſuch Particles: therefore one of theſe Particles is to the Grain of Sand, as this is to a Globe, whoſe Diameter exceeds 16 Feet, and this Globe would not contain a greater Number of theſe Grains of Sand, than the Grain of Sand contains of ſuch Particles. But this Globe contains 4096 Globes of one Foot Diameter.

CHAP.

CHAP. V.

Concerning the Cohesion of Parts; where I shall treat of Hardness, Softness, Fluidity, and Elasticity.

ALL Bodies that are perceived by our Senses, consist of very small Parts, no one of which is indivisible in itself; but all of them are in respect to us: For all the Division we can make, is only a Separation of Parts. 64.

If a great Force is required to make such a Divison, or a Separation is made upon a small Motion of the Parts, so that the Body is broke upon a small bending, it is called a *hard* Body.

If the Parts yield more easily and fall in by being pressed, such a Body is said to be *soft*.

But this great and less Force, in the common Signification, determine nothing; for a Body that is hard, in respect to one Man, seems soft to another.

DEFINITION 1.

A Body is said to be hard, in a philosophical Sense, when its Parts mutually cohere, and do not at all yield inwards, so as not to be subject to any Motion in respect to each other, without breaking the Body. 65.

We know of no such perfectly hard Body, but Bodies are said to be harder, as they approach nearer to this perfect Hardness. 66.

DEFINITION 2.

A Body is said to be soft, in a philosophical Sense, when its Parts yield inwards, and slip in upon one another, even though it may require a Blow with a Hammer to do it. 67.

DEFINITION 3.

A Body whose Parts yield to any Impression, and by yielding are easily moved, in respect to each other, is called a Fluid. 68.

All these Things depend upon the Cohesion of Parts; the closer a Body is, the nearer it approaches to Hardness. 69.

But the Hardness of the smallest Parts does not differ from their Solidity; it is an essential Property of a Body flowing from its Nature, 70.

ture, which is no more to be explained, than why a Body is extended, or a Mind thinks.

71. I do not know whether all Bodies conſiſt of Parts that are equal and alike: And there are alſo ſeveral Things very difficult, in relation to the Cauſe of the Coheſion of the ſmall Parts of Bodies.

The Laws of Nature, which are admitted here, are deduced from Phænomena.

72. *It is a particular Law of Coheſion, that all the Parts have an attractive Force*; i. e. if they are near to each other, they tend to one another ſpontaneouſly; the Cauſe of which Motion we do not know, but as we obſerve this Motion generally to take place, and that all
* 7. Particles are ſubject to it, we rank it among the Laws of Nature *.

Definition 4.

73. *By the word Attraction I underſtand, any Force by which two Bodies tend towards each other*; though perhaps it may happen by Impulſe. We give this Name to the Phænomenon, and not the Cauſe of it.

We do not here alter the common Signification of this Word. For we generally ſay, that a Body is moved by Attraction as often as it tends to another Body, if the Preſence of this laſt is required to produce this Motion. In this ſenſe we ſay, that a Loadſtone draws Iron, that a Man draws towards himſelf a Body, which is tied to a Cord, and tends towards him, by his Action. Upon this account
74. in many Caſes we don't ſcruple to refer thoſe Motions to *Attraction*, in which there is a manifeſt Impulſe; we expreſs the Effect itſelf and nothing elſe, by the word Attraction, without having any regard to the Cauſe of it. *But we refer that Attraction alone to the Laws of Nature, which we find to obtain in the ſmalleſt conſtituent Parts of Bodies.*

75. But *this Attraction of the ſmalleſt Particles is ſubject to theſe Laws; That it is very great, in the Contact of the Parts; and that it ſuddenly decreaſes, inſomuch that it acts no more at the leaſt ſenſible Diſtance; nay, at a greater Diſtance, it is changed into a repellent Force, by which the Particles fly from each other.*

By Help of this Law, ſeveral Phænomena are very eaſily explained and that Attraction and Repulſion is fully proved by a vaſt Number of chymical Experiments. That there is ſuch a Thing appears alſo from the following Experiments.

Ex-

EXPERIMENT 1.

We ſee that in all fluid Bodies all the Parts attract one another, from the ſpherical Figure that the Drops always have; and alſo becauſe there is no Liquor whoſe Parts do not in a manner ſtick together, which is evidently true even in Mercury itſelf.

EXPERIMENT 2.

But this mutual Attraction of Particles is much better proved; 77. because in all Liquids, two Drops, as A and B, as ſoon as they Plate I. touch one another ever ſo little, they immediately run into one Fig. 2. large Drop, as F. All which things, as they alſo happen in liquified Metals, it follows, that the Parts of which they are compounded do attract one another, even when they are ſeparated by the Motion of the Fire.

This Motion is to be attributed to a Force acting either upon the 78. external Surface of the Drop, or upon each of the ſmall Particles, of which the Drop is made.

It is manifeſt that it can't be aſcribed to a Force acting upon the Surface, unleſs we ſuppoſe the Preſſure to be equal upon every part; but I have deduced from the Laws of the Preſſure of Fluids, *in Chap.* 3. *Book* II. that the Figure of the Drop cannot be changed by ſuch a Preſſure.

For it appears at firſt View that the Preſſure upon the Surfaces Plate I. *a b* and *c d* of the oval Drop *a b c d* exceeds the Preſſures upon the Fig. 3. Surfaces *a c*, *b d*, if the Drop has every part equally preſſed. Yet the Drop can't become round, without the leſſer Preſſures overcoming the greater, which is abſurd.

Therefore the Force acts upon every one of the ſmall Particles. Whereby they each either tend to thoſe next to them, or are removed from them: They are not ſeparated; therefore the Motion can only be aſcribed to that Action, whereby the Particles tend each to thoſe next to them, which Motion we call Attraction *. * 73.

This being laid down, the greater the Number is of the Particles 79. which attract one another between two Particles, the greater is the Action with which they are carried towards one another; which produces a Motion in the Drop, 'till the Diſtances between the oppoſite Points in the Surface become every where equal; which can only happen in a ſpherical Figure.

He, who ſhould diſcover the Cauſe of this Attraction, would do great ſervice to Phyſics; I only affirm that it is, and that it is the immediate Cauſe of Coheſion: And I deduce its Univerſality from
* 10 11. what is ſaid before *.

80. Several Bodies act upon other extraneous Bodies by this Attraction, by an immediate Application of Parts. I ſhall give a few Examples, in which the Effects of it are moſt remarkable.

Experiment 3.

81. Plate III. Fig 1. Having ſcraped two leaden Balls a little, ſo as to make two ſmall, plain, and bright Surfaces, if we apply theſe to each other, and bring the Parts cloſe by Compreſſion, upon account of the Softneſs of the Metal, we ſhall have a ſtrong Coheſion between the Balls, which will be ſo much the greater, as more Parts touch one another; and how ſmall ſoever it is, yet will it always very much exceed that ſmall Coheſion which may be attributed to the Preſſure of the Air.

Experiment 4.

82. Plate III. Fig. 2. The ſmall Glaſs Tubes, *tt*, *tt*, *tt*, open at both Ends, are immerg'd in Water, as they are repreſented in the Figure. The Water aſcends ſpontaneouſly into them; and ſo much the higher, as the Diameter is leſs. If the Tubes are very ſmall, they are called Capillary Tubes; but the Experiment will alſo ſucceed in the larger, whoſe Diameters, for Example, are equal to the ſixth Part of an Inch.

It is plain from the following Experiment, that this Effect is not to be attributed to the Preſſure of the Air.

Experiment 5.

83. Plate III. Fig. 3. The ſmall Tubes *tt*, *tt*, &c. are fixed to a piece of Cork; which Cork is faſtened to a ſmall braſs Wire AE, paſſing through the Hole O of the Glaſs Receiver R; then the Air is exhauſted from the Receiver R by help of an Air-pump. The Receiver being placed on the Air-pump Plate, the Tubes are immerged in the Water, contained in the Glaſs CD, by moving the Cylindric braſs Wire AE; in this Caſe the Water aſcends into them juſt in the ſame manner as in the foregoing Experiment. How the Wire may be moved without letting in the Air, ſhall be ſhewn in what follows.

Ex-

EXPERIMENT 6.

The two Glass Planes ABCD touch one another at AB, but are a little separated at CD, by thrusting some thin Plate between them; their Edges CB are immerged in some colour'd Water in such manner, as to have their Sides AB and CD kept vertical; the Planes being moisten'd first in the Inside with the same Liquor. 84. Plate III. Fig. 4

The Water rises between these Planes by their Attraction, and it rises highest where the Planes are nearest together; and as their Distance continually decreases from CD to AB, the Water rises up to different heights in every Place, and makes the Curve Line *efg*.

EXPERIMENT 7.

ABCD are two Glass Plates, or Planes, touching one another at AB, but a little separated at CD, by thrusting a thin Plate of any kind between them. They are sustained by a wooden Frame. The Plates may be inclined to the Horizon, by raising the End AB, where they are joined. This is performed by means of the Screw HI; whose exterior Part is the Solid L, joined to the Frame, and by moving the Screw the Inclination of the Planes may be changed at pleasure. 85. Plate I. Fig. 4.

A Drop of Water or Oil, G, is put between the Planes, so as to touch both the Planes, which must before-hand be moisten'd with the same Liquor; this Drop is attracted by both Planes, but the Attraction has a greater Effect upon the Drop, where their Distance is the less: that is, a greater at *e* than at *f*, therefore the Drop is moved towards *e*, that is, ascends, and moves upwards the faster, in proportion as it is higher, the Surfaces of the Drop that touch the Glasses growing larger, where the Distance between the Planes is diminished. The Angle of Inclination of the Planes may be so increased, that the Gravity of the Drop shall balance the Attraction, and then the Drop is at rest; and in that Case, if you raise the End AB of the Planes still higher, the Drop will descend by its Gravity, which will then overcome the Attraction.

EXPERIMENT 8.

Quicksilver unites itself to Gold and Tin; also Water and Oil stick to Wood and clean Glass. 86.

87. We have Inftances of Repulfion between Water and Oil, and generally between Water and all unctuous Bodies; between Mercury and Iron; as alfo between the Particles of any Duft.

Experiment 9.

88. Plate I. Fig. 5, 6. If a Piece of Iron is laid upon Mercury, its Surface is depreffed about the Body immerfed, as it is reprefented about the Balls A and B *(Fig. 6.)* and after the fame manner, where Attraction obtains, the Surface of the Liquor is higher about the floating Bodies, as about the Balls A and B *(Fig. 5.)* and does not run to a Level by its Gravity; fo here, where the repellent Force exerts its Action, Fluids, notwithftanding their Gravity, do not run down to fill up the Cavities which are made round the floating Bodies.

89. But we muft not attribute this Cavity to the Attraction of the Particles of Mercury. I confefs, fuch a Cavity may be formed by this means, tho' the Mercury fhould be attracted by Glafs, if the Attraction were lefs than that, whereby the Particles of Mercury attract each other. But it appears from the following Experiment that this is not the Cafe.

Experiment 10.

90. Plate III. Fig. 5. Let Mercury be pour'd into a recurve glafs Tube BAE, having one of its Legs narrow, not exceeding in Diameter E, the twelfth Part of an Inch, the other Leg BC being wider. When the Mercury is at reft, the Height in the two Legs will be unequal, and greater in the wider Leg.

Who does not fee that the Depreffion of the Mercury in the narrower Leg can't be attributed to Gravity? Neither can it be attributed to the Attraction of the Parts, whereby the Weight can't be alter'd. Therefore this Effect depends upon the Action of the Glafs, whofe Attraction raifes Water, and whofe Repulfion depreffes Mercury.

91. Plate I. Fig. 5. We muft attribute to Attraction and Repulfion the Phænomena of Balls floating upon Fluids; when they attract the Fluid, it afcends all round them, as at *f, g, h, c, (Fig. 5.)* when they repel the Fluid, it is made hollow on every fide, as at *f, g, h, c, (Fig. 6.)* *.

* 88.

If in the Veffel, in which the Experiments are made, the Fluid is attracted by the Sides of the Veffel, it is fuftained all over, and is higher near the Sides, as at *e l, (Fig. 5.)* when the Veffel is fo filled, that

that the Fluid runs down from all Parts, then, by the mutual Attraction of the Parts, it ſtands higher in the middle than at the ſides, and forms the Convex Surface C B D in the Veſſel A. Plate I. Fig. 7.

From theſe Principles only are the following Experiments explained.

EXPERIMENT 11.

When a glaſs Veſſel is not quite full of Water, a glaſs Bubble always runs to the Side, and there remains, provided it be not laid on too far from it. The Bubble is equally preſs'd every way by the Water, but when it comes to the Side of the Veſſel, ſo that the two Elevations *e*, *f*, concur, the Force, by which the Water is rais'd there, does in part take off the Preſſure; and the oppoſite Preſſure overcomes; but the Elevations are extended more than appears. After the ſame manner two Bubbles at a certain Diſtance mutually draw near one another. 92. Plate I. Fig. 5.

EXPERIMENT 12.

When the Glaſs is ſo full as to be ready to run over, the Bubble goes off of itſelf from the Side to the Middle of the Glaſs; becauſe the Force, by which the Water is rais'd in the Middle, diminiſhes the Preſſure from that Part. 93. Plate I. Fig. 7.

EXPERIMENT 13.

Two Iron Balls alſo, laid upon Mercury in a Glaſs, mutually attract each other. And theſe are likewiſe carried towards the Sides of the Glaſs: the Reaſon of both Phænomena is this; there are Cavities round the Balls, and the Sides of the Glaſs *; where the Cavities are join'd, the Preſſure is diminiſh'd, and both Balls are carry'd towards that Part. 94. * 91.

We have alſo notable Inſtances of Attraction in Cryſtalliſations. 95.

DEFINITION 5.

That Property of Bodies is call'd their Elaſticity, by which they return of themſelves to their former Figure, when it has been alter'd by any Force, and this Force ceaſes. 96.

If any compact Body be dented in without the Parts falling into that Dent, it will return to it's former Figure, from the mutual Attraction of it's Parts.

It

97. It ſhall be alſo ſhewn in it's proper Place, that that Property of the Air, which is call'd it's *Elaſticity*, ariſes from the Force whereby it's Parts repel one another.

98. And leſt any one ſhould imagine, becauſe we don't give the Cauſe of the ſaid Attraction and Repulſion, that they muſt be look'd upon as occult Qualities: We ſay here with Sir *Iſaac Newton*, 'That we conſider thoſe Principles not as *occult Qualities*, which 'are ſuppos'd to ariſe from the *ſpecifick Forms* of Things; but as '*univerſal Laws* of Nature, by which the Things themſelves are 'form'd: For the Phænomena of Nature ſhew that there are 'really ſuch Principles, tho' it has not yet been explain'd what 'their Cauſes are. To affirm that the ſeveral Species of Things 'have *occult ſpecifick Qualities*, by which they act with a certain 'Force, is juſt ſaying nothing. But from two or three Phænomena 'of Nature to deduce general Principles of Motion, and then explain in what Manner the Properties and Actions of all Things 'follow from thoſe Principles, would be a great Progreſs made in 'Philoſophy, tho' the Cauſes of thoſe Principles ſhould not yet be 'known.'

SCHOLIUM I.

Concerning the Effect of the Attraction of Glaſs upon Water, conſider'd in general.

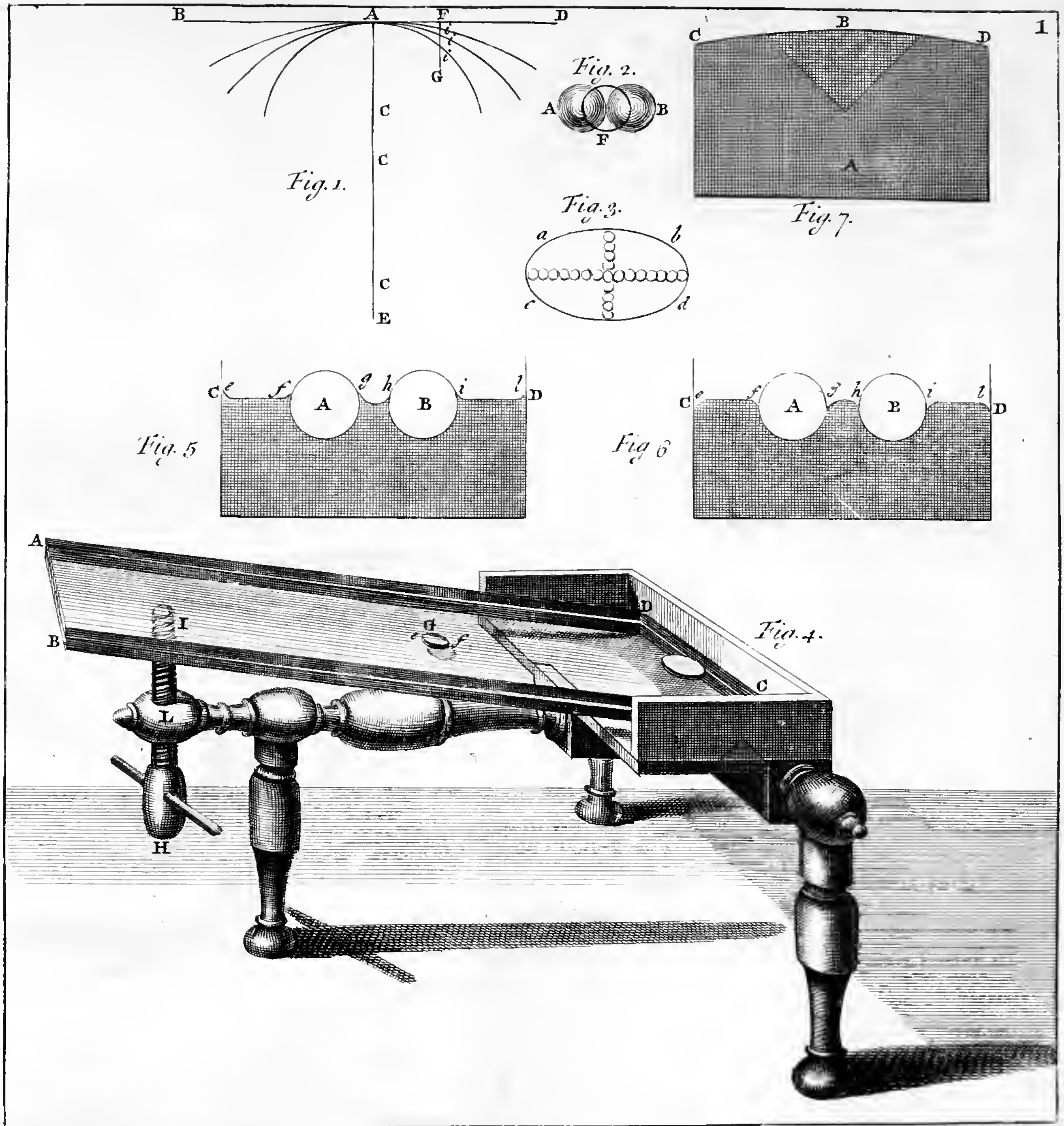
99. ALL the aqueous Particles are attracted by the Glaſs, at a ſmall Diſtance from it; *i. e.* they tend in right Lines towards all the Particles of the Glaſs, whoſe Diſtance is not greater than the Power of Attraction between the Glaſs and Water reaches. Let the Surface of the Glaſs be A B; a Particle C; this tends towards the Glaſs in the Line C D, perpendicular to the Surface; it alſo tends towards the Point *e*, and at the ſame time it tends with equal Force towards all the Points in the Surface that are equally diſtant from D with *e*, i. e. thoſe that are placed in the Circumference of a Circle, whoſe Diameter is *e f f:* by reaſon of the Equality of all theſe Forces the Point cannot be carried more to one Point than to another; therefore, as all theſe Forces act together, the Particle is drawn in the Line C D. By applying a like Demonſtration to other Particles of the

Plate II. Fig. 5.

100. Glaſs, acting upon *the Particle of Water*, it will appear, *that it tends to the Glaſs in a Line perpendicular to it's Surface.*

Plate II. Fig. 6.

Let there be upon the Glaſs Plane A B the Drop G. All the Particles, near the Glaſs, tend directly to it, and draw the Particles with which they cohere along with them; whence there ariſes a Motion in the Drop like to that, which would be in it, if it were preſs'd towards A B, by a Plane C D parallel to it: for the Effect of this Preſſure agrees with the Effect of Attraction;


1
Fig. 1.
B A F D
G
C
C
C
E
Fig. 2.
A B F
Fig. 3.
a b c d
Fig. 7.
C B D
A
Fig. 5
C e f g h i l D
A B
Fig 6
C e f g h i l D
A B
Fig. 4.
A B I G f D C L H

J. Mynde sc.

Attraction; but by this Preſſure the Drop would expand itſelf *every way*; therefore this *Expanſion is alſo the Effect of Attraction.* 101.

Let A B be the Surface of the Water; let a Part of the Glaſs Plane 102.
F D, whoſe Thickneſs is here repreſented, be immerg'd perpendicularly Plate II.
in it. The Water is attracted by the Plane *, and endeavours to expand Fig. 7.
itſelf every way upon the Plane, as if it were preſs'd in the Direction B D †. * 100. † 101.
By this Motion the Particles are driven only towards D, the contrary Motions beneath the Surface deſtroying one another; therefore the Water will be rais'd, and the aſcending Water will be follow'd by that, which coheres with it, and will be ſuſtain'd by the Glaſs ſo, that the Weight of the Water rais'd will be equal to the Force by which it is rais'd.

Let this Height be D C, which we repreſent higher than it would really be; but the Water in C D G is ſuſtain'd by that Force alone, by which the Particles are driven upwards towards C: for where the Water is at reſt, the Forces, by which the Water between C and D endeavours to expand itſelf *, mutually deſtroy one another: for Example, a Particle at * 101.
e is preſs'd equally upwards and downwards. Therefore the Force which 103.
ſuſtains the Water, is in *the Proportion of the Breadth of the Surface, along which the Water aſcends, to the Height of the Water, meaſur'd in a Line parallel to its Surface:* which ſame Ratio *the Weight of the Water rais'd follows.*

SCHOLIUM II.

Concerning Capillary Tubes.

WE ſee that Water riſes ſpontaneouſly in the ſmaller Glaſs Tubes *, * 82.
which how it is done now plainly appears. But *the Quantity of the Water ſuſtain'd*, follows the Ratio of the Circumference of the Surface 104.
of the Water rais'd *; and this Circumference, *in cylindric Tubes, immers'd* * 103.
perpendicularly, is increas'd or diminiſh'd with the Diameter of the Tube.

Let there be two Tubes having the Diameters D, *d*; the Heights of 105.
Water *in the Tubes* A, *a*; the Quantities of the Water rais'd will be to
each other as $D^q \times A$ is to $d^q \times a$ *; therefore $D^q \times A$, $d^q \times a$: : D, *d* †; * 2. 11. 14. El. XII.
by dividing the Antecedents by D^q, and the Conſequents by d^q we have, † 104.
A, *a* : : $\frac{1}{D}$, $\frac{1}{d}$, *i. e. the Heights are inverſely as the Diameters.*

SCHOLIUM III.

Concerning the Aſcent of Water between the Planes, of which Mention was made in N° 84.

LET A C, B C, be two Lines repreſenting an horizontal Section of 106.
the Planes in the Surface of the Water; let us ſuppoſe the Space, Plate II.
contain'd by the Angle A C B, to be divided by Lines as *d e*, *f g*, *h i*, Fig. 8.
l m,

l m, &c. at a ſmall and equal Diſtance from one another; it is manifeſt
* 103. that equal Quantities of Water are rais'd in the Spaces *d f e g*, *h i m l* *; and therefore that there are equal Priſms there, whoſe Heights are inverſely
* 34 El. XI. as their Baſes; but theſe Baſes, becauſe they may be look'd upon as Parallelograms, and have the Heights, *d f*, *h l*, equal, are to one another
* 1 El. VI. as *d e* is to *h i* *; which are as *d* C is to *h* C.

Plate III. Fig. 4. * *La Hire* S. C. *l.* IV. p. 2. † *Ibid. l.* v. p. 13. From this I deduce that the Curve *e f g* is an Hyperbola, whoſe Aſymptotes are the Lines A B, where the Glaſſes touch one another, and the Surface of the Water, B C *. Upon account of the right Angle A B C the Hyperbola is equilateral †; for I have examin'd the Caſe in which the Line, where the Glaſſes mutually touch one another, is perpendicular to the Surface of the Water.

The Height in the Tube is alſo eaſily compar'd with the Height between the Planes.

107. Plate II. Fig. 8. Let there be a Section M *of a cylindric Tube*, whoſe *Semi-diameter is equal to the Diſtance* e d *between the Planes.* It is manifeſt that the Force, which ſuſtains the watery Priſm, whoſe Baſe is *d e f*, follows the Proportion of the Line *d f*; for the Force, which ſuſtains the Parallelopiped, whoſe
* 103. Baſe is *d f e g*, is proportional to the two Lines *d f* and *e g* *.

In the Tube the Force, which ſuſtains the Priſm whoſe Baſe is *n o p*, is proportional to the Arc *n p*; becauſe the whole Circumference is proportional to that, which ſuſtains the whole aqueous Cylinder, contain'd in the Tube. If *n p* and *d f* are equal; the Forces are equal which ſuſtain the Priſms; and therefore the Priſms themſelves are equal; and in this caſe the Baſes *n o p*, *d e f*, are alſo equal, wherefore the Heights of the Priſms don't differ, and *the Water aſcends to the ſame Height both in the Tubes and between the Planes.*

The Experiment of the Aſcent of Water between the Planes may be made ſeveral ways.

108. Plate II. Fig. 9, 10. It would be tedious and uſeleſs to conſider all that relates to this Matter; it is ſufficient to have examin'd the principal Caſe; I ſhall only obſerve in two other Caſes in which the Angle A B C, which the Line, where the Glaſſes are join'd, makes with the Surface of the Water, is either an acute or obtuſe one, the Glaſs Planes remaining perpendicular to the Surface of the Water, that the Water is alſo terminated by an hyperbolical Line, one of whoſe Aſymptotes is the Surface of the Water: the other is had by drawing B F perpendicular to C B, in the Point B; the Aſymptote requir'd will be B E; which divides F D into equal Parts, perpendicular to B F in any Point, and terminated by the Line B A.

If D F paſſes thro' the Point D of the Hyperbola, B F will be the Semi-diameter conjugate with the Semi-diameter B D.

In *Fig.* 10. the Hyperbola is not continued beyond F; yet the Water aſcends higher, but is terminated by another Curve.

In *Fig.* 9. tho' the Hyperbola cuts the join'd Sides of the Glaſs Planes in D, the Aſcent of the Water is not terminated there, but the Curve is alter'd from the Hyperbola, at a certain Diſtance, and different from A B, according

according to the different Angle made by the glass Planes, and the Ascent is continued along B A. For *where there is but a small Distance between the glass Planes the opposite Attractions mutually help one another*, whereby the Ascent of the Water is increas'd. A like Increase of Action is mention'd in the following Number; and it also takes place in Light attracted by Bodies, as I observe in the last Number of *Chap.* 1. *Book* IV. 109.

SCHOLIUM IV.

Concerning the Motion of the Drop mention'd in N° 85.

LET us suppose the Planes, between which the Drop is mov'd, to be cut by another Plane, perpendicular to the said Planes, and to the Line which joins them: this Section is represented by the Figure; but as the Motion of the Drop depends upon the Inclination of the Planes to each other, I may represent this greater than what it really is, as likewise the Distance between the Planes, and the Distance at which the Glasses act upon the Oil. 110. * Plate II. Fig. 11.

Let A B, C D, be the two Planes; *e e f f* the Drop; *g h* the Distance at which the Glass attracts the Oil: All the Oil therefore between *e i h f* is drawn towards the Plane, and endeavours to expand itself every way upon the Plane*; but it cannot do so, by reason of the Coherence of the Parts of the Drop, and the opposite Forces at *e* and *f*, which destroy each other; and the Drop would not be mov'd, if the Planes were parallel. But now, because the Direction of the Action of the Attraction is perpendicular to the Glass, the Oil, in the Space *f l h*, is attracted by the Surface *f g*; and yields, because this is destroy'd by no contrary Action; by which Motion the whole Drop is mov'd, whose Parts cohere together. Therefore the Drop tends towards that Part in which the glass Planes meet, as long as the Inclination of the Planes towards the Horizon is such, that the Force of Gravity, whereby the Drop endeavours to descend along the Plane, is less than its Attraction, by which it is carried upwards.

* 101.

But when the Distance between the Planes is small, the opposite Attractions help each other, and the Force is increas'd in a greater Proportion than the Diameter of the Drop; which Increase in the Ratio of the Diameter, may easily be deduc'd from what is demonstrated above.

CHAP. VI.

Concerning Motion in general, where I ſhall ſpeak of Place and Time.

111. *Motion is a Tranſlation from one Place to another Place, or a continual Change of Place.* Every Body has an Idea of it, which is ſimple, and can't be explain'd by Words.

112. *Place is the Space taken up by a Body*, whoſe Idea is alſo ſimple. It is twofold, true or abſolute, and relative.

DEFINITION 1.

113. *True Place is that Part of immoveable Space, which a Body takes up.*

DEFINITION 2.

114. *Relative Place*, which only can be diſtinguiſhed by our Senſes, is *the Situation of a Body in reſpect of other Bodies.*

True Place is often chang'd, whilſt relative Place is not, and ſo reciprocally.

115. Whence ariſes a *true and abſolute Motion, and another Sort call'd a Relative Motion.*

Whilſt a Body moves, Time goes on.

116. *Time* alſo *is twofold*; *true or abſolute, and relative.*

117. *True* Time has no Relation to the Motion of Bodies, nor to the Succeſſion of Ideas in an intelligent Being, but *by its Nature always flows equally.*

DEFINITION 3.

118. *Relative Time is Part of the true Time meaſur'd by the Motion of Bodies*, this is perceiv'd by a Succeſſion of Ideas.

All Motion may become ſwifter, as likewiſe a Body may move ſlower than it did before; a Succeſſion of Ideas alſo admits Acceleration and Retardation; whence it follows, that relative Time differs from true Time, for this laſt never flows faſter, or ſlower.

DEFINITION 4.

119. *That Affection of Motion, by which a Body runs thro' a certain Space in a certain Time, is call'd Celerity or Velocity*; which therefore

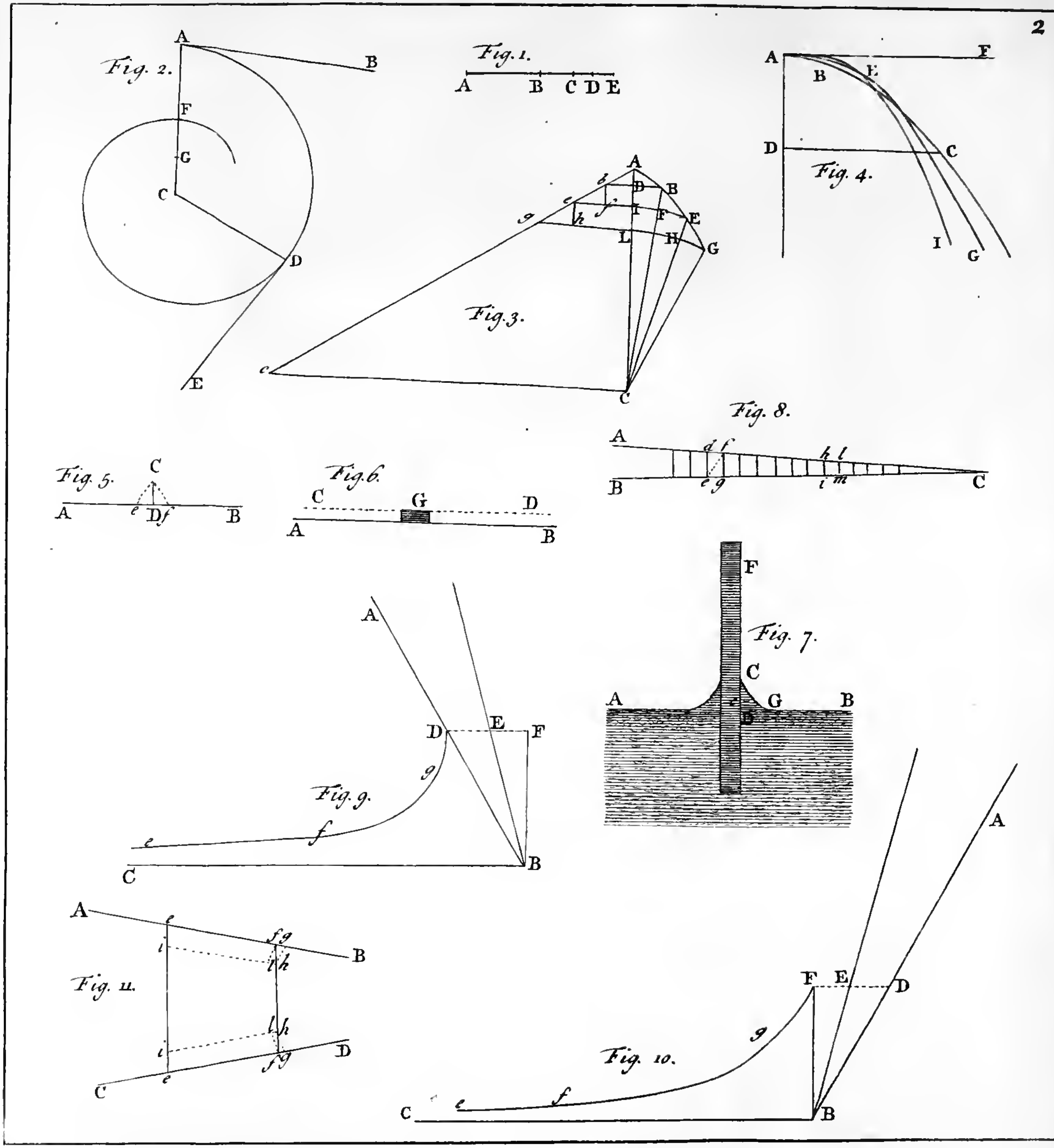


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.

fore is greater or leſs, according to the Largeneſs of that Space, *and is always proportional to the Space.*

The Space run thro' is alſo *increaſed with the Time, if the Velocity is the ſame.* 120.

Therefore, in general, the Space paſs'd thro' is in the Ratio compounded of the Time and Velocity. 121.

Several Bodies being given, if in each of them the Velocity be multiplied by the Time, the Products will be as the Spaces paſſed through.

Definition 5.

The Direction of Motion *is in a right Line, which we ſuppoſe drawn towards the Place whither the moving Body tends.* 122.

Definition 6.

A Power, or Preſſure, is a continued Force acting upon a Body to remove it from its Place, and which can exert an Action upon a Body, when it is at reſt, or when the Motion impreſs'd on it is not chang'd. Namely, if the Action of the Preſſure is deſtroy'd by a contrary Preſſure. 123.

The Preſſure can act in a Place, whereby it is diſtinguiſh'd from the Action of a Body acting by its innate Force, which Action is always from one Place to another.

BOOK I.

Part II. Concerning the Actions of Powers.

CHAP. VII.

How to compare the Actions of Powers.

124. IT appears, at firſt View, that thoſe *Preſſures*, i. e. Actions of Powers, *are equal*, which *produce equal Effects in equal Times*.

No one will doubt of one Preſſure being able to overcome another contrary to it. If it is not an Axiom, it may eaſily be deduc'd from
125. the Propoſition foregoing, *That equal Preſſures, acting in a contrary Direction, mutually deſtroy each other, and that thoſe are equal which do deſtroy each other*.

126. Whence it appears, *That the Preſſures are to one another as the Effects produc'd in equal Times*.

127. *If an Obſtacle is preſſed, and* does not leave its Place, *the Preſſure* is deſtroy'd *by a contrary Preſſure*; otherwiſe it would produce no Effect. If therefore it *is not deſtroy'd* by a contrary Preſſure, *the Obſtacle yields*. Here we are not to conſider that Force which is communicated to the Obſtacle upon certain Occaſions, whereby it
* 18. 128. continues in Motion *: *I ſpeak only, in this whole ſecond Part, of the Tranſlation, which is the immediate Effect of the Preſſure*, and which always takes place only in the firſt infinitely ſmall Part of Time, when the Obſtacle is moving by the Action of the Power.

129. When the Effect of the Preſſure, not deſtroy'd by a contrary Preſſure, becomes the Tranſlation of the Obſtacle, it follows, *That the Actions of different Powers*, not deſtroy'd by contrary Preſſures, can *only differ from one another in reſpect of the Obſtacles upon which the Powers act, and in reſpect of the Spaces paſs'd through by the Obſtacles in a certain Time*.

De-

Definition.

I call the Magnitude of the Preſſure, when it is conſider'd with 130.
relation to the Action upon a quieſcent Obſtacle, but permitted to move, i. e. the Capacity of acting, when the Preſſure is not deſtroy'd by a contrary Preſſure, *the Intenſity of the Power.*

Therefore *the Intenſities of the Powers are, as the Actions upon the* 131.
Obſtacles, which are remov'd by the Preſſures.

If the Obſtacles paſs through equal Spaces in equal Times, the Inten- 132.
ſities of the Powers are as the Obſtacles *. * 126. 129.

If the Powers act upon equal Obſtacles, the Intenſities of the Powers 131.
are as the Spaces through which the Obſtacles are carried in equal 133.
Times †. † 126. 129.

But *if both the Obſtacles and the Spaces paſs'd through by theſe in* 131.
equal Times differ, the Intenſities of the Powers are as the Obſtacles, 134.
and as the Spaces paſſed through *, that is, in a Ratio made up of * 132. 133.
theſe.

For Example, if the Action of one Power acts upon a double Obſtacle, and it be mov'd thro' a triple Space; the Action, and therefore the Intenſity of the Power will be twice triple, or three times double, that is, ſextuple. This compound Ratio is had, if, Numbers being given in a Ratio of the Obſtacles, and others in a Ratio of the Spaces paſs'd through, in every one of the Powers, the Obſtacle be multiplied by the Space paſs'd through by it, for the Products will give the Compound Ratio that is ſought.

If therefore Numbers are given, which expreſs *the Intenſities of* 135.
different Powers, theſe will be as the Products of the Obſtacles by the Spaces; if therefore each of the given Numbers *be divided by the Space paſs'd through by its Obſtacle, the Quotients will be as the Obſtacles themſelves.*

Therefore the Obſtacles are ſo much the greater, as the Intenſities are greater, and the Spaces paſs'd through leſs; that is, the Obſtacles are in a Ratio compounded of the direct Ratio of the Intenſities, and the inverſed Ratio of the Spaces gone through. 136.

If the Numbers, that expreſs the Products of the Obſtacles by the Spaces, that is, that expreſs the Intenſities of the Powers, be
each divided by the Numbers, which denote the Obſtacles, the 137.
Quotients will be as *the Spaces;* which therefore *are directly as the Intenſities, and inverſely as the Obſtacles.*

The

138. *The Intensities of the Powers are equal, if the Spaces paſs'd thro' are in an inverſe Ratio of the Obſtacles.* Foraſmuch as one Power exceeds another in reſpect of the Obſtacle, ſo much is it exceeded in reſpect of the Space paſs'd through. For Example, if the Obſtacles are as Eight and Six, the Ways paſs'd through as Three to Four, each of the Intenſities will be expreſs'd by the Number
* 134. Twenty-four *.

All theſe Things relate to Actions upon Obſtacles left to themſelves, and which reſiſt by their *Inertia* only.

I muſt now ſpeak of Preſſures mutually deſtroying one another. This only happens to contrary Preſſures, and theſe are contrary, when one reſiſts another, and becomes an Obſtacle in reſpect of it.

139. In this Caſe *equal* Preſſures mutually deſtroy one another *: but
* 125. there is this Equality when *oppoſite Preſſures equally reſiſt.* For each acts upon the oppoſite Preſſure by its Reſiſtance. Theſe Reſiſtances are determin'd, firſt, if we conſider the Intenſities; for the Reſiſtances are as the Intenſities, when the Circumſtances are the ſame:
140. for *the Intenſity of the Power being chang'd,* if other things are not alter'd, *the Force by which it reſiſts will be chang'd in the ſame Ratio.*

But Secondly, whilſt a Preſſure is overcome, and the Point, to which it is applied, is remov'd to a certain Diſtance, ſome determinate Action is requir'd, that this may be done in a certain Time; this muſt be doubled, if the ſame Thing is to be done twice in the ſame Time; that is, if the Point muſt be remov'd to a double Diſtance, in the ſame time. Then the Preſſure which is overcome, is twice overcome after the ſame manner, and twice reſiſts, that is,
141. its Reſiſtance is double, therefore *the Reſiſtance of a Power, whoſe Intenſity is not chang'd, increaſes, as the Space paſs'd through, in a certain time, by the Point to which it is applied.*

142. *And the Actions of different Powers, by which they reſiſt contrary Preſſures, are to one another in a Ratio made up of the Intenſities of the Powers, and the Spaces, which may be paſs'd through, in the ſame*
* 140. 141. *time, by the Points, to which theſe Powers are applied* *.

143. From theſe I deduce that *Preſſures whoſe Intenſities are equal, acting in a contrary Direction, deſtroy each other in this Caſe only, when the Points, to which they are applied, if they are ſuppoſed to be*
* 141. *in motion, paſs through equal Spaces* *.

And

And *these Spaces being put equal, they will not destroy one another, if the Intensities differ* *. 144.

* 140.

But Powers, whose Intensities differ, may exert equal Pressures, if they are applied to Points, which, being mov'd, pass through unequal Spaces, and in such manner, that as much as one Resistance exceeds the other in Intensity, so much must it be exceeded in respect of the Way to be pass'd through *, in which Case the Inequalities are compensated.

* 143.

Therefore *opposite Pressures* are equal, and *destroy one another, if the Intensities of the Powers are inversely, as the Spaces to be pass'd thro' by the Points, to which they are applied, in the same time, supposing them to be in Motion.* 145.

We may determine in general from the same Premises what is requir'd, *that many Powers acting on one Part, whilst one or more act in a contrary Direction at another Part, may destroy them.* 146.

The Intensity of each of them must be multiplied by the way to be pass'd thro', in a certain Time, by the Point, to which it is applied; and the Products will be to one another as the Actions of each of the Powers, whereby they resist contrary Pressures. Now *if the Sum of the Products of one Part be equal to the Sum of the Products of the other*, the opposite Resistances will be equal, and *the opposite Actions will mutually destroy one another.*

CHAP. VIII.

General Things concerning Gravity.

PHÆNOMENON 1.

ALL Bodies near the Earth, if hinder'd by no Obstacle, are carried towards the Earth. 147.

DEFINITION 1.

The Force, by which Bodies are carried towards the Earth, is call'd Gravity. 148.

DEFINITION 2.

That Force, in respect to a Body acted upon by it, is call'd the Weight of a Body. 149.

PHÆ-

Phænomenon 2.

150. *The Force of Gravity acts equally, and every Moment of Time, near the Earth's Surface.*

There is indeed a ſmall Difference of Gravity in different Countries, which we ſhall take notice of in *Chap.* 17. *Book* V. but it is too ſmall to be conſider'd here, eſpecially becauſe it is wholly inſenſible in neighbouring Countries.

151. When the Deſcent of a Body is hinder'd by an Obſtacle, it always preſſes the Obſtacle equally by its Weight tending towards the Center of the Earth; therefore *it may be look'd upon as a Power acting upon an Obſtacle*, and what has been demonſtrated concerning Powers in the foregoing Chapter, takes place here alſo.

Phænomenon 3.

152. *Bodies that deſcend by the Force of Gravity move equally ſwift, if all Reſiſtance is taken away.*

This Phænomenon appears from the following Experiment.

Experiment.

153. Several glaſs Cylinders are put upon the Air-pump (by help of which the Air is taken out of Veſſels) in ſuch manner, as to make one Cylinder ſix or ſeven Feet high, whoſe Diameter is four or five Inches. The Air is exhauſted; and a Piece of Gold and a Feather are let down from the upper Part of this exhauſted Veſſel, at the ſame Time; which come alſo to the Bottom exactly at the ſame Time.

154. If a little Air be let in, and the Experiment is repeated, there may be obſerv'd a Difference in the Deſcent, ariſing from the Reſiſtance of the Air.

I ſhall ſhew in *Part* I. *Book* III. where I ſhall treat of the Air-pump, and what relates to it, how this Experiment is to be manag'd, and how it may with Conveniency be repeated ſeveral Times.

This ſame Phænomenon is deduc'd from another Experiment alſo, to be mention'd in what follows.

155. Hence it follows, that all Obſtacles whatſoever are carried by Gravity thro' equal Spaces, in equal Times, by the immediate Action of Gravity; for it appears that Bodies are mov'd in the ſame manner in the firſt Moment of Time, and that they are accelerated in the ſame manner in every one of the following Moments; therefore the Actions of Gravity upon Bodies are as the Bodies

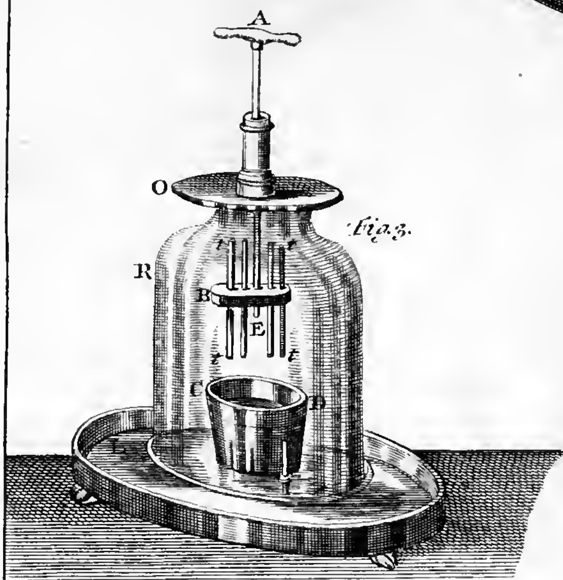


Fig.3.

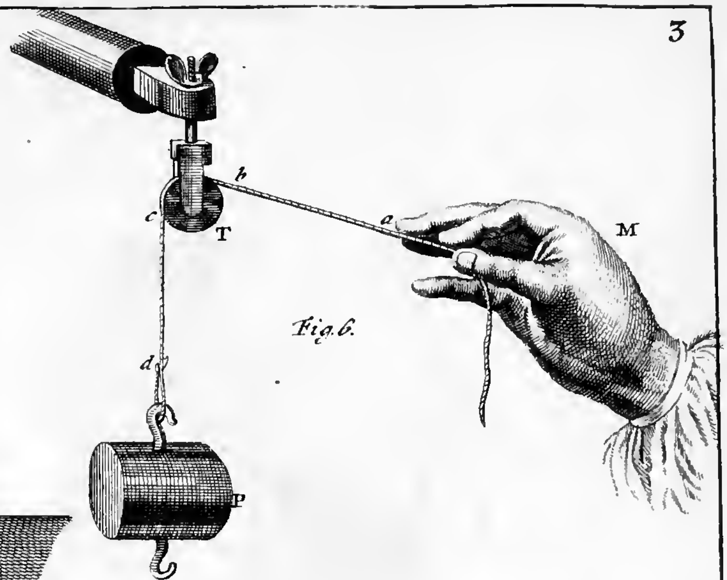


Fig.6.

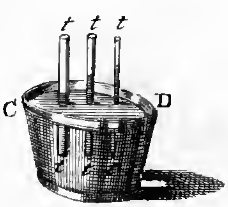


Fig.2.

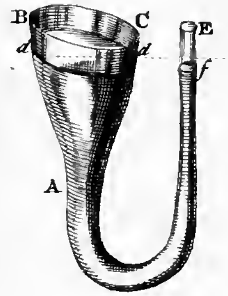


Fig.5.

Fig.1.

J. Mynde sc.

Bodies themselves *, that is, *the Weights are as the Quantities of Matter*; and every one of the equal Particles of Matter weigh equally, of whatsoever Body they are the Particles. * 132. 156.

When a Weight is consider'd as a Power, the Intensity of the Power is proportional to the Quantity of Matter, in the weighing Body, and the Direction of the Power is towards the Center of the Earth. 157.

It was necessary to make these Remarks on Gravity, because we make use of Weights in the Experiments about Pressures.

CHAP. IX.

Concerning certain Machines, which are made use of in several Experiments.

DEFINITION I.

A Single Pulley is a little Wheel moveable about an Axis, about which a Rope being put, is call'd the drawing or running Rope. The Pulley is T, the drawing Rope *a b c d*. 158. Plate III. Fig. 6.

By this Machine the Direction of the Power is chang'd, nor is it of any other Use, when fix'd; for in this Case, *a Force, or Power, applied to the drawing Rope, as* M, *which is equal in Intensity to the Obstacle* P, *balances it* *: for the Obstacle is a contrary Power, which, if it be mov'd, passes thro' a Space equal to that which is pass'd thro' by the opposite Power in the same Time. 159. * 144.

I often make use of Pulleys in those Experiments, by which I illustrate the Actions of Powers; for Weights are applied, whose Directions, (as they all tend downwards) are frequently to be chang'd.

Many Pulleys are join'd to the Machines themselves, others are separate, and may be applied to different Machines, such are represented at T, and T. But there are several Figures of each not represented; five or six are sufficient. Plate IV. Fig. 1, 3.

The PULLEY, *whose Box turns about its Axis.*

The Sheave or Wheel *m* of this Pulley, turns after the usual Manner; but besides, when the Pulley is fix'd, its Box is moveable, and supposing in Tail vertical the Plane of the circular Piece or Sheave *m*, may have every vertical Situation possible. 160. Plate IV. Fig. 1, 2.

The Parts of this Pulley are to be ſeen ſeparate in *Fig.* 2. and are, 1. The Tail R, to which the Screw D ſhould alſo be referred. 2. The Box S, Part of which is the thin Plate *g*. 3. And laſtly, the circular Piece itſelf *m*, call'd a Sheave.

By help of the Tail, the Pulley is join'd to Machines, and fix'd; then the Part *c* of the Tail, is put thro' a ſquare Opening, which it fills exactly, and the Plate *b* is made faſt to the Surface of the Body, by turning the Screw D.

The Sheave *m* is ſhut up in its Box, and is ſuſpended in it between the Plates *g* and *f*, the Ends of the Pulley's Axis being ſo plac'd in the Holes *i*, *i*, that the Axis turns in them. In this Caſe the Plate *g* is applied to the ſolid Piece *e*, and faſten'd by the Screws *l*, *l*.

The ſame ſolid Piece *e* is perforated at *n*; having a cylindric Cavity, which is made very ſmooth in the Inſide. Thro' this the iron Pin *q* goes, which is exactly cylindrical, well poliſh'd, and fills the Cavity. At its Extremity *n* it is ſomewhat larger, to receive the Head of the Pin.

This Pin goes into the Tube *a*, of the Tail R; then the Orifice of the Tube is applied to the Solid *e*, which is join'd to the Tail by the little Pin *p*, which paſſes thro' the Tube *a*, and the larger Pin in the Tube, and there ſticks.

All the Parts being thus join'd together, the Box, containing the Pulley, or Sheave, turns about the Pin *q*.

The Pulley, *fitted with a flat Tail.*

161. Plate IV. Fig. 3, 4. The Conſtruction of this Pulley is ſufficiently manifeſt from a bare Inſpection of the Figure. The Parts are ſhewn ſeparate in *Fig.* 4. *l* is the Tail, this is thruſt into a Groove, when the Pulley is to be fix'd in any Place.

In the *Figures* 1, 2, 3, 4. of *Plate* IV. all the Dimenſions are reduc'd to the Half of the true Bigneſs.

A Pillar, *fitted for making many Experiments, and ſupporting Machines.*

162. Plate IV. Fig. 5. The wooden Pillar C is ſet up upon a Table, and faſten'd by the Screw B, which is applied to the Tail A, paſſing thro' a round Hole, beneath the Table.

This Pillar is perforated from *a* to *b*; having its fore and hind Parts made plane near the Sides of the Aperture, which is every where of the ſame Breadth.

To

To this Pillar there is often join'd a ſmaller G, this is done for the moſt part by putting the wooden Ring E between, which the Screw D paſſes thro', which alſo goes into the Cavity *d* of the ſmaller Pillar, which is eaſily faſten'd to it, becauſe it has a Screw that anſwers to D. 163.

After the ſame Manner ſometimes the Head H is put upon the Top of this ſmaller Pillar G, already join'd to C, which is faſten'd by help of the Screw I, paſſing into the Head itſelf. 164.

The Nut F fits the Screws D and I, and the Uſe of it will be ſeen preſently.

The Dimenſions of this fifth Figure are reduc'd to a ſixth Part.

Different Arms are applied to the ſaid Pillar, and exhibited in *Fig.* 6, 7, 8, 9. the Dimenſions being reduc'd only to an Half.

I. The firſt of theſe Arms is repreſented at Q. To join it to the 165.
Pillar, the Cylinder or Screw D (*Fig.* 5.) is put thro' the Hole *f*, Plate IV.
which fills the Hole, and about which the Arm may be turn'd Fig. 6.
in ſuch manner, that it may be fix'd in any Situation, by applying the Screw F, or ſmaller Pillar G.

At the Extremity of the Arm, there is a round Hole *e*, thro' which the Tail of the Hook V paſſes, which may be turn'd in the Hole, and is faſten'd by the Screw R, the thin Copper-plate *l* being put between, that the Wood mayn't be damag'd by the Compreſſion of the Screw. 166.

II. A ſecond Arm is delineated at M; it's Tail N is thruſt into 167.
the Aperture *n b* of the Pillar (*Fig.* 5.) whoſe Breadth this exactly Plate IV.
fits, being moveable in it, in ſuch Manner, that the Arm may be Fig. 7.
faſten'd in any Part of the Aperture or Channel, by help of the Screw O, P.

At the Extremity of the Arm, which is made flat, there are two Holes *d* and *c*, one round, the other ſquare.

In the firſt may be faſten'd the Hook V of the firſt Arm, which I mention'd of the Hole *e*, (*Fig.* 6 *.) * 166.

The other Hole *c* is of uſe, when the Pulley T, (*Fig.* 1.) is to 169.
be applied to the Arm †; for the Hole fits the Tail of the Pulley. † 160.

III. A third Arm A, is join'd to the Pillar in the ſame manner 170.
as the following, when the ſmaller Pillar G is put upon it, (*Fig.* Plate IV. Fig. 8.
5 *.) whoſe Screw I paſſes thro' the Hole of the Arm, and is * 163.
faſten'd as the firſt Arm is †. † 165.

The Arm A, of which I am ſpeaking, is broader at the fore 171.
Part B C. This Part is arm'd with a Braſs Plate E C B D, bent at

the Extremities of the Arm towards its Sides; to ſuſtain the Pulleys O, O, which turn upon the Axes D, and hang between the Arm and Plate.

172. This Plate is broader in the middle in ſuch manner, as to riſe above the Wood, in which prominent Part there are three Holes, one in the middle *g*, equally diſtant from it, *e*, *e*. The Hook V may be ſuſtained to this Arm alſo; but I ſhall ſpeak of its Situation when I treat of its Uſe.

173. *A fourth Arm* A, *is joined to the Pillar as the third is* *.

Plate IV. Fig. 9. * 170. † 160. It has a Prominency E in its forepart, in which may be faſtened the Pulley T (*Fig.* 1.) †; the Tail of this paſſes thro the ſquare Hole *c*, which the ſquare part of the Tail fits.

174. The Plate D B is applied to the Arms, to this are faſtened the five Hooks V, V, V, V, V; of whoſe Diſtance, and Situation I ſhall ſpeak, when I explain their Uſe.

175. Another braſs Plate F G H I is alſo joined to the Arm. Its part G H is diſtant from the Wood a little more than half an Inch. In this part of the Plate there are three narrow Holes; two of whoſe Extremities appear at *t*, *t*; the middle one is covered by the Plate M L.

176. Two Plates Q R, L M, are applied to the Side of the Arm. The firſt is fixed, and there is a narrow Hole *s*, at its Extremity, which is ſo bent, as to make a right Angle with the Plate.

177. The Plate L M is held by a broad-headed Nail, about which it turns; yet it may wable a little, becauſe of the Screw *n*, going through the Hole L, which nevertheleſs, by reaſon of the Bigneſs of that Hole, does not hinder all Motion. But the Plate is fixed by that Screw, when you put upon it the Nut N.

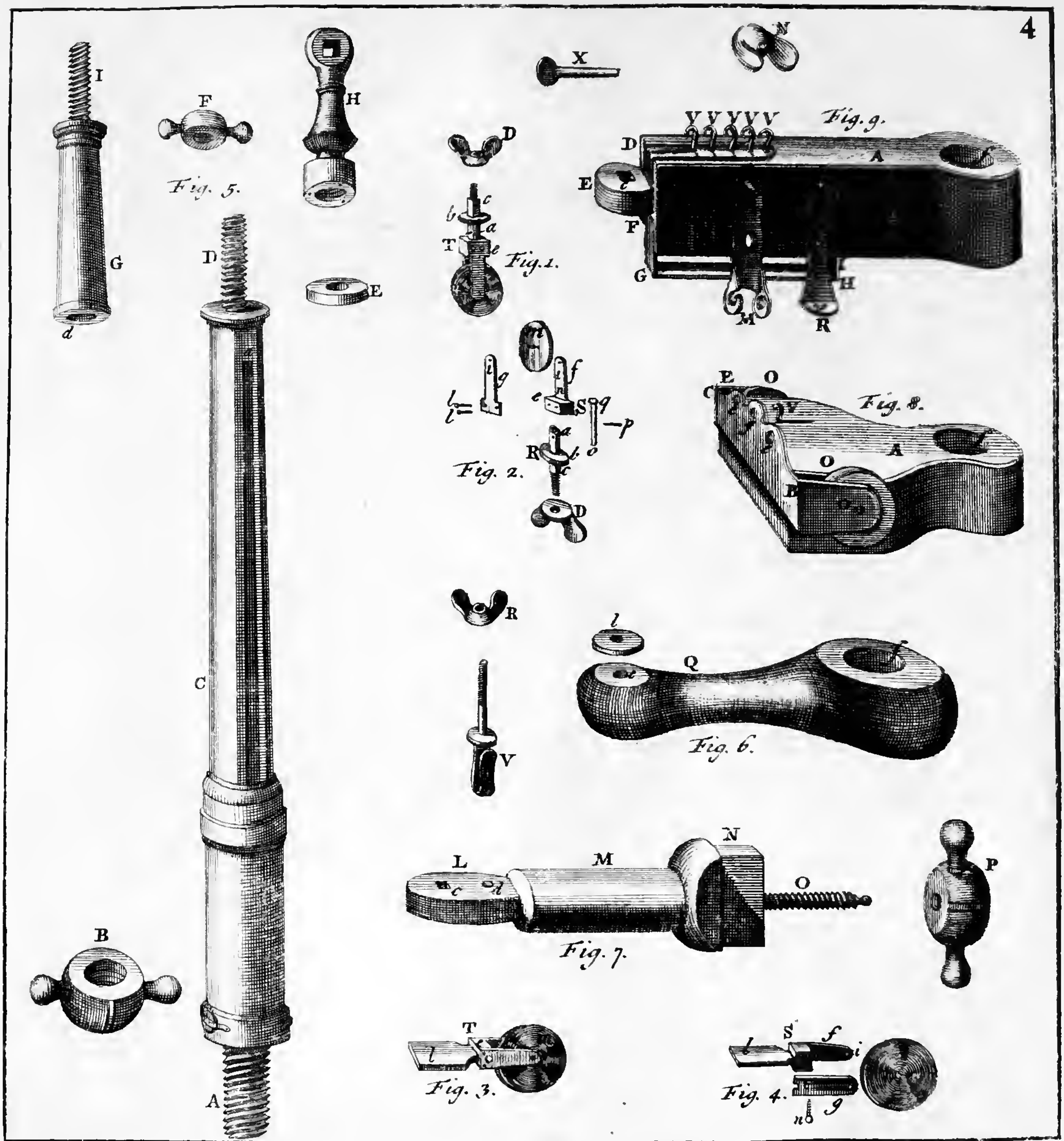
178. In this Arm there are four Holes, of which you may ſee one at P; the other three are in the oppoſite Surface: Into theſe are thruſt Pins like X, and to them are fixed the Threads which go through the Holes *t*, *t*, *t*, *s*.

CHAP. X.

Of the Balance and Center of Gravity.

179. * 156. WEIGHTS are tried, that is, the Quantities of Matter in Bodies are compared *, by making uſe of the Balance, or Pair of Scales, a known Inſtrument.

De-



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DEFINITION 1.

The Line about which the Balance moves, or rather rolls, is called the Axis of the Balance. 180.

DEFINITION 2.

When we consider the Length of the Brachia or of the Beam, the Axis is consider'd but as a Point, and called, the *Center of the Balance.* 181.

DEFINITION 3.

We call Points of Suspension, or Application, those Points in which the Weights either actually are, or from which they freely hang; or from which the Scales hang in which the Weights are put. Concerning this Instrument, we are to observe what follows: 182.

A Weight gravitates on a Point, at any Height that it hangs from it, in the same manner as if you suppose it placed in that Point. 183.

For the Weight of a Body at all Heights, equally draws the Rope that it hangs by *. * 149, 150.

EXPERIMENT 1.

The Weight P, by help of the Rope B D, is applied to the Balance A B, at different Heights; and thereby the Situation of the Balance is not changed.

The Action of a Weight to move a Balance is so much the greater, as the Point loaded by the Weight, is farther from the Center of the Balance; and this Action follows the Proportion of the Distance of the aforesaid Point from that Center. 184. Plate VI. Fig. 1.

When a Balance rolls, in that Motion of the Balance the Point B goes through the Arc B *b*, and the Point A the Arc A *a*, of which the Cap is the greatest; therefore in that Motion of the Balance the Action of the same Weight is various, according to the Point to which it is applied, and follows the Proportion of the Space gone through by that Point *; and therefore at A, it is as A *a*, and at B, as B *b*; but these Arcs are to one another as C A, C B. Plate VI. Fig. 2. * 151, 141.

EXPERIMENT 2.

Each Arm of the Balance A B, the Length of whose Beam is of 2 Foot, is divided into 100 equal Parts, counting the Divisions from the Center of the Balance. 186. Plate V. Fig. 3.

The

187. The Action of one Ounce applied to the 60th Division, is equal to the Action of three Ounces applied to the 20th Division.

For the conveniently making that Experiment, with some others that follow, we have several Brass Weights of one Ounce, as P, which may be applied to the Divisions of the Beam, and have an Hook in their lower Part. There are also several Scales as L, which with their Strings and Hook to hang by, weigh also exactly an Ounce.

That the Actions of Weights on a Balance, do differ, *cæteris pari-*
188. *bus*, as the Weights themselves differ, is plain. But these Actions can only differ in respect to the Weights, or of their Distances from the Center; whence we deduce, *that the Action of a Weight for mov-*
189. *ing a Balance follows a Ratio compounded of the Weight itself, and of*
* 185. *its Distance from the Center of the Balance.* Multiplying the Weight by its Distance from the Center, the Produce expresses the Action.

Definition 4.

A Balance is said to be in Equilibrio, when the Actions of the
190. *Weights for moving the Balance, on each Brachium are so equal, as to destroy one another, as they did in the last Experiment.*

Definition 5.

When a Balance is in Equilibrio, the Weights on each side are said to
191. *equiponderate.*

192. *Unequal Weights may equiponderate*; there is such an Equilibrium, when the Distances from the Center are reciprocally as the Weights. For in that Case if each of the Weights be multiplied by its Distance,
* 189, 190. 16. El. 6. the Products will be equal *. This Proposition is confirm'd by the foregoing Experiment.

193. The *Roman* Balance or Steel-Yard, by which Bodies are weigh'd by one Counterpoise, is built on this Principle.

Experiment 3.

194. The Steel-Yard A B has two very unequal Brachia; the Scale is applied to the shortest; the other is divided into equal Parts, beginning the Division at the Center of Motion; the greater Divisions are numbered, and each of them are again divided into 8 less Parts also equal to each other. Let such a Weight be applied to it, that when it hangs at the first of the large Divisions, it may keep in Equilibrio half a Pound in the Scale: Then the Body to be weighed is to be put into the Scale, and the Weight above-mentioned is to be mov'd

mov'd along its Arm till you have an Equilibrium; the larger Diviſions intercepted between the Weight and the Center of the Balance will ſhew the Number of the Half-Pounds the Body weighs, and the ſmall Diviſions the Ounces. You may alſo apply a leſs Weight to diſcover the ſmaller Differences between the Weight of Bodies.

Upon this Principle alſo is made the falſe Balance, whoſe Brachia 195.
are unequal.

Experiment 4.

To the forementioned Balance * muſt be applied two Scales of 196.
unequal Weight, the one at the 190th and the other at the 96th * 186. Plate VI.
Diviſion, to have an Equilibrium. If then any two Weights be Fig. 5.
given which are to one another as 24 to 25, for Example the firſt of 12 Ounces, and the ſecond of 12 Ounces and a half, and that be put into the firſt, and this into the ſecond Scale, they will equiponderate.

Many Weights applied to different Diviſions on the ſame Brachium, 197.
may equiponderate with one ſingle Weight. It is required that the Product of that Weight by its Diſtance from the Center, may be equal to the Sum of all the Products of the other Weights, each having been ſingly multiplied by its Diſtance from the Center.

Experiment 5.

On one of the Brachia of a Balance hang the following Weights, 198.
two Ounces at the 20th Diviſion, one at the 30th, and at laſt three Ounces at the 60th Diviſion, and you will have an Equilibrium by hanging five Ounces on the other Arm at the 50th Diviſion.

Multiplying 50 by 5, you have 250. On the other Brachium we have three Products 20 × 2, that is 41; 30 × 1, that is 30; and 60 × 3, that is 180. Now collecting 40, 30, 180, into one Sum, you have alſo 250.

Many Weights, unequal in Number, applied on either ſide, may 199.
equiponderate. In that caſe, if each be multiplied by its Diſtance from the Center, the Sum of the Products on either ſide will be equal; and if thoſe Sums are equal, there will be an Equilibrium.

Experiment 6.

A Sight of the Figure will make this plain. Multiplying the ſin- 200.
gle Weights by their Diſtances from the Center, we have on one Plate V.
Side the Products 15, 40, 110, 80, 90, 500; and on the other 70, Fig. 2.
105, 300, 360; whoſe Sum taken on each Side is 835.

201. A perfect Balance must have these Requisites. 1. The Points of Suspension for the Scales, or Weights, must be exactly in the same Line with the Center of the Balance. 2. They must be exactly equidistant from that Center. 3. The Brachia must be as long as they can conveniently be made. 4. There must be as little Friction as possible in the Motion of the Beam and Brachia. 5. The Parts of the Axis which are separated by the Beam must be exactly in the same right Line. 6. Lastly, the Center of Gravity of the Beam must be a little below the Center of Motion.

Definition 6.

202. *That Point in a Body, about which all its Parts, in any Position of the Body, are in Equilibrio, is called the Center of Gravity.*

203. Every single Body, or several Bodies joined, whether they be contiguous or separate, have one common Center of Gravity, as we shall demonstrate in the first following *Scholium*.

204. *When the Center of Gravity is sustained, a Body may be at Rest*; because there is an Equilibrium between the two opposite Parts.

Experiment 7.

205. Plate VI. Fig. 1. The Body A is sustained and at rest, because its Center of Gravity *c* is sustained by the Prop F.

206. *When the Center of Gravity is not sustained, the Body will move till that Center be sustained:* Because the opposite Parts cannot be in Equilibrio about any other Point.

Experiment 8.

207. Plate V. Fig. 3. The Body A set upon a Table will fall, and the Body B cannot remain in the Situation in which it is represented, because their Centers of Gravity are not sustained.

This is the Reason, why some Bodies, laid upon inclined Planes, roll, and others only slide.

Experiment 9.

208. Plate V. Fig. 4. The Body A slides, because its Center of Gravity is sustained by an inclin'd Plane; that is, the vertical Line which goes through that Center, cuts the inclined Plane in the Base of the Body. But the Body B rolls, because the vertical Line which passes through the Center of Gravity, cuts the inclin'd Plane without the Body.

From

From what has been said, it also follows that a Body descends, as long as its Center of Gravity descends, that is, moves towards the Center of Gravity of the Earth. 209.

Sometimes in that case, a Body seems to rise; and indeed, considering its whole Bulk, it does rise; when the Center of the Figure of the Body does not coincide with the Center of Gravity.

EXPERIMENT 10.

Let the two vertical Planes IHLM, and FDE be so placed as to contain an Angle; so the Distance EL is less than the Distance DH; but the Points D, H, are raised up higher than the Points E, L. 210. Plate VII. Fig. 1.

Between these Planes is plac'd the Wheel A, whose Axis B is made up of two Cones, whose Bases are applied to the Wheel. The Wheel is sustained by the Sides DE, HL, of the Planes, and goes of itself towards DH, the highest Place.

EXPERIMENT 11.

The wooden Cylinder A has within it a leaden Cylinder near its Side, fix'd in a wooden Box *b d*. The Center of Gravity is in a Section parallel to the Base, dividing the Cylinder into two equal Parts, and in a Place answering to the Point of the Base *c*. 211. Plate V. Fig. 5.

This Cylinder plac'd any how, will move, till the above-mention'd Center of Gravity be in the lowest Place that it can come to.

If it be laid upon an inclin'd Plane, in the Situation in which it is drawn here; the Center of Gravity will descend, while the Body itself ascends up the Plane, in a proper Inclination of the Plane.

The Body ascends while it rolls towards the superior Part of the Plane; but while it rolls thus, care must be taken that it does not slide along the Plane. Now it is retain'd by a Rope which goes round part of the Cylinder, whose End is joined to the Cylinder at F, the other End E remaining fix'd to the Plane.

Besides, we deduce from what has been said concerning the Center of Gravity; *that a Point in any Body, or Machine, which sustains the Center of Gravity of a Body, sustains all its Weight; and that the whole Force, by which the Body tends towards the Earth, is as it were collected into that Center.* 212.

EXPERIMENT 12.

If the Ruler AB, be suspended at the Brachium of a Balance, and makes an Equilibrium with the Weight P, it will equiponderate in any Situa- 213. Plate V. Fig. 6.

tion; because the Center of Gravity C is sustained after the same manner, and always answers to the same Point of Suspension.

SCHOLIUM I.

Concerning the Center of Gravity.

* 202. WE have said that the Center of Gravity is a Point in a Body, about which all its Parts, in any Situation of the Body, are in Equilibrio *. We have with many Writers who have treated of Mechanics supposed there was such a Point in every Body; but now we will prove it.

214. Let A and B be two heavy Points, of a Gravity any how unequal; let us conceive them joined by an inflexible right Line without Weight: Let there be given in that Line a Point C, such as C A may be to C B, as the Weight of the Point B to the Weight of the Point A. These Weights will be in Equilibrio about C, and that in any Situation, as may be deduced
* 185. from what has been before demonstrated *. Therefore if the Point C be sustained, the Points A and B will also be sustained, and their Action will be as it were collected into the Point C.

Let there be given another heavy Point D, of any Weight; let D and C be joined by another streight inflexible Line without Weight; in that Line let the Point E be so fix'd, that E C may be to E D, as the Weight of the Point D, to the Sum of the Weights of the Points A and B.

If A and B were joined at C, there would be an Equilibrium about E,
* 185. suppose the Line C D in any Situation *. But A and B, as we have demonstrated it in any Situation of the Line A B, act as if they were joined at C; therefore the three Points A, B, D, being joined by inflexible Lines, in any Situation, are in Equilibrio about the Point E; which therefore is the Center of Gravity of the three Points. It is also plain from the same Demonstration, that these Points have no other Center of Gravity, than the Point E.

If there was a fourth heavy Point, it must be joined by a right inflexible Line with E, and then it will appear by a like Demonstration, that the four Points have a common Center of Gravity, and but one.

215. But as this Demonstration may be referr'd to any Number of Points, it may be applied to all the heavy Points that make up every Body: Therefore every Body *has a Center of Gravity, and but one such Center.*

Of the Investigation of the Center of Gravity.

216. Let any Number of Bodies be given, whose common Center of Gravity
Plate VI. is C; let us conceive an horizontal Plane to go thro' this, which shall be
Fig. 8. the Plane of the Figure. Let the Centers of Gravity of the Bodies themselves be A, B, D, E, F; if those Centers don't happen to be in the abovementioned horizontal Plane, they must be referr'd to it by vertical Lines, and the Bodies will gravitate on that Plane in the same manner as if their
* 183. Centers of Gravity were in the Points where the vertical Lines cut the Plane *.

Let

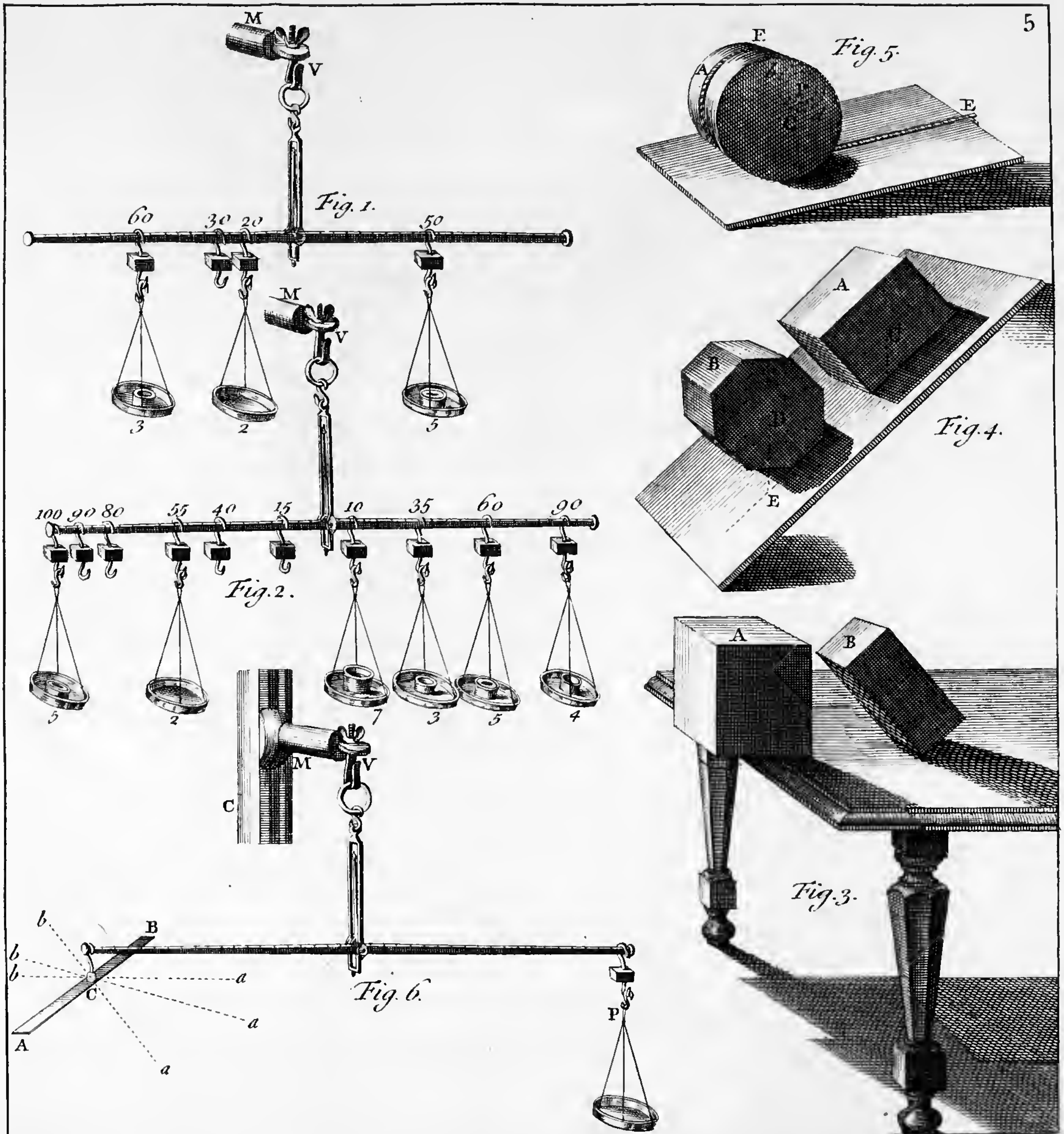
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Fig. 1.
M
V
60
30
20
50
3
2
5
Fig. 2.
100
90
80
55
40
15
10
35
60
90
5
2
7
3
5
4
C
Fig. 6.
B
b
b
b
a
a
a
C
A
P
Fig. 5.
F
A
E
Fig. 4.
A
B
E
Fig. 3.
A
B

J. Mynde sc.

Let the Plane be sustained by the Line G H; *you will have the Actions of the Weights for moving the Plane about the Line* G H *, *by multiplying each Weight by its Distance from the Line* G H *, and the Sum of the Products gives the whole Action, whereby all the Weights together press the Plane, in order to move it about G H. 217. * 189.

But all the Weights act, as if they were at C †; therefore you have also their Action, by multiplying the Sum of the Weights by the Distance of the Point C from the Line G H: Therefore *if the* above-mentioned *Sum of the Products*, which, as appears, is equal to this last Product, *be divided by the Sum of the Weights, the Quotient will give you the Distance of the Center of Gravity from the Line* G H. † 212.

In regard to the Weights, which are referr'd to the horizontal Plane by vertical Lines, the Distances of the Points, to which the Weights are referr'd, from the Line G H, are equal to the Distances of the Centers of Gravity of those Bodies from a vertical Plane passing thro' G H.

But since this Demonstration holds good in any Situation of the Bodies; if the Bodies hold together by inflexible Lines without Gravity, one can conceive no Plane which may not, keeping its Situation in respect to the Bodies, become vertical; whence it follows, that *any Bodies* and Plane being given, the *Distance of the Center of Gravity from the Plane is found, by multiplying the Weight of each Body by the Distance of its Center of Gravity from the Plane, and dividing the Sum of the Products by the Sum of the Weights of all the Bodies.* 218.

If we apply the like Demonstration to a Plane, which passes between the Bodies, the Difference between the Sums of the Product on either side, must be divided by the Sum of the Weights, to find out the said Distance of the Center of Gravity from the Plane. 219.

From thence we deduce a Method, for finding out the Center of Gravity; seeking its Distance from three Planes *; *which Method may also be applied to any particular Body, by referring to its Parts, what has been demonstrated of different Bodies.* 220. 221. * 218.

If Bodies, whose common Center of Gravity is required, *have their Centers of Gravity in the same Plane, that Center sought is determined by discovering its Distance from the two Lines* * *drawn any how in that same Plane.* 222. * 217.

When the particular Centers of Gravity are in the same Line, the common Center of Gravity is found by only one Operation, whereby its Distance from any Point taken in that Line, is determin'd; namely, multiplying each Weight by its Distance from the Point assum'd, and dividing the Sum of the Products by the Sum of the Weights, the Quotient will give the Distance of the required Center from the Point assum'd, if all the Weights are on the same Side. But if the Point assum'd be between the Weights, the Products on one Side must be substracted from the Products on the other Side of the assum'd Point; and this Difference, divided by the Sum of the Weights, will give what we seek. 223.

SCHOLIUM II.

Mechanical Arithmetick.

224. THE four Rules of Arithmetick, Addition, Subſtraction, Multiplication and Diviſion, may be eaſily work'd by means of the Balance above-mention'd *, whoſe Brachia are divided into equal Parts; and the Demonſtration of the Operation is very eaſily deduc'd from what has been before ſaid: therefore it will be ſufficient to illuſtrate the Rules themſelves by Examples.

* 186.

Let any Weight repreſent one, (or Unity); for Example an Ounce: one may in the ſame manner make uſe of the 10th Part of an Ounce.

225. Let the Number 364 be to be applied to the Balance. I apply 3 Ounces to the 100th Diviſion, and 1 Ounce to the 64th.

226. Let the Brachium of the Balance be loaded any how; we determine what Number is the Value of that Action, by ſuſpending at the 100th Diviſion of the oppoſite Brachium a Weight that may be increaſed ſufficiently by an Ounce at a time, till it prevails: Suppoſe that 9 Ounces do not yet make an Equilibrium, but that 10 exceed it; leaving the 9 on, by moving one along the Brachium, I ſeek for the Equilibrium, which I find where the Ounce comes to 47; ſo that the Action requir'd will be 947 in Value.

227. Addition. Let there be to add 34, 54, 268, 407, 45, 65. I apply thoſe Numbers ſeparately to the ſame Arm of the Balance *, I ſeek the Value of this Action †; and I find 873 the Sum requir'd.

* 225. † 226.

Substraction. Out of the Sum of the Numbers 567, 258, we are to ſubſtract 489 and 56. I apply the firſt Numbers to one Brachium *; and to the other apply thoſe that are to be ſubſtracted †, and I ſeek the Value of the Action whereby I can make an Equilibrium *: And I find the Difference required 280.

* 225 † 225. * 226.

229. Multiplication. Let the Number 67 be given to be multiplied by 15. I hang the Number 15 at the 67th Diviſion, and ſeek the Value *, whereby I find the requir'd Product * to be 1005.

* 226.

Division. The Number 1005 is to be divided by 15. I apply the Number to be divided to the Balance †, and moving the Weight 15 along the Beam I ſeek for the Equilibrium, which is had when the Weight is come to the 67th Diviſion, which denotes the Quotient.

† 225.

231. The Rule of Three, is perform'd by Multiplication and Diviſion; but by this Machine one Operation is ſufficient, 77 : 132 : : 63, being given, a fourth Number proportional is required. I apply 13 Ounces and 2 decimal Parts to the 63d Diviſion, (that is, I apply a Weight equal to 132 to the 63d Diviſion.) Then to the 77th Diviſion of the other Brachium, I apply a Weight which I change till I have the Equilibrium; by thus trying, I find that a Weight of 10 Ounces and 8 decimal Parts is requir'd, which ſhews the Number ſought to be 108.

CHAP.

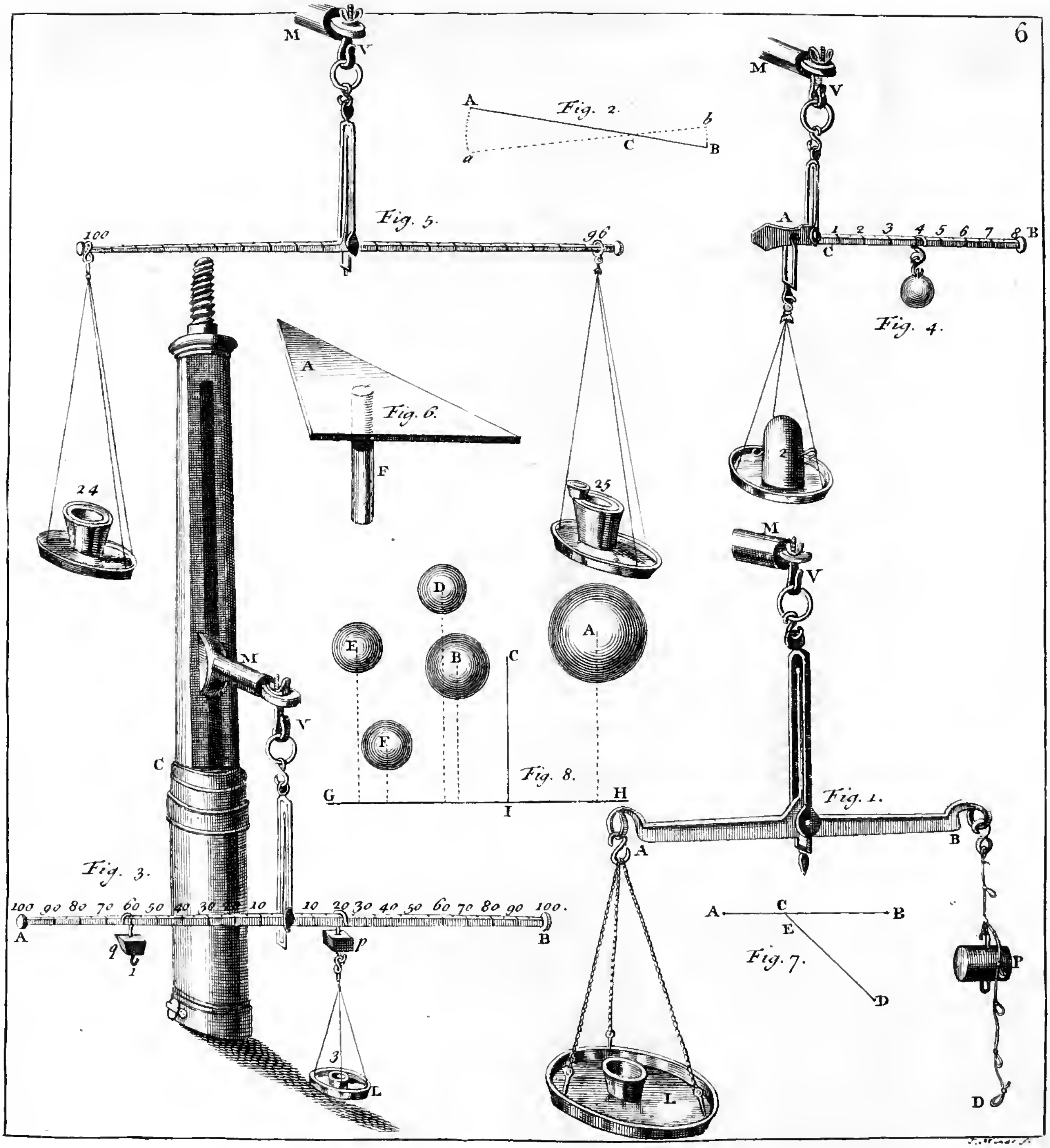
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Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
M
V
A
B
C
D
E
F
G
H
I
L
P
a
b
p
q
100
96
24
25
1 2 3 4 5 6 7 8
100 90 80 70 60 50 40 30 20 10
10 20 30 40 50 60 70 80 90 100.
1
2
3

CHAP. XI.

Of the Lever, the first of the simple Machines.

DEFINITION I.

THE Lever is call'd by Mathematicians an inflexible, right Line, of no Weight, or at least of equable Weight, as AB, *of use to sustain, or raise Weights.* 232. Plate VII. Fig. 2, 3, 4.

When Weights are to be rais'd, this Line is applied to a Fulcrum or propping Point, that it may move about this Point.

It is the first, and most simple of those, that are call'd simple Machines; and is of use, to raise Weights to a small Height.

There are four more simple Machines, which are treated of in the three following Chapters.

Three Things are to be consider'd concerning the Lever; 1. The Weight to be sustain'd, or rais'd, P. 2. The Power, which sustains, or raises it, which is here shewn by the Weight Q, and is commonly the Action of a Man. 3. The Fulcrum F, by which the Lever is sustain'd, and upon which it moves, or rather turns round, whilst the Fulcrum remains unmov'd itself. 233. Plate VII. Fig. 2, 3, 4.

There are three kinds of Levers.

1. The Lever is of the first Kind, when the Fulcrum is between the Weight and the Power. 234. Plate VII. Fig. 2.

2. It is said to be of the second Kind, when the Weight is applied to the Lever between the Fulcrum and the Power. Plate VII. Fig. 3.

In the Lever of the third kind the Power acts between the Weight and the Fulcrum. Plate VII. Fig. 4.

In all Cases the same Rules take place, which follow from what has been said of the Balance *, and which shews the Analogy between the Balance and Lever. The Lever of the first kind is like the *Roman* Steel-yard fitted to raise Weights. * 185.

The Action of the Power, and Resistance of the Weight, increase in the Ratio of the Distance from the Fulcrum *; and therefore, *that the Power may be able to sustain the Weight, it is requisite, that the Distance of the Point in the Lever, to which it is applied, be to the Distance of the Weight, as the Weight is to the Intensity of the Power* *, *which if it is a little increas'd, or remov'd further from the Fulcrum, raises the Weight.* 235. * 185. * 192.

Ex-

Experiment 1, 2, and 3.

236. Plate VII. Fig. 2, 3, 4. This Rule is confirm'd by Experiments in the three foremention'd Levers, as appears in *Fig.* 2, 3, and 4. *Plate* VII. for there is an Æquilibrium, when the Weight P, and the Weight Q which reprefents the Power, as alfo the Diftances from the Fulcrum F, have fuch a Proportion, as is given between the Numbers in the Figures.

237. We make ufe of a Ruler or ftrait Piece of hard Wood, which, that it may be in Æquilibrio, there is applied the Weight D to it, which muft be fo much the greater, as the Fulcrum is nearer the End of the Ruler; wherefore there is a prominent Screw at A, by help of which the Brafs Cylinder D is join'd to the Lever, and may be varied upon occafion.

But that the Breadth of the wooden Ruler may be of no prejudice to the Experiment, there are Holes in the Middle of its Breadth, fo that the Powers and Weights may be applied to a Line drawn thro' the middle of it. There are alfo two Slits *a* and *b*, into which the Fulcrum enters in fuch manner, as that the faid Line is immediately fuftain'd.

The reft needs no further Explanation; C is the Pillar often made ufe of*.

* 162.

238. Plate VIII. Fig. 1. * 235. What I have faid of comparing the Power with the Weight to be rais'd *, may be applied to the bended Lever alfo. Let there be fuch a Lever A C B made of two Rulers A C, C B, making an Angle at C. The Point C is applied to the Fulcrum, and about this the Lever turns: it is manifeft that in the Motion of the Lever, from the Situation A C B into the Situation *a* C *b*, the Angles A C *a*, B C *b*, are equal, and that the Spaces pafs'd thro' A *a* and B *b* from the Points A and B, are to one another as the Diftances, A C, B C. Upon this Account, that the Power at B perpendicularly applied to B C, may fuftain the Weight at A, it is requifite, that the Ratio between this and the Intenfity of the Power may be the fame, but inverfe, as that which is given between A C and B C *.

* 145.

Experiment 4.

239. Plate VIII. Fig. 2. The bended Lever A B, being put upon a Fulcrum, is at reft in fuch a Situation, as to have the Line *c a* horizontal; which is done by the Weight D, join'd to the End A of the Lever. The Weight P of three Pounds, which hangs at *a*, is fuftain'd by the Power

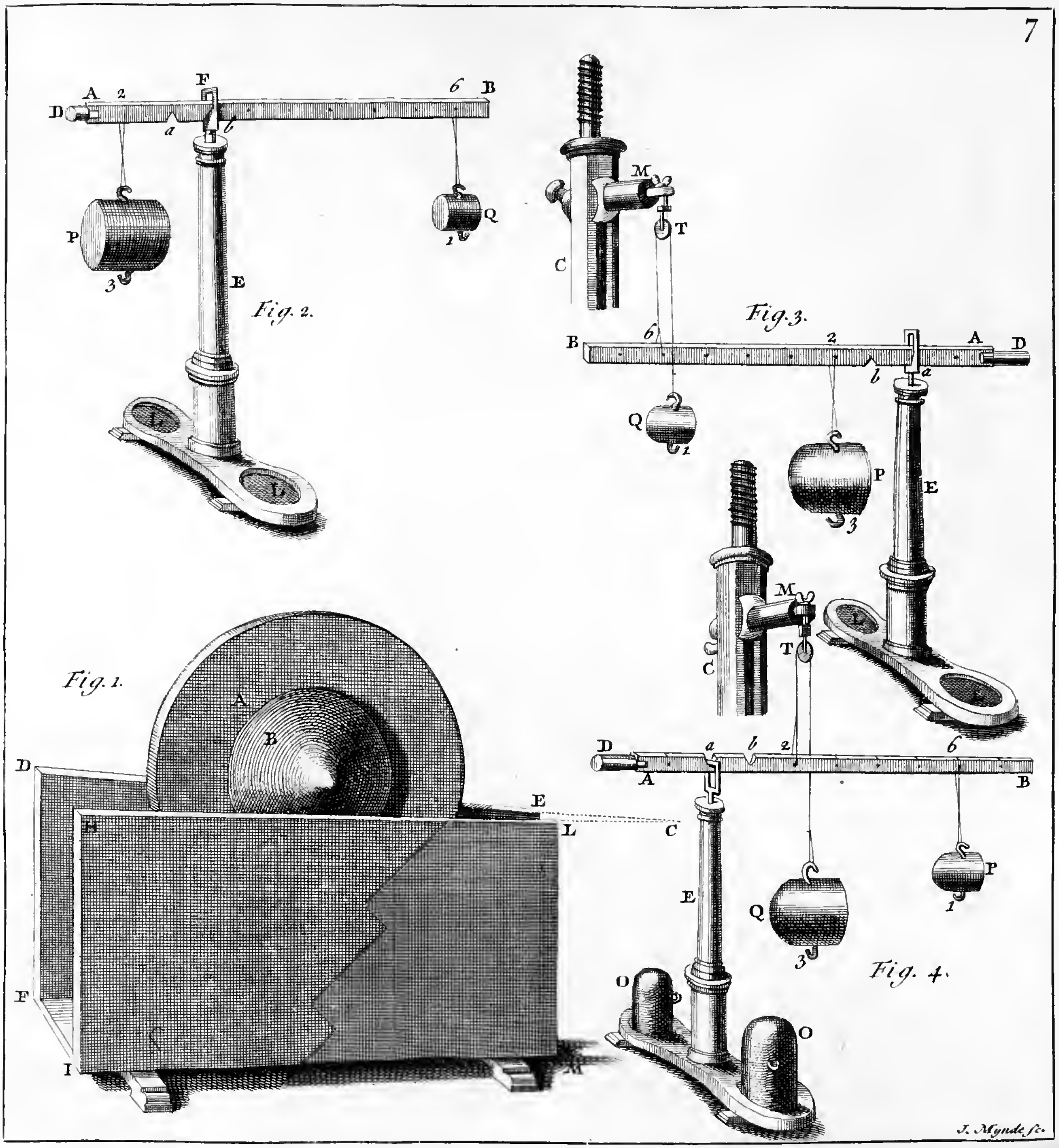
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
J. Mynde sc.

Power Q, which weighs one Pound; becauſe *a c*, is to *c b*, as 1 is to 3.

The Thread to which Q is tied, makes a right Angle with the Side *c b* of the Lever; which how it is done the Figure ſhews. The Pulley T has a Tail *, which is faſten'd, by being thruſt into a Cavity, in the Side of the Table. * 161.

Workmen make uſe of the Lever alſo in carrying Weights; and there are various Caſes of this Uſe of the Lever, worthy Notice, and whoſe Demonſtration is eaſily deduc'd from what has been ſaid. 240.

In all Caſes we may obſerve in general, *that the Intenſity of the Power, or joint Intenſities of the Powers, when many are given, ſhould be equal to the Gravity of the Weights to be carry'd, or ſuſtain'd:* for in all Caſes the Powers and Obſtacles paſs thro' equal Spaces. 241.

If a Weight is to be ſuſtain'd, or carry'd by two Powers, it muſt be plac'd between the Powers, and the Diſtances of the Powers from the Weight on both Sides, muſt be in an inverſe Ratio of the Intenſities of the Powers. 242.

For the Actions of the Powers will mutually diſturb each other, unleſs there be an Æquilibrium between them about the Point of Suſpenſion of the Weight; which Æquilibrium being given, in this Point the Actions of the Powers are united, and act contrary to the Weight; and therefore ſuſtain it, by reaſon of the Equality between the Powers and the Weight.

A MACHINE, *whereby the Experiments which are made on the Lever, by which Weights are mov'd, are demonſtrated.*

The wooden Ruler D E, almoſt one Inch thick, and two Inches broad, has a Slit in its fore-part from *m* to *n*, and is alſo divided into equal Parts. 243. Plate VIII. Fig. 3.

The ſolid Piece F is join'd to the middle of this Ruler, by help of which it may be applied to the Pillar C, and fix'd at different Heights, as I ſaid above of the Arm M, (*Plate* IV. *Fig.* 7 *.) * 167.

The Pulleys with Tails may be ſo faſten'd in the Slit †, that they may anſwer to any one of the Diviſions. † 161.

Another wooden Ruler A B is made uſe of; this repreſents the Lever, and is ſmaller; the Weight of this ought to be fix'd, mine weighs an Ounce and an half.

This

This is alſo divided, and its Diviſions anſwer to the Diviſions of the Ruler D E. The Ruler A B is perforated in the two Points *i*, *i*, which are not far, but equally diſtant from its Ends, and anſwering to two Diviſions, taken at pleaſure; two Threads, or ſmall Cords, are tied to the Ruler by theſe Holes, and put round the Pulleys T, T, which in the upper Ruler anſwer to the Points *i*, *i*, that the Ruler A B may be ſuſtain'd by the Weights of the Scales L, L, each of theſe Scales in my Machine weighs $\frac{3}{4}$ of an Ounce; and the Lever is conſider'd as having no Weight; and the Weight of theſe Scales is not conſider'd in the Experiments. Every one of the other Scales made uſe of in the Experiments weighs one Ounce, which Weight is not to be neglected in the Computations.

Experiment 5.

244. A Weight of eight Ounces is put into one of the Scales L, L, and into the other one of four Ounces. Theſe Weights, which repreſent the Powers, drawing the Lever upward in the Point *i* and *i*, are to one another as one to two; and together make twelve
* 241. Ounces; and can therefore ſuſtain a Weight of twelve Ounces *. This they will do if we put eleven Ounces into the Scale weighing one, and apply it to the Lever at O, that there may be an Æquilibrium between the Actions of the Powers, leſt the Lever ſhould turn about. In this Figure the Diſtance between the Points *i*, *i*, is divided into 30 Parts, and the Point O divides this Diſtance into two Parts, which are to each other as two to one, *i. e.* inverſely as the Powers.

Experiment 6.

245. *When two Weights are to be ſuſtain'd by one Power, the Power muſt be plac'd between the Weights, and then what has been ſaid of*
* 242. *the two Powers *, muſt be applied to the Weights.* For the Weights can't be ſuſtain'd, unleſs their common Center of Gravity is ſuſ-
† 206. tain'd †.

Oftentimes many Weights are ſuſtain'd or carry'd by one, or more Powers. Concerning which we muſt take notice, *that all*
246. *the Weights have a common Center of Gravity*, which Center is ſuch, that, if every Weight on either ſide be multiplied by its Diſtance from that Point, the Sum of the Products on both ſides
* 199, 200. will be the ſame *.

The Powers alſo howſoever diſpos'd have a common Center of Ac-
* 151. *tion*; for they may be repreſented by Weights *, and here the Intenſity

ensity of every Power ſhould be multiplied by its Diſtance from the Center, and the Sums of the Products on both ſides will then be equal.

That the Powers may be able to ſuſtain the Weights, it is requiſite that the Center of Action of the Powers may agree with the Center of Gravity of the Weights. Then the Actions of all the Weights, and Powers are reduc'd to one and the ſame Point, which is drawn upwards and downwards by equal Forces, and is therefore ſuſtain'd.

EXPERIMENT 7.

From what has been ſaid it is eaſy to explain the Figure, in which O denotes the Center of Gravity of the Weights, and the Center of the Actions of the Powers. 247. Plate VIII. Fig. 4.

What has been before ſaid alſo takes place, if the Lever is drawn both ways by the Powers; for theſe are ſo to be diſpos'd, that the Center of the Actions, of thoſe acting on one ſide, may coincide with the like Center of the oppoſite Actions: and we ſhall have a Lever, which will be at reſt, if the Sum of the Intenſities of the Powers, on one ſide, is equal to the Sum of the Intenſities of the oppoſite Powers. 248.

This Propoſition is eaſily confirm'd by an Experiment, by making uſe of a ſmall wooden flat Board, of about one Foot long, and two Inches and an half broad. This is ſuſtain'd in an horizontal Situation by a Stand, and has a Groove long-wiſe on either ſide, that the Pulleys with Tails * may be applied to it; about which horizontal Ropes are put, which draw the horizontal wooden Ruler at oppoſite Parts. By this means the Experiment may be varied at pleaſure. I have often made uſe of ſuch a Machine; but, tho' it is very ſimple, I have neglected it; becauſe another may be made uſe of in this Caſe, which is of ſervice in making Experiments on oblique Forces, which will be mention'd afterwards, wherefore there is no need of a peculiar Machine in this Caſe. 249.

* 161.

A MACHINE, *whereby Experiments are made on oblique Forces, and the Lever which is drawn horizontally.*

A ſmall wooden Table G, which is ſquare, or a little oblong, is ſuſtain'd by Feet. Upon this is put the wooden Rectangle MNPQ, which is repreſented by itſelf at MNPQ, (*Fig.* 2.) Its Bigneſs is ſuch, that it will exactly include the Table; wherefore, that it may be put upon it, the Supports E, E, E, E, are to be added, which are mark'd in *Fig.* 1. by the Letters *e*, *e*, *e*, *e*. By which 250. Plate IX. Fig. 1.

the Rectangle, whilſt it is applied to the Table G, is alſo rais'd above it about an Inch.

In the Experiments of which I am now ſpeaking, it muſt be rais'd higher yet, which is done by the four ſmaller Supports *f*, *f*, *f*, *f*. One is repreſented ſeparately at F, (*Fig.* 2.) the ſmall Cylinder or Peg *h* is join'd to this, which goes into a Hole, in the Corner of the Table G.

251. In the ſhorter Sides of the Rectangle there are Slits I *i*, I *i*, going thro' the Wood; the other two Sides have two ſquare Holes *c*, *c*, and the ſmaller Slits *d* D, *d* D, &c. In all theſe the Pulleys may be faſten'd, whoſe Boxes turn round *. The Slits D *d*, D *d*, are ſo determin'd with the Hole *c* in each Side, that, when the Pulleys are faſten'd at *c*, and at the Extremities of the Slits *d*, D, *d*, D, &c. the Diſtances between the Pulleys are equal.

* 160.

Experiment.

252. A wooden Ruler is made uſe of, divided into equal Parts in ſuch manner, that the Diviſions anſwer to the Pulleys faſten'd, as has been mention'd *.

* 251.

This Ruler is drawn horizontally by two Powers on one Side, and three at the oppoſite Side: the Sum of the two is equal to twenty Ounces, and the Sum of the three oppoſite ones is equal to this. The Point in which the Centers of the oppoſite Actions meet is O, about which there is an Æquilibrium on either Side, as is eaſily deduc'd from the Numbers mark'd in the Figure.

The Ruler A B, now hanging in the Air, is eaſily driven to one, or the other Side, *i. e.* towards N P, or M Q.

CHAP. XII.

Concerning the Axis in Peritrochio, the ſecond of the ſimple Machines.

THE Lever is of uſe when Weights are to be rais'd to a ſmall Height, as was ſaid in the Beginning of the foregoing Chapter; when the Height is greater, the Axis in Peritrochio is made uſe of.

De-

DEFINITION.

The Axis in Peritrochio is a Wheel turning round with its Axis. 253.

The Power in this Machine is applied to the Circumference of the Wheel, by whofe Motion, the Rope, to which the Weight is faften'd, is wound round the Axis, by which the Weight is rais'd.

Let B D be the Wheel; A E its Axis; F the Weight to be rais'd; Q the Power: the Wheel is mov'd by its Action, the Points B and A defcribe fimilar Arcs by that Motion; thefe Arcs are the Ways gone thro' by the Power and the Weight, and are to one another, as B C to A C, that is, as the Diameter of the Wheel to the Diameter of the Axis, from whence the following Rule is deduc'd. 254. Plate VIII. Fig. 5.

The Power has the greater Force, the larger the Wheel is, and its Action increafes in the fame Proportion as the Wheel's Diameter. The Weight refifts fo much the lefs, as the Diameter of the Axis is lefs, and its Refiftance is diminifh'd in the fame Ratio as the Diameter of the Axis. And to have an Equilibrium between the Power and Weight, it is requir'd, that the Diameter of the Wheel be to the Diameter of the Axis in an inverfe Ratio of the Power to the Weight *. 255.

* 145.

Take notice, that the Diameter of the Rope is to be added to the Diameter of the Axis.

EXPERIMENT.

This Rule is confirm'd feveral Ways by help of the Machine here drawn, in which there are three Brafs Wheels *a*, *b*, *c*, which may be mov'd by a Weight tied to a Thread reprefenting the Power. Thefe Wheels cohere, and to them is join'd the wooden Wheel A, to fhew how the Power may be applied to the Handles *d*, *d*. 256. Plate VIII. Fig. 6.

The Axis at B is twice as thick as at D, that by that means Experiments may be varied.

The Ends of the Axis are fmall, and of Steel, to have the lefs Friction.

The Wheel is fuftain'd by the wooden Pillars E E, and thofe fupported by a Stand.

When the Diameter of the Axis is the 16th Part of the Diameter of the Wheel, one Ounce fuftains 16 Ounces P, and fo on.

257. When the Power is applied to the Handle as at *d*, the Distance of the Point where it is applied from the Center, must be consider'd as the Wheel's Semidiameter.

CHAP. XIII.

Of the Pulley, the third of the simple Machines.

258. UPON many Occasions the Axis in Peritrochio cannot serve for raising Weights; in those Cases we must make use of Pulleys, and the Machine which is made with them is very compendious, and portable from Place to Place.

* 158 What a Pulley is, has been already said *.

If a Weight be so join'd to a Pulley that it is not hindred from turning about, and it be raised with the Weight, each End of the running Rope sustains half the Weight. Therefore *when one End is tied to an Hook, or any other way fix'd, a moving Force equal to half*
259. *the Weight applied to the other End, will sustain the Weight.*

EXPERIMENT I.

260. Plate IX. Fig. 3. P, a two Pound Weight, is so joined to the Pulley O, that the Rotation of its Wheel is not hinder'd thereby; *f* the End of the Rope is fasten'd to the Hook V, and the Rope is also carried round
* 159. the fix'd Pulley T, to change the Direction *; then the Weight Q of one Pound, applied to this End, sustains the Weight P.

The Hook D, by its Weight sustains the moveable Sheave O, lest it should disturb the Experiment by its Weight.

261. Many Sheaves may be join'd together, and a Weight hang on them; then if one End of the Rope be fix'd, and it be carried round all those Sheaves and as many fix'd ones, a great Weight may be raised by a small Power; in that case, the greater the Number of Sheaves join'd to the Weight (for the Action of the Power is not
* 159. altered by the fix'd Sheaves *) the less may the Power be to sustain
262. the Weight; *and a Power, which is to the Weight, as one to twice the Number of Sheaves moveable with the Weight, will sustain that Weight.* For this is the Number of Ropes, whereby the Weight is sustained, and the Power is applied but by one Rope, as appears by the following Experiments.

The

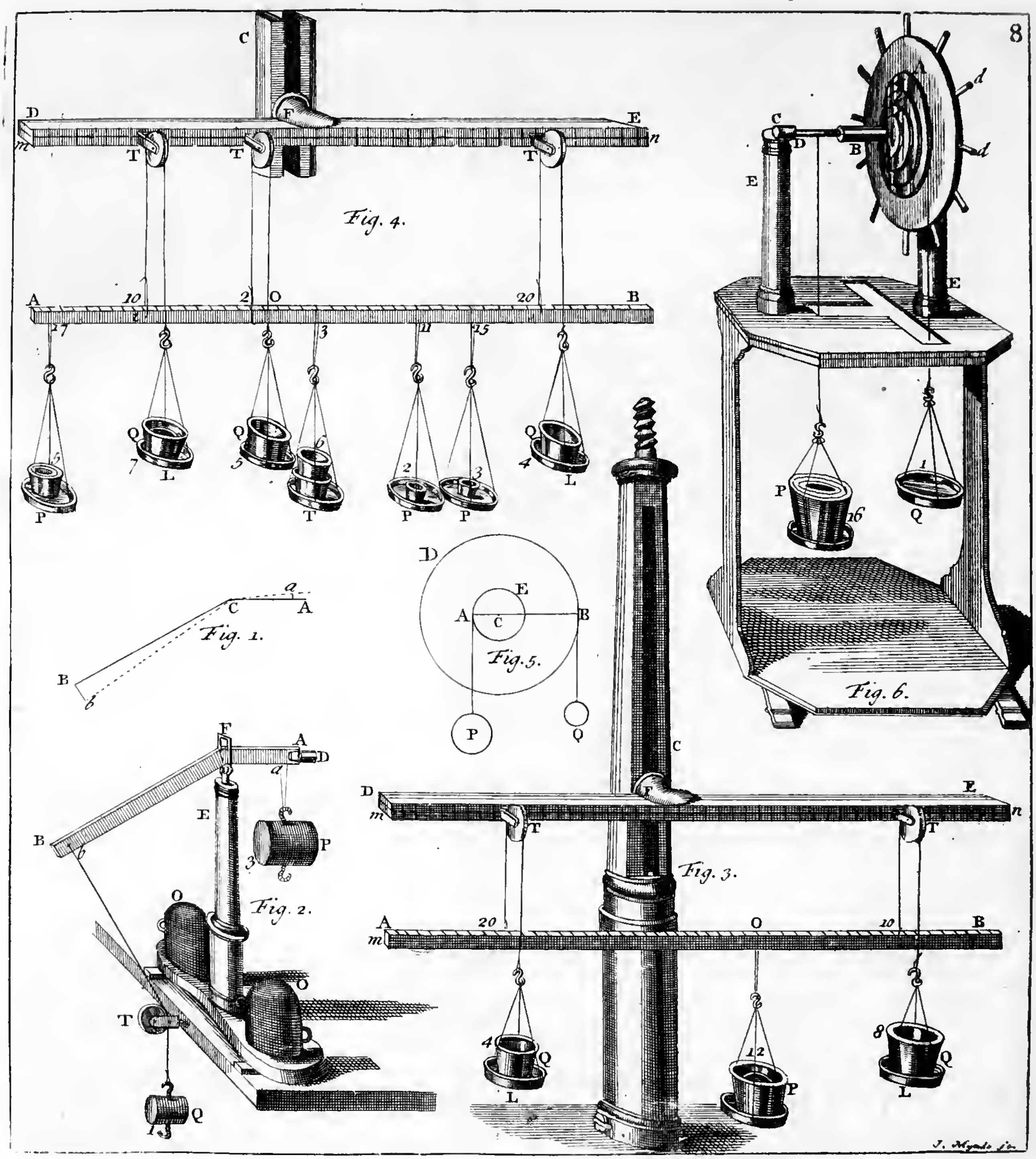
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Fig. 4.
Fig. 6.
Fig. 1.
Fig. 5.
Fig. 2.
Fig. 3.

The Pulley in that Case, consists of two Parts; the first is suspended and contains fix'd Sheaves; the second is moveable with the Weight. These considered together are call'd a Pulley. The Parts are also call'd by the same Name, whence one is an upper and the other a lower Pulley. But they are rather call'd a Tackle. 263.

EXPERIMENT 2.

P, a Weight of 6 Pounds is join'd to the Ruler DE, in which three Sheaves O, O, O, roll freely. *f* the End of the Rope is tied to the Hook V, and the Rope goes round those three Sheaves, and as many more fix'd ones; at the other End Q the Weight of one Pound is suspended, and you have an Equilibrium. 264. Plate IX. Fig. 4.

The last of the fix'd Pulleys T might have been left out; but then in that case the Power must have drawn upwards the Part *i h* of the Rope, by an Action equal to the sixth Part of the Weight F, because that Weight is sustain'd by six Ropes equally stretch'd. 265.

In this, as well as in the foregoing Experiment, and those that follow in this Chapter, the Weight hangs upon a Hook as D, by which all the moveable Sheaves are sustained.

EXPERIMENT 3.

No matter how the Sheaves are join'd; the foregoing Disposition is not conveniently applied for raising Weights; therefore Workmen make use of unequal Sheaves disposed as in Fig. 5. for the Bigness of the Sheaves does not alter the Demonstration. 266. Plate IX. Fig. 5.

In that case all the Ropes are not vertical, unless the Diameter of the Sheaves are in an arithmetical Progression of the natural Numbers 1, 2, 3, 4, *&c.* The odd Numbers 1, 3, 5, express the Diameters of the Sheaves moveable with the Weights; and the even Numbers 2, 4, 6, the Diameters of the fix'd Sheaves.

But because by this Method, we should often be brought to use too big Sheaves, *in the Practice the Parallelism of the Ropes is neglected*; because the Error arising from a small Obliquity of the Ropes, may be neglected; but yet we must observe in that case, that there is occasion to use unequal Sheaves, that the Ropes may be separated. See *Fig.* 8. But what follows from the Obliquity of Ropes, we shall see in the 16th *Chapter* of this Book. 267. Plate IX. Fig. 8.

When the Diameters of the Sheaves are as the natural Numbers *, all of them perform one Revolution in the same time; wherefore all the superior and all the inferior Sheaves may be join'd, so that we may have only two moveable Axes; we always suppose the 268. * 266.

Axes

Axes fix'd to the Sheaves to be of Steel, and to turn in Braſs, as
* 160. we have explain'd above *.

Experiment 4.

269. Plate IX. Fig. 6. We often make uſe of equal Sheaves, parallel to each other, which Conſtruction is very compendious.

The End of the Rope is fix'd to the Hook V, then the Rope goes down to the firſt lower Sheave, winds about it, then riſes up to the firſt upper Sheave, from which it goes down to the ſecond lower one, from which it riſes again to the ſecond ſuperior Sheave, &c.

270. Plate IX. Fig. 7. That in theſe Circumſtances all the Ropes may be vertical, and parallel, we muſt obſerve, that the Axes of the upper Sheaves being put in the ſame Line, the inferior Sheaves muſt not be diſpos'd in the ſame manner, and ſo *vice verſâ*. But the propereſt Diſpoſition of the Axes is ſuch as we ſhew here. The Axes are repreſented at T and O, as in horizontal Planes, which are ſuppoſed to paſs at T thro' the ſuperior Axes, and at O thro' the inferior.

If the Paralleliſm of the Rope be neglected, Care muſt be taken leſt the lower Pulley comes too near the upper, that is, the Weight won't be rais'd to the ſame Height as if the Ropes were parallel.

Experiment 5.

271. Plate IX. Fig. 8. When the End of the running Rope, which was fix'd in the former Experiments, is join'd to the Weight, or to the Box carrying the Sheaves moveable with the Weight, the Ratio of the Power to the Weight is no longer as 1 to twice the Number of the Sheaves join'd to the Weight; but this double Number muſt be increas'd by 1. And here, when two Sheaves are join'd to the Weight, the Ratio is as 1 to 5; for the Weight is ſuſtain'd by ſo many Ropes.

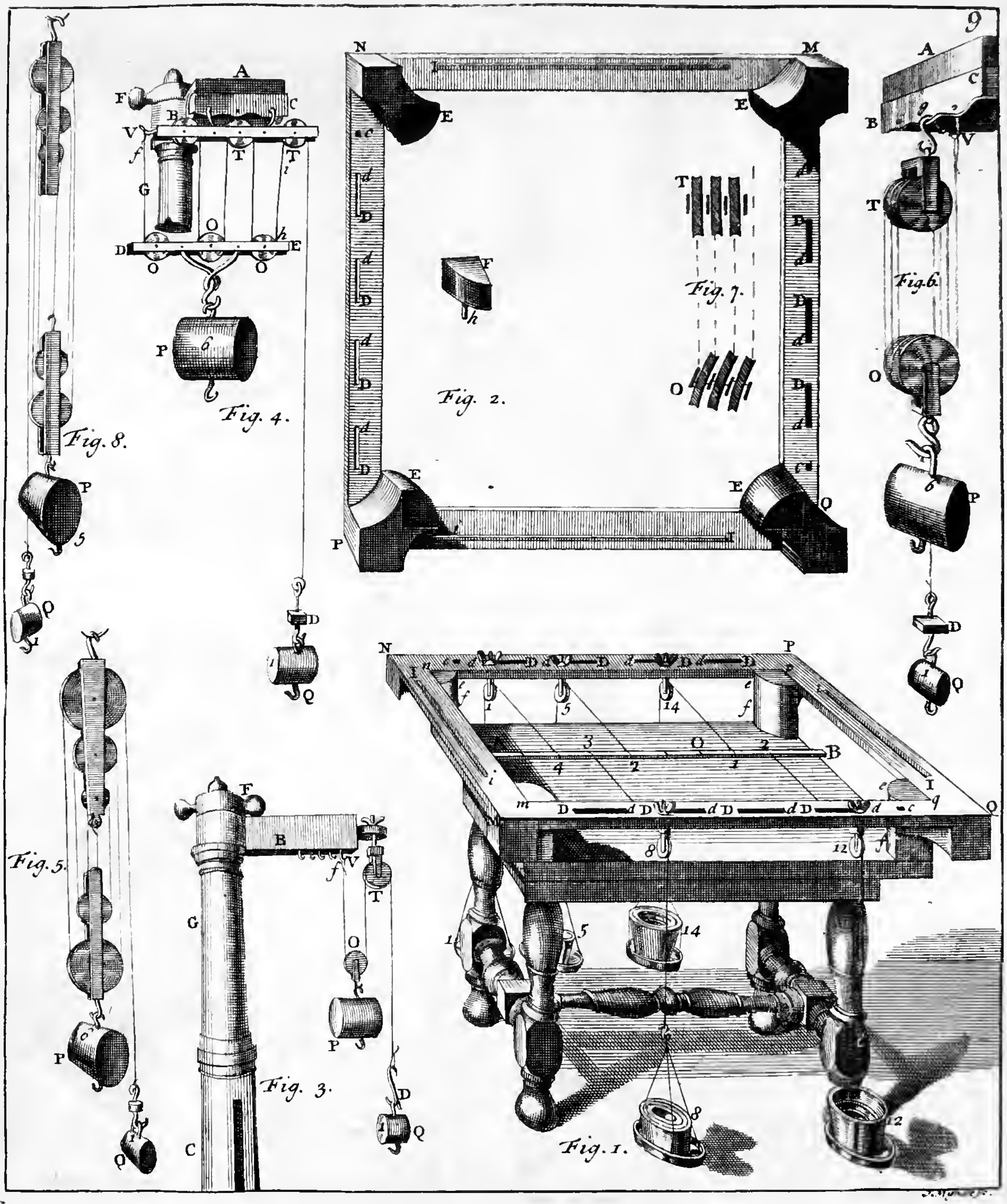
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Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.

CHAP. XIV.

Of the Wedge and Screw, the fourth and fifth of the ſimple Machines.

FROM what we have already ſaid, it appears ſufficiently, how by help of a ſmall Power a great Weight may be ſuſtain'd or rais'd. The mechanical Art is not reſtrain'd to thoſe Uſes alone; the Powers, whoſe Intenſities are ſmall, may be applied to the overcoming of any great Reſiſtance. A remarkable Example of it may be ſeen in the *Wedge*, an Inſtrument ſerving to cleave Wood, and for many other Uſes.

DEFINITION 1.

A Wedge is a Priſm of a ſmall Height, whoſe Baſes are equicrural Triangles; one of them is repreſented at B C D. 272. Plate X. Fig. 1.

DEFINITION 2.

The Height of the Triangle is the Height of the Wedge; as C E. 273.

DEFINITION 3.

The Baſe of the Triangle is alſo call'd the Baſe of the Wedge; as B D. 274.

DEFINITION 4.

The Edge of the Wedge is the right Line which joins the Vertices of the Triangles; as C *c*. 275.

The Edge of the Wedge is applied for cleaving of Wood and ſeparating Bodies, and is often driven in by the Blows of a Mallet, inſtead of Preſſion. 276.

When the whole Wedge is driven in, the Space gone thro' by the Baſe to which the Power is applied, is the Height of the Wedge E C, which therefore muſt be taken for the Space gone thro' by the Power; but the Space which the Bodies that are ſeparated, recede from one another in the ſame Time, is the Baſe of the Wedge, whence it follows,

That the Power is to the Reſiſtance of Bodies to be ſeparated, when it is equipollent to it, *as the Baſe of the Wedge to its Height* *. 277.

* 145.

278. When Wood is to be cloven, this Rule does not hold good; becauſe all the Parts of the yielding Wood do not go thro' equal Spaces. What relates to the cleaving of Wood will be explain'd in the next Scholium I.

A Machine, *whereby the Properties of the Wedge are demonſtrated.*

279. The ſmall wooden Table A is to be fix'd about 3 Foot and an half above M.

Plate X. Fig. 2. * 162. † 164.

For this purpoſe, to the Pillar C *, which has another leſs Pillar fix'd to it, muſt be added the Head H †. In this Figure we could not draw the whole Pillar C, ſo we omitted its middle Part, as well as the Middle of the Ropes that we ſhall conſider preſently.

There is a ſquare Hole in the upper Part of the Head H, (ſee *Plate* IV. *Fig*. 5.) thro' which paſſes the Tail of a wooden Cylinder B, which fits the Hole ſo exactly, that the Cylinder is made faſt by the help of a Screw F, as well as the Board A, which is join'd to the Cylinder. This little Table or Board is 6 Inches long, and $4\frac{1}{2}$ wide, and muſt be fix'd in an horizontal Poſition.

At its Angles are the four Holes *a*, *a*, *b*, *b*, thro' which the Ropes paſs, and which are fix'd in the Holes; theſe are equal to one another, and about 30 Inches long.

To theſe Ropes are faſten'd four Braſs Plates, as *d* and *d*, of which only two can be ſeen.

By their Help are ſuſpended two wooden Cylinders G I, G I, the Length of each of which is equal to the Diſtance *a b*, that the two Ropes by which the ſame Cylinder is ſuſtain'd, may be parallel. The Axes of the Cylinders are of Steel, and ſlender as *e e*; they go thro' *d d*, the greater Holes of the Plates, and fill them, yet ſo as to turn freely.

To diminiſh the Friction the Baſes of the Cylinders are a little prominent in the middle, wherefore the Length of the Cylinders, if it be meaſur'd between the Circumferences of the Baſes, falls ſhort a little of the Diſtance above-mention'd *a b*, between the Ropes that ſuſtain the Cylinders.

The Diameters of the Cylinders at I, and G, are $2\frac{1}{2}$ Inches; the middle Part O is ſmaller, 3 Inches long, whoſe Diameter is $1\frac{1}{2}$ Inch. This ſmaller Part is encompaſs'd with two Rings, as *r*, made of the Wood itſelf, ſo that the Plate D E, which is applied to this ſmall Part, may touch only the Rings.

In

In the Board A there are alſo two other Holes between *a*, *a*, and *b*, *b*, namely *c*, *c*, thro' which the Ropes paſs, which are faſten'd by Pins as *s*, to the upper Part of the Board. Two Braſs Pulleys, as T, are ſo ſuſpended by theſe Ropes, as to turn freely about their Steel Axes.

A Pulley, as T, is join'd to one Cylinder, whilſt it is faſten'd to the Plate *d* by the Ropes *m*, *m*; the Rope *n n* joins with the oppoſite Plate *d*, which goes round the Pulley T, and is drawn by the Weight P. Such another Weight is ſuſpended on the other ſide of the Cylinders in the ſame manner; by which two Weights the Cylinders are drawn together.

By turning the Pins *s*, of which only one is viſible in this Figure, the Pulleys are rais'd or depreſs'd, till the Ropes *n* and *m* are horizontal.

The Wedge is form'd of two thin wooden flat Boards, D E, D E, join'd by Hinges, ſo as to make any Angle with each other.

Thro' theſe paſſes the Screw L L, which is bent in an Arc of a Circle, over which two outward Screws, as *i*, are mov'd. By theſe are ſeparated the Planes D E, D E, and the Angle which they make ſo fix'd that it may not be alter'd.

Hanging up the Scale with the Weight Q, the Wedge is put between the Cylinders; that Weight is ſuſpended by the Rope *f* in the middle Point of the Edge of the Wedge.

That the Ratio between the Baſe and the Height may be determin'd in the Wedge, ſmall iſoſceles Triangles muſt be made of Wood, a little truncated at the Vertex, as A B C; on which the Height and Length of the Baſe are mark'd, any Meaſure being given. It is convenient to expreſs the Height by the Number 16, if the Cylinders are drawn towards one another by whole Pounds. Such a Triangle is put between the Boards that make the Wedge, that the Situations of the Plates, as *i*, may be determin'd. Plate X. Fig. 3. Fig. 2.

Experiment.

Things being diſpos'd, as has been ſaid in the Deſcription of the Machine, if the Weight by which the Wedge is puſh'd in between the Cylinders, (that is, the Weight of the Wedge, of the Screw L L, and the Scale with the Weight in it Q) is to the Sum of the Weights P, P, as the Baſes of the Wedge to its Height, you have an Æquilibrium between the Force by which the Cylinders are ſeparated, and that by which they are drawn together. This is prov'd, becauſe the Wedge is rais'd or depreſs'd by the leaſt Agitation. 280.

The *Screw* has a great Affinity with the Wedge. It confifts of two Parts.

DEFINITION 5.

281. Plate X. Fig. 4. The firft, which is call'd *the infide Screw, is a Cylinder cut in the Form of an Helix, as* A B.

282. The fecond, which is call'd *the outfide Screw*, and whofe Figure differs according to the different Ufe of the Machine, *is a Solid hollow'd cylindrically, whofe concave Surface is cut in fuch manner, that its Eminencies may agree with the other's Cavities*, as D E.

Thefe two Parts may mutually move in each other, which is requir'd in the Ufe of this Machine. It ferves chiefly for the Compreffion of Bodies, which are to be join'd and comprefs'd firmly; for in this Machine a very fmall Power compreffes Bodies very ftrongly. The Screw alfo may be ufed for raifing Weights.

In each Revolution of this Machine, one Part being at reft, the other is pufh'd forward to the Diftance between the two Threads of the Screw, near each other. The Power by which the Screw
283. is mov'd, is applied to the Handle; and *the Power is to the Compreffion which it generates, as the* faid *Diftance between the two neareft Threads of the Screw is to the Periphery of a Circle, run thro' by the Handle to which the Power is applied:* for the Way gone thro' by the Point or Plane, by which the Refiftance is overcome, is in that Ratio to the Way of the Power. This Way would be a little greater, if the Power was applied in a Direction parallel to the Thread, but this often would be very difficult; therefore in Practice the Power almoft always acts in a Plane perpendicular to the Axis of the Cylinder, which forms the infide Screw, and this is the Cafe which we have confider'd.

284. Here we muft obferve, that when the Power in any Machine is equipollent to the Weight, or Refiftance, by increafing the Power, the leaft that is, it will overpower, if the Machine has no Friction; but when there is a Friction, that muft alfo be overcome by the Power: but we cannot determine by a mathematical Reafoning how much is requir'd to do this.

285. *In Screws the Friction is very fenfible, and alfo very ufeful*; for by it the Machine is kept in its Situation; and when the Power ceafes to act, it does not return to its former Situation by the Action of the Bodies that are comprefs'd, or the Gravity of the Weights.

SCHOLIUM

SCHOLIUM I.

Concerning the cleaving of Wood.

LET a Piece of Wood be given, whose Parts already separated make an Angle E F L; let this moreover be to be cloven by means of the Wedge A C B, whose Base is A B, and whose Height is C D. 286. Plate X. Fig. 5.

Where the Parts are separated, tho' ever so little, all Resistance is taken away: but before the Parts are separated at F, the Points E, L, must be mov'd a little, that is, the Angle E F L must be inlarg'd; we must therefore determine the Force, by which this Angle is to be inlarg'd.

Let us suppose this Angle to be inlarg'd, so that it may become *e* F *l*; the Wedge has enter'd, and is at *a* C B; the Parts of the Wood E, L, have been carried thro' E *e*, L *l*, but those that are the least distant from F, move thro' the least Space, and the Lines E F, and L F, by their Motions describe the Areas of equal Triangles *e* F E, *l* F L.

Having drawn *e* F and *f* F, parallel to E F, and *e* E, let the Parallelogram *e* E F *f* be form'd; the Triangles *e* F E, and *f* E F are equal *; and the Parallelogram is equal to the two Triangles *e* F E and L F *l* taken together: therefore the above-mention'd Motions of the two Lines E F, L F, join'd, amount to as much as the Motion of the Line E F alone, thro' the Space E *e* or F *f*; which little Line therefore represents the Distance, whereby the Parts of the Wood are separated from one another: but considering this Separation, this little Line is the Space gone thro' by the Obstacle that is to be overcome, whilst the Space which the Power goes thro', is C *c*, namely the Space thro' which the Wedge has been carried. * 34 El. I.

Therefore the Force, by which the Wedge is thrust in, is to the Resistance of the Wood, when they are equipollent, as *e* E is to C *c* *. * 145.

Let C *g* be drawn parallel to E *e*, those Lines will be equal †, because the Side A C of the Wedge was carried by a parallel Motion; therefore the Ratio is found, which is given between *g* C and C *c*. † 34 El. I.

The little Line E *e*, and therefore also *g* C, is perpendicular to F E; for E *e* is an Arch of a Circle, so small, that it may be taken for a right Line; the Radius of whose Circle is F E.

Let *the Line* D H be *drawn thro' the Point* D *in the middle of the Base, reaching* A C *the Side of the Wedge at* H, *and making a right Angle with* F E, *the Side of the separated Wood continued*; this is parallel to C *g*. 287.

Because of the Sides *c* C, C D, and the other Parallels, the Triangles C *g* C, D H C, are similar; therefore D H is to D C, that *is to the Height of the Wedge*, as *g* C to C *c*; that is, *as the Force by which the Wedge is thrust in, to the Resistance of the Wood, when neither can overcome the other*; the Power being a little increas'd, the Parts of the Wood are separated.

288. When the Parts of the Wood are not feparated, except fo far as the Wedge goes in, the Lines A C and E F come together, and the Angle
* 8 El. VI. D H is a right one, therefore the Triangles C H D, C A D, are fimilar *; and D H is to D C, as A D to A C. In that Cafe therefore *the Force by which the Wedge is pufh'd in, is to the Refiftance of the Wood, where they are equipollent, as the half Bafe of the Wedge to its Side.*

SCHOLIUM II.

The Examination of a certain Machine.

289. OThers have defcrib'd a Machine for demonftrating the Properties of
* 279. the Wedge, different from that above defcrib'd *; I formerly caus'd fuch an one to be made, upon the fame Principle with it, which differ'd from it but a little; but I fhall fhew in a few Words in what it is fallacious.

In that Machine, the Wedge like that which is made ufe of in our Machine, was drawn by a Weight between the Cylinders I G, I G, in the fame manner as we faid was done in ours; but the Cylinders were mov'd along Brafs Rulers, upon which prominent Steel Axes were plac'd. The Cylinders were drawn by Weights hanging at Ropes, which went over fix'd Pulleys that were only moveable about their Axes.

In that Machine an Equilibrium is given, if the Force, by which the Wedge is thruft in, is to the Sum of the Weights P, P, as the half Bafe of the Wedge to its Height, which Proportion does not hold in the
* 277. Wedge *.

This Machine does not reprefent what happens in the Action of the Wedge, whereby Bodies are feparated; for the Weights drawing the Cylinders do not reprefent the Force, whereby the Cylinders cohere together; but each of the Cylinders is drawn to the fix'd Pulley by half of thofe Weights: but in our Machine, the Cylinders cohere by the Force of the whole Weights P P.

CHAP. XV.

Of Compound Machines.

290. EVERY compound Machine may be reduc'd to fimple ones; for it is made up of fimple ones join'd. When two are join'd, the Power is applied to one, and the Action of that Machine acts upon the other inftead of a Power; therefore that fame Action, in computing the Effect of the fecond Machine, is taken for the Intenfity of the Power, which moves this fecond Machine.

If

If the Action is quadrupled, by help of the first Machine; and tripled by the second Machine only, it is manifest that a quadruple Action is tripled, and is duodecuple or twelve-fold: but this may be applied to any Number of Machines; wherefore it is an universal Rule, that *in any compound Machine, the Ratio of the Intensity of the Power to the Resistance, with which it is in Æquilibrio, is compounded of all the Ratios, which would take place separately in the simple Machines.* 291.

Which Ratio we also discover, by comparing the Spaces pass'd thro' by the Power and Weight, in the same time, and the same Motion of the Machines; for these Spaces are inversely, as the Power is to the Weight *. * 145.

I will illustrate these Rules by Examples.

Experiment 1.

Three Levers A, B, C, are so dispos'd, that the Weight P, applied to the Lever A, is sustain'd by the Power Q, acting upon the Lever C. 293. Plate XI. Fig. 1.

The Fulcrum F of the Lever A is put upon the transverse Piece of Wood L L, which is supported by two Pillars, that the Application of a greater Weight, as P, may not be hinder'd.

The other Levers, B and C, are each sustain'd by one Pillar only; and are, when separate, in Æquilibrio, as well as A, by having smaller Weights join'd to the Arms, D, D, D, *&c.* In the Lever A, if it is applied alone, the Ratio of the Power to the Weight is 1 to 5. In the Lever B, 1 to 4. In the Lever C, 1 to 6.

A Ratio made up of these three is as 1 to 120. That is, the single Ounce Q sustains F, that weighs seven Pounds and an half, *i. e.* an hundred and twenty Ounces; which may also be determin'd by comparing the Spaces pass'd thro', in the same time, when the Machine is mov'd.

By these Levers join'd together a compound Steel-yard is made, whereby Bodies are weigh'd, by applying a smaller Weight. 294.

Experiment 2.

A B is a Lever of the first kind, which turns upon the fix'd Point C; this communicates Motion to a Lever of the second kind F H, which moves upon this last Point; at G the Scale L is suspended: the Weight D makes an Æquilibrium. 295. Plate XI. Fig. 2.

The Arm CB of the firſt Lever is divided into equal Parts; I here take notice of the greater Diviſions, which are ſubdivided into ſmaller: now it is manifeſt that by the ſmall Weight Q, moveable upon this Arm, it may be determin'd how much Bodies weigh as P, put into the Scale.

As many Levers are join'd, ſo ſeveral Wheels may be alſo join'd.

296. A Weight is hung upon the Axis of a Wheel, and a Rope is put round its Circumference, which that it may be drawn is join'd to the Axis of another Wheel, to whoſe Periphery the Power is applied.

After the ſame manner many Wheels might be made uſe of, and the Computation ſhould be made according to the general Rule *; but it is more convenient to transfer Motion from one Wheel to another, by the help of Teeth.

* 291.

297. If the Axis of a Wheel has Teeth, the Motion will be communicated to a ſecond Wheel, whoſe Periphery has Teeth, which anſwer to the firſt. By this means Motion may alſo be communicated to a third Wheel, and farther; but as theſe can't be applied ſeparately, the Rule N°. 291. does not take place here, but this other, whoſe Demonſtration is eaſily deduc'd from a Compariſon of the Spaces paſs'd through.

298. *The Ratio of the Power to the Weight, when the Actions are equal, is compounded of the Ratio of the Diameter of the Axis of the laſt Wheel, to which the Weight is faſten'd, to the Diameter of the firſt Wheel, to whoſe Circumference the Weight is applied; and the Ratio of the Revolutions of the laſt Wheel to the Revolutions of the firſt, in the ſame time.*

Experiment 3.

299. Plate XI. Fig. 3. A Power is applied to the Wheel A, which ſuſtains a Weight tied to a Cord, going round the Axis of the Wheel B, whoſe Circumference has Teeth, which enter between the Teeth of the Axis of the Wheel A; for this Axis CD has Teeth at D.

The Diameter of the Axis of the ſecond Wheel is an eighth Part of the Diameter of the firſt Wheel, and the Circumference of B, contains 35 Teeth, whilſt there are only ſeven in D; ſo that A turns round five times, whilſt B turns once. Therefore the Ratio of the Power to the Weight, is made up of the Ratio of 1 to 8, and 1 to 5, and is 1 to 40. Half a Pound ſuſtains twenty Pounds.

In the Use of Pulleys we see how *many Sheaves* join'd together make a simple Machine; *if they are moveable separately, they belong to compound Machines.* 300.

EXPERIMENT 4.

The five Sheaves O, O, O, O, O, are movable separately, and each has its peculiar Rope, one of whose Ends is fasten'd to a Hook of the Arm A * of the Pillar CG; the other is join'd to the Hook of the next Sheave; if we except the last which goes over the fix'd Pulley T, that the Power Q may be applied to it. This Power sustains a greater Weight according to the greater Number of Sheaves, for it is doubled by each. In this Case, one Ounce sustains thirty two, and a Quarter of a Pound sustains eight Pounds. 301. Plate X. Fig. 6. * 173, 174.

The Hooks of the Arm A are so to be dispos'd, that all the Ropes may be parallel.

If instead of single Sheaves, we make use of Tackle *, containing several Sheaves, the Increase will be greater; but several fix'd Sheaves are also requir'd for each. 302. * 263.

In the two following Experiments I make use of Machines of a different kind join'd together.

EXPERIMENT 5.

The drawing Rope of the Pulley is join'd to the Axis in Peritrochio, the Power is applied to the Wheel; and here, when the Force is increas'd six times by the Pulley, and when the Diameter of the Axis is the sixteenth Part of the Diameter of the Wheel, the Ratio of the Power to the Weight is made up of the Ratios of 1 to 6 *, and 1 to 16 †; therefore it is as 1 to 96; and therefore the single Ounce L sustains the Weight P of six Pounds. 303. Plate XI. Fig. 4. * 261. † 254.

The Axis in Peritrochio may be mov'd by applying a Screw; in this Case the Wheel must have Teeth, which Teeth must be inclin'd, to agree with the Thread of the Screw. Such is the Wheel A, which is mov'd by Help of the Screw DC. This is call'd *an endless Screw*, and a very small Power, by help of this, produces a wonderful Effect; for there are required so many Revolutions of the Screw, in each of the Revolutions of the Wheel, *i. e.* of the Handle whereby the Screw is mov'd, as the Wheel has Teeth. If to this Wheel another Wheel with Teeth be also added, the same Power will be able to overcome a greater Obstacle. 304. Plate XI. Fig. 5.

Experiment 6.

305. Plate XI. Fig. 5. The Machine exhibited here, conſiſts of two Wheels, and an endleſs Screw, which is mov'd by the Handle DE. In this, the Ratio of the Power to the Weight, when they are equipollent, is made up of the Ratio of the Semidiameter of the Axis of the laſt Wheel B, to the Length of the Handle DE, and the Ratio of the Revolutions of this Wheel to the Revolutions of the Handle, or Screw, in the ſame time. The firſt Ratio, in this Machine, is 1 to 30; the ſecond is collected from the Number of Teeth. The laſt Wheel B has 35 Teeth in its Periphery, the Axis of the firſt Wheel A contains 7 Teeth; therefore the firſt Wheel turns round five times, while the ſecond turns once; but this firſt contains 36 Teeth, therefore the Screw makes ſo many Revolutions, while this Wheel turns round once *: the Ratio compounded of theſe two is, 1 to 180, which is the ſecond Ratio ſought for; and the Ratio made up of this and the firſt, 1 to 30, is the Ratio of 1 to 5400, which is the Ratio of the Power to the Weight in caſe of an Æquilibrium: and if you increaſe the Power ever ſo little the Weight would be rais'd, if there were no Friction; which as it is not to be neglected in all theſe Machines, the Power muſt be increaſed pretty ſenſibly, before it can overcome the Weight; yet a very great Weight is rais'd by a very ſmall Power. The Length of the Piece ED may be doubled, or increas'd more, whereby the Action of the Power is doubled, or augmented more; in this Caſe a Weight of an hundred Pounds, and greater, is eaſily rais'd by a ſmall Hair.

* 304.

306. Innumerable other compound Machines may be conſtructed, whoſe Actions are determin'd after the ſame manner by Computation, according to the Rule mention'd in the Beginning of this Chapter *; or by comparing the Space paſs'd through by the Power with that paſs'd through by the Weight, or any other Obſtacle: for the Ratio of theſe is the inverſe Ratio of the Power and Weight, or Obſtacle, when the Action of the Power is equipollent to the Reſiſtance of the Obſtacles *.

* 291.

* 145.

307. Preſſures, which acting contrarily deſtroy one another, are always equal; if therefore the Intenſity of a Power is leſs than that of an Obſtacle, the Power ſhould exceed the Obſtacle in reſpect of the Way paſs'd through, and indeed as often as its Intenſity is leſs than that of the Obſtacle; for the Effects of the Preſſures can differ no other way *, and therefore there can be given no other Compenſation.

* 142.

C H A P.

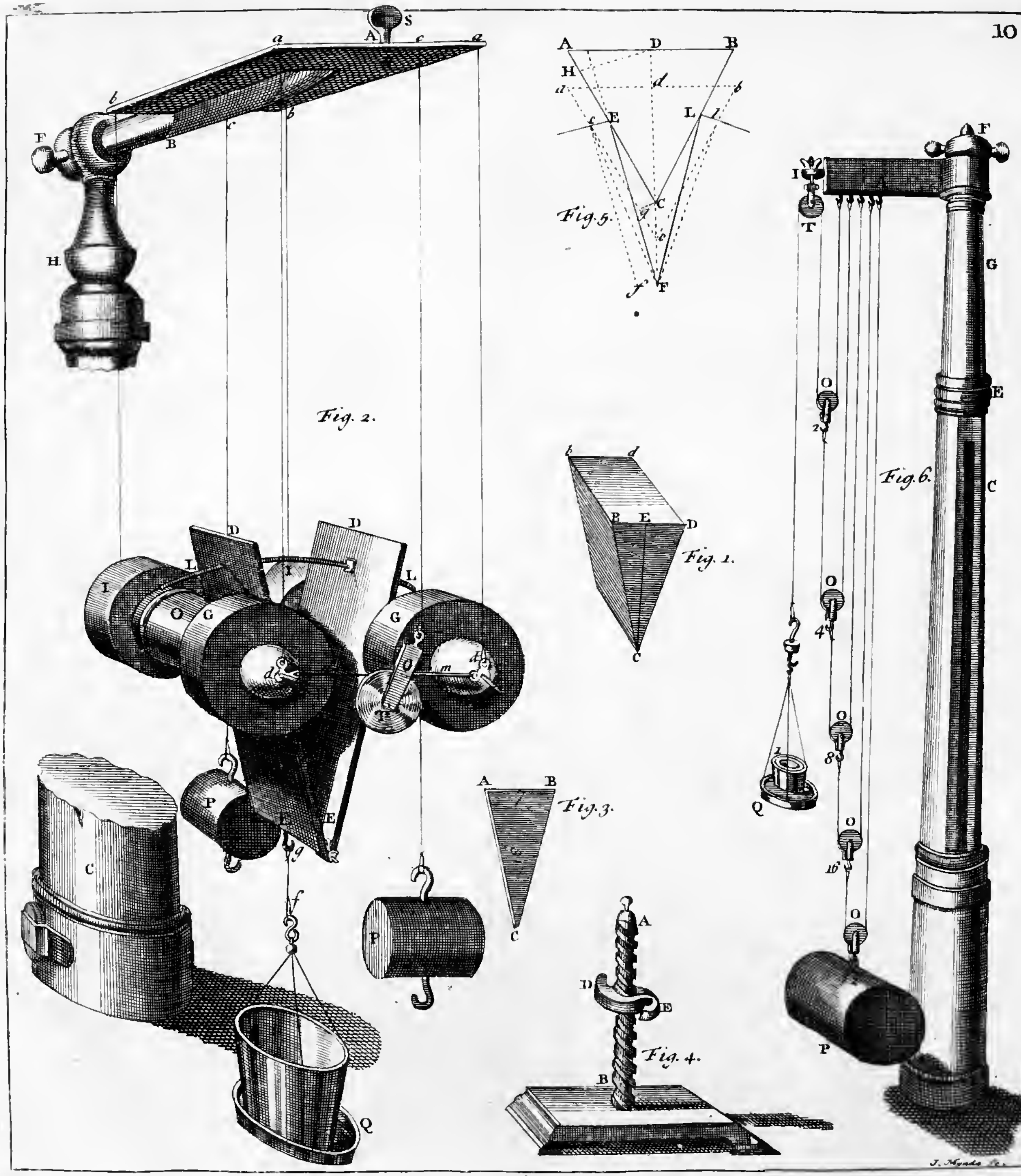


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
J. Mynde Sc.

CHAP. XVI.

Of Oblique Powers.

Definition 1.

I Call that a direct Power, which presses, or draws the Point, to which it is applied, in the Direction of the Line, in which it can yield. So that the Point, when mov'd, follows the Direction of the Power. 308.

I have treated hitherto of such an Application of Powers.

Definition 2.

In every other Case the Power is said to be oblique. And the Point, when mov'd, goes in a way different from the Direction of the Power. 309.

Let A be a Point, moveable only in the Line A D, and drawn towards D; let this be retain'd by an Action, whose Direction is along A E. If this Power were applied along A B, it is manifest, that the Power must be such, whose Intensity is equal to the Action, in the Line A D; but it is applied along A E, and its Intensity is requir'd. 310. Plate XI. Fig. 6.

Let A C be rais'd perpendicular to A D, and from a Point in this taken at pleasure, let C F be drawn, perpendicular to E A, continued if requir'd.

Let us suppose A C, C F, to make a bended Lever turning about C; and the Point A to be join'd to the End of the Leg C A.

If the Lever is turn'd about C, the Point A is mov'd, in the first Moment, (which only is spoken of, when the Æquilibrium is to be determin'd) in the Line D A B. But this Motion may be communicated to the Lever, by applying a direct Power at F along F E; or by a like Power acting along A B, and these are, when they produce the same Effect, as C A to C F *. * 238.

But the first coincides with the oblique Power at A applied along A E, which we seek; and the second with the Power directly destroying the Action which draws the Point A along A B, which we suppose to be known.

Let there be rais'd a Perpendicular to A B, *in a Point taken at pleasure, cutting the oblique Direction* A E *at* E, and making with 311.

* 28 El. I. it the Angle A E B equal to the Angle F A C *; wherefore by reaſon of the right Angles A B E, A F C, the Triangles C F A, † 4 El. VI. A B E, are equiangular, and C A is to C F, as A E is to A B †: Therefore, *if* A B *repreſents the Action, which can retain the Point* A *directly*, A E *will repreſent the oblique Power, which produces the ſame Effect along* A E: and it is eaſily determin'd how much a Power, applied to a Machine, is to be increas'd upon account of its Obliquity, as appears in the following Example.

Experiment I.

312. Plate XII. Fig. 1. To the Lever A B, which is in an horizontal Poſition, and whoſe Arms B C, A C are as 3 to 1, there is applied at A the Weight P of two Pounds; and at B a Power acting obliquely along *e h*, repreſented by the Weight Q. Let there be conceiv'd a Line *e i* perpendicular to the Lever, and ſhewing the Direction of the Agitation of the Point B, or *e*; if, the rectangular Triangle *e i h* being made, *e i* is to *e h*, as two to three, and the Weight Q of one Pound, there will be an Æquilibrium.

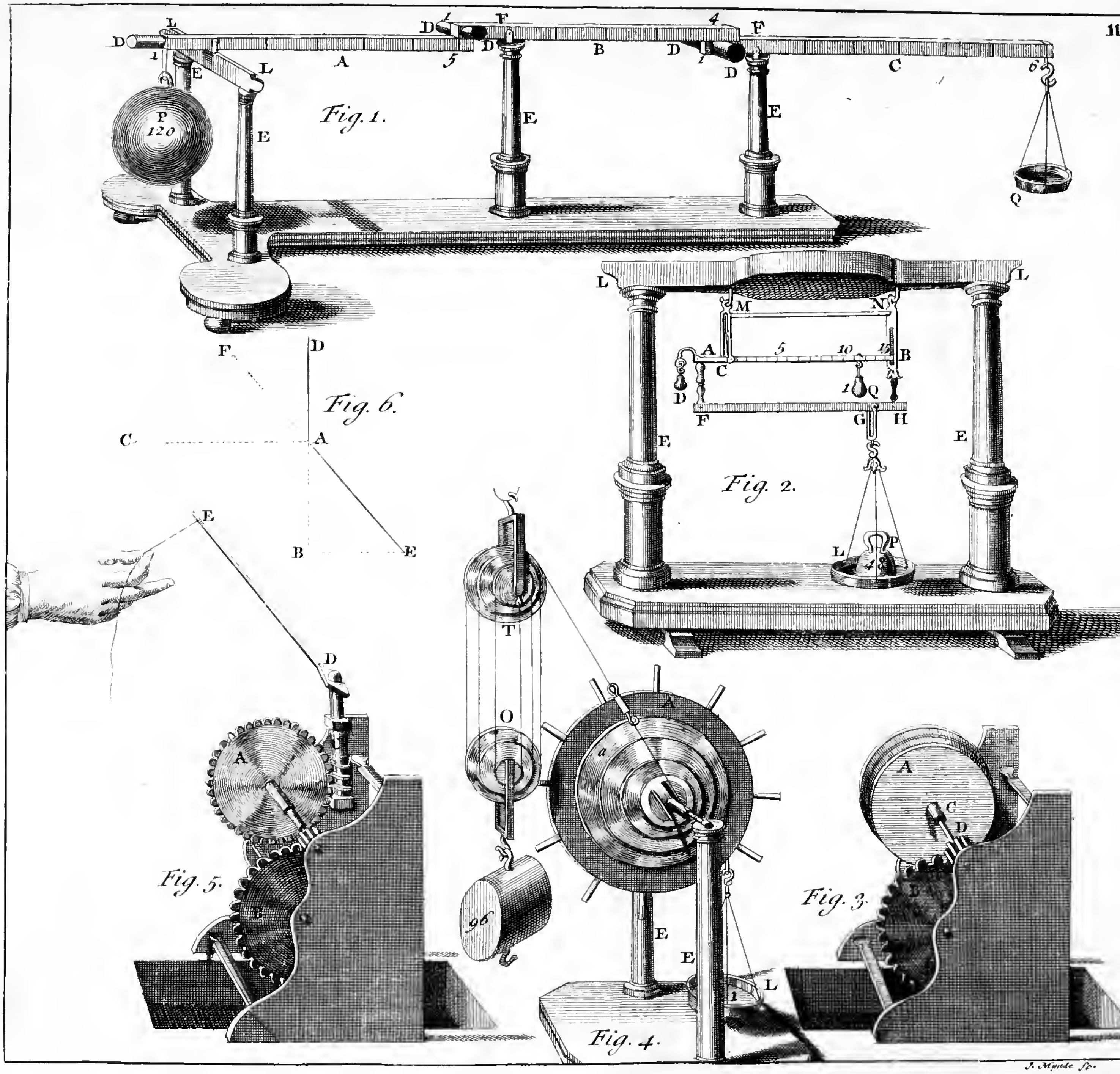
The oblique Power along *e h*, equal to three, exerts its Effect * 311. along *e i*, equal to two *; but the Action along *e i*, equal to two, † 235. ſuſtains in this Lever a Weight, equal to ſix †: therefore the oblique Power is to the Weight, as three to ſix, *i. e.* as 1 to 2.

313. In this Experiment the Lever before ſpoken of is uſed *, and * 237. the Rope *e h* is put round the Pulley, fix'd in the ſide of the Ta- † 161. ble †. I determine the Obliquity of the Power by the wooden Triangle L M N, which has a Plumb-Line faſten'd to it, but we give the Obliquity to the Thread by removing the Foot of the Lever more or leſs from the ſide of the Table. The Lever is retain'd by Weights, put upon its Foot; for this oblique Power draws the Lever in the Direction C B, and it ſhould be ſo retain'd, as yet to be moveable about C: but I determine the Force, which retains the Lever, which is equal to the Action of the Power along C B, by the rectangular Triangle E H I, and it is to the ob- * 311. lique Power as E H is to H I *.

314. We ſhould reaſon in the ſame manner when we ſpeak of an oblique Power, applied to any Machine.

I ſhall now conſider oblique Powers more univerſally.

315. Plate XII. Fig. 2. Let there be given a Point A drawn by three Powers, and at reſt. Let the Directions be A B, A D, A E, and let us ſuppoſe theſe Lines to be to one another, as the Powers are, when they retain the Point, *i. e.* when any two deſtroy a third. Then the Powers



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Powers A D, A E, draw the Point A along A *b*, with as much Force, as it is drawn along A B, by one Power only.

D *d*, E *e*, being drawn perpendicular to A *b*; the Force, whereby the Point A is drawn towards *b*, by the Power A D, is equal to A *d* *; and the Action of the Power A E, in the same Line A *b*, is * 311.
equal to A *e*. Therefore the Line A B, which expresses the third Power, destroying these two Actions, is equal to the Sum of the Lines A *d*, A *e*.

But this Equality is not sufficient, that the suppos'd Æquilibrium may be given between the three Powers; *f* A *g* being drawn perpendicular to A B, and D *f*, and E *g*, parallel to it; it is manifest that the Point A is drawn by the Powers A D, A E, along A *f* and A *g*, by Actions proportional to these Lines *, and that the * 311.
Point will not be at rest, unless these Actions mutually destroy one another, *i. e.* are equal.

Therefore what is requir'd to make the Æquilibrium, without 316.
which it cannot be, and which will always make it, is, that the Lines A *f*, A *g*, or D *d*, E *e*, be equal, and that A B be equal to the Sum of the Lines A *e*, A *d*. But we have these if, having compleated the Parallelogram with the Sides A D, A E, the Direction A B be the Production of the Diagonal A *b*, and equal to it; which appears, if we attend to the Triangles A *e* E, D *d b*, which agree in all things *. * 29. 34. 26. El. I.

Now if we attend only to the Triangle A D *b*, when the Side D *b* is parallel to the Line A E, and equal to it †, it follows, that † 34 El. I.
a Point, drawn by three Powers, is at rest, in this Case only, if 317.
the Powers are to one another, as the Sides of a Triangle made by Lines, parallel to the Directions of the Powers.

This Proposition shews how *the Actions of two Powers may be* 318.
reduc'd to one only. The two Powers A D, A E, are equal to one which should act along A *b*, and be proportional to this Line.

It appears from this also, that *the Action of a Power may be re-* 319.
solv'd into two Actions; and indeed innumerable Ways, by reason of the innumerable Triangles, which may be form'd, the same Side being kept.

It matters not whether a Body be drawn by a Power along A *b*, whose Intensity is express'd by this Line, or by two Powers along A D and A E, the Intensities *of which* are respectively proportional to these Lines; and this Resolution of a Power into two, is indeed arbitrary, but only in respect of one of them; for *if one is given,* *the other is determin'd*: for the Triangle is determin'd, when two Sides and the Angle contain'd by them are given.

Concerning the Propoſition of Number 317, we obſerve further, that from a known Property of Triangles, it is deduc'd, that the
320. Sides are to one another, as the Sines of the oppoſite Angles: *Three Powers are in Æquilibrio, which are to one another as the Sines of the Angles, form'd by the Directions of the oppoſite Powers.* That is, the Power, which acts along A E, is the Sine of the Angle B A D, and ſo of the reſt.

321. We have explain'd above the Machine, by which we demon-
* 250. ſtrate the Experiments of the oblique Forces *; now we make uſe of it, ſetting aſide the ſmaller Supports *f*, *f*, *f*, *f*, (*Plate* IX. *Fig.* 1, and 2.) then the Threads which are plac'd horizontally, and go over Pulleys, drawn by Weights hanging on them, will ſcarce be rais'd a quarter of an Inch above the Board.

Experiment 2.

322. Plate XII. Fig. 3. Three Pulleys T, T, T, are join'd to the Machine in Places taken at pleaſure; three Threads join'd in one Point by a Knot, go over the Pulleys, the Weights of ſix, nine and twelve Ounces hanging to them.

Theſe of themſelves endeavour to come to an Æquilibrium, to which they return, if the Knot be remov'd out of its Situation. Each of the Pulleys muſt be turn'd ſo, that its Groove may follow the Direction of the Thread.

On a ſeparated Board, of five or ſix Inches long, and as wide, made of thin Wood, or of Paſteboard, the Figure is drawn, which we have repreſented upon the Board of the Machine itſelf.

For the Triangle A D *b* is form'd, whoſe Sides are to one another as the Weights, that is, as ſix, nine, and twelve, or in ſmaller Numbers, as two, three, and four. The Side *b* A is continued to A B; at D *b* is drawn the Parallel A E, and the three Lines A B, A D, A E, will determine the Situation of the Threads, as appears, if, without diſturbing the Situation of the Threads, this thinner Board be thruſt in between the Threads and the Board of the Machine.

Experiment 3.

323. Plate XII. Fig. 4. The Triangle L M N, made of Braſs, equilateral, is drawn by three Threads, join'd to the Triangle at its Angles. If the Experiment with theſe Threads be made in the ſame manner, as the foregoing

foregoing Experiment with the Threads join'd by a Knot, these Threads will dispose themselves in the same manner, and agree with the Lines drawn on the Board.

EXPERIMENT 4.

This Experiment has only one Circumstance different from the foregoing Experiment. In this case the Threads are join'd to the Brass Plate, in the Points L, N, M. 324. Plate XII. Fig. 5.

The Use of the Propositions, N° 317, 318, 319, is very extensive. Let the Point A be drawn by the four Threads, A D, A E, A F, and A G, by Powers, respectively proportional to the Lines A D, A E, A F, and A G. The Triangle A F B being made, or the Parallelogram A F B G, the fore-mention'd Powers along A F, and A G, are reduc'd to one acting along A B, and which is proportional to this Line *, and there is an Æquilibrium given, if the three Powers along A D, A E, and A B, have a Relation determin'd for three Powers; in which Case, if the Powers along A D and A E be also reduc'd to one A *b*, A B and A *b* will be equal and in the same Line. 325. Plate XIII. Fig. 1.

* 718.

EXPERIMENT 5.

This Experiment is manag'd as the three foregoing, by making use of four Pulleys, and four Threads join'd by a Knot. 326. Plate XII. Fig. 3.

Let there be applied the Weights of six, fifteen, twelve, and nine Ounces; that is, which are to one another, as two, five, four, three. The Knot resting, on the Paper, put upon the Table C, Lines are to be drawn in the Directions of the Threads.

Let there be A B, A E, A F, A G, so determin'd, that A D may contain two such Parts, A E five, A F four, A G three; and the Parallelograms D E, G F, being made, their Diagonals A B, A *b*, will be in the same Line, and equal. Plate XIII. Fig. 1.

I han't drawn the Figure by itself as in the foregoing Experiments, not attending to the Threads; because if there are three Powers, the Weights being given, and their Order, there is an Æquilibrium one way only; but these being given, if there are four Powers, the Angles made by the Threads may be varied many ways, the Æquilibrium remaining. 327.

But if we would make use of the Figure, before delineated at pleasure, it must be put upon the Table G of the Machine, and the Threads being plac'd according to the Directions of the Lines, the Places of the Pulleys should be determin'd. Plate XII. Fig. 3.

What is ſaid of four Powers, might have been ſaid of five, and more; for if two of five be reduc'd to one, we have the foregoing Example.

Plate XIII. Fig. 2. The Point A is drawn by three Powers according to the Directions A B, A D, A E, A F and A G, and whoſe Intenſities are proportional to theſe Lines. The Powers along A D and A E are reduc'd to the ſingle Power A *c*; the Powers acting along A F and A G are reduc'd to one along A *h*; laſtly, theſe two new Powers, along A *c* and A *h*, are reduc'd to one along A *b*, which if it be equal to a fifth along A B, and acts in the ſame Line with it, but in a contrary Direction, there is an Æquilibrium, and not otherwiſe *. * 317.

Experiment 6,

329. Is made with five Pulleys, and five Threads join'd in a Knot, otherwiſe it does not differ from the 5th Experiment *. * 326, 327.

Another Machine, *by which thoſe Things which relate to a Point drawn different ways by Threads, are demonſtrated.*

330. Plate XIV. Fig. 1. This Machine conſiſts of a round flat Board, of about 8 Inches Diameter, which is horizontal and ſtanding upon a Foot; it has a Groove in the middle of its Thickneſs going round it to have Pulleys * join'd to it in any Part of its Circumference, the Tail of the Pulley going into the Groove. * 161.

This Board is a little hollow'd in the upper Part to receive another round Board, but leſs, and of about $\frac{1}{4}$ of an Inch in Thickneſs, ſtanding up a little above the firſt Board; ſo that the Thread that goes over the Pulley, fix'd to the Machine, in the Manner deſcrib'd, when extended horizontally may preſs upon the Surface D B E.

A great many ſuch little Boards are requir'd for different Experiments. They muſt be cover'd on each Side with Paper, that the Lines mention'd in the Experiments may be eaſily drawn upon them.

I formerly made uſe of this Machine, and it is very compendious; and tho' I mention'd a more perfect one, yet I did not think it unneceſſary to give an Account of it here. In this, tho' you only uſe three Powers, yet the Pulleys cannot be plac'd at pleaſure; but in all the Experiments, the Situation of the Pulleys is determin'd by extending the Threads from the Center of the circular Board along the Threads drawn upon it *. * 327.

Innumerable

Innumerable and very complex Problems, concerning different Forces acting differently, may be propos'd, whose Solutions may be drawn from what has been said before, and by help of our first Machine *, only by increasing the Number of Pulleys for the Experiments; but these complex Cases being seldom of use, we return to those that are more simple. 331.

* 250.

The Weight P, join'd to the Pulley, is sustain'd by Powers applied on either side to the running Rope; but drawing obliquely along C A and C B; the Powers are equal to one another, because every Rope that goes round a Pulley, is not at rest, except it be drawn equally on either side *; the Weight P is the third Power, and the Point C is drawn by those 3 Powers. Let the Line C E be suppos'd perpendicular to the Horizon, and the Line E F parallel to the Line C A: then will C E be to F E or F C, (for these two Lines are equal, by reason of the mention'd Equality of the Powers drawing along C B, C A) as the Weight P to either of the Powers applied to the Rope *. 332. Plate XII. Fig. 6.

* 159.

* 317.

If one End of the running Rope be fasten'd to a Nail, the Weight P is sustain'd by one such Power alone.

If the Weight P is not join'd to a Pulley, but the Ropes C A and C B be join'd to the Weight itself, it may be sustain'd by two unequal Powers; in which Case the Sides C F, F E, of the above-mention'd Triangle, are unequal, and to one another as the Power. 333.

Here it is to be observ'd, from the given Inclinations of the Ropes C A, and C B, to the Horizon, that the Proportion of the Powers to the Weight P may be determin'd from Trigonometrical Tables. 334.

If in the Triangle F C E, the Line F G be conceiv'd drawn thro' F, and parallel to the Horizon, G C will represent that Part of the Weight which is sustain'd by the Power C F *; and G E will be the Part that the other Power sustains. When a Pulley is made use of, as in this Figure, G C and G E are equal.

* 311.

Let F be the Center of the Circle, whose Radius is G F, F E will be the Secant, and E G the Tangent of the Angle, which F E or C A makes with the Horizon; and C F will be the Secant, and C G the Tangent of the Angle of Inclination of the Thread C B to the Horizon: whence it follows, that the Powers are proportional to the said Secants, and the Weight P follows the Proportion of the Sum of the above-mention'd Tangents.

A Machine, *whereby Experiments are demonſtrated of Weights ſuſtain'd by oblique Powers.*

335. Plate XIII. Fig. 3. A Board half an Inch thick, of the Figure FAIBE, is repreſented in the Plate. To the Side of the Machine are faſten'd the vertical Pulleys T T, and parallel to the Surface of the Board, from which they are but little diſtant. On the broader extreme Parts A F, BE, which are cover'd with white Paper, Lines are drawn, each of which agree with the Thread that is extended and goes over the Pulley.

At D a Tail is join'd to this Board behind, which is ſeen at *g h*, by whoſe Help the Machine is join'd to the Pillar C *, at any Height. The Tail goes through the Opening of the Pillar, and is encompaſs'd with a Screw, that by the Help of the Nut *i*, the Board may be fix'd; as the Tail is round, it may turn in the Opening of the Pillar, and ſo the Machine may be fix'd eaſily, by Help of the plumb Line IL, ſo that the Line AB may be horizontal.

* 162.

At the End of the Lines above mention'd drawn on the Surface of the Board at BE, and AF, there are Numbers written, which expreſs the Tangents of the Angles, which thoſe Lines make with the Horizon, when the Machine is fix'd as we have ſaid. The Secants of the ſame Angles are mark'd in the middle Points of the ſame Lines; ſo that in each Line, the Number in the Middle is to the Number at the End, as the Power plac'd along the Direction of the Line, to that Part of the Weight which it is able to ſuſtain *.

* 334.

Experiment 7.

336. The Weight P is ſuſtain'd by two Threads join'd at *s*, and carried over the Pulleys of the Machine, and which are ſo drawn by the Powers O and Q, that the Threads agree with the Lines T *n* and T *r*.

Now O and Q are to one another, as $11\frac{1}{2}$, and $13\frac{3}{4}$, that is, as the Numbers in the middle of the Lines; and P is expreſs'd by the Sum of the Numbers, mark'd at the End, which is equal to $15\frac{1}{4}$.

If O is equal to $11\frac{1}{2}$ Ounces; Q $13\frac{3}{4}$ and P $15\frac{1}{4}$; there will be no Reſt till the Threads return to the Inclinations here mark'd.

Experiment 8.

337. If a Pulley be made uſe of, the Powers O and Q muſt be equal, and for the reſt the Experiment is like the foregoing. But in determining

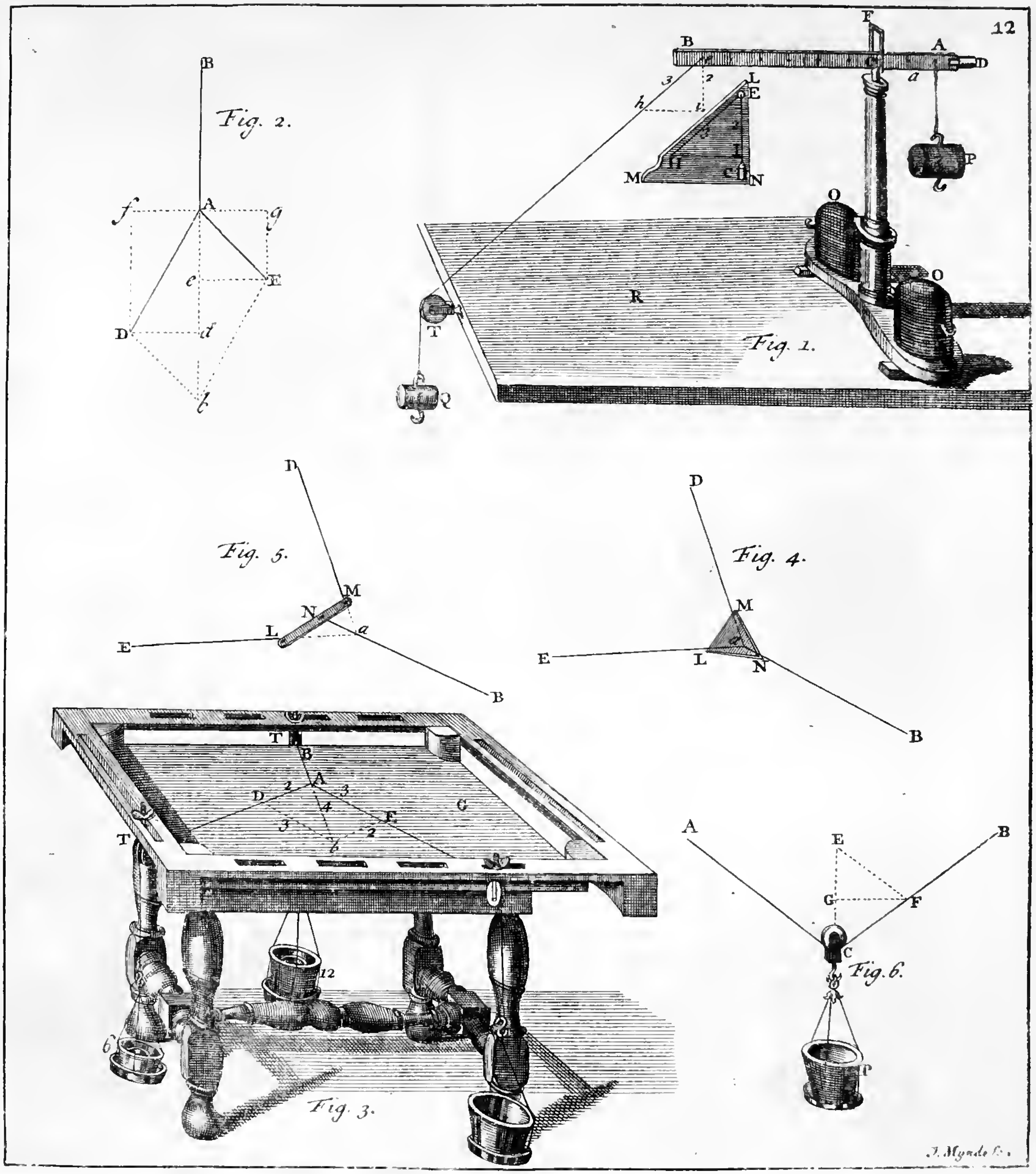
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Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
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termining the Weight P we must have regard to the Weight of the Pulley: and to do this conveniently, we make use of such an one, which, together with its Hook and Box, weighs just half an Ounce.

The Force with which a Body endeavours to descend upon an inclin'd Plane, is determin'd by what we have said of the oblique Power *. * 311.

Definition 1.

An inclin'd Plane is such an one as makes an oblique Angle with the Horizon. 338.

C B represents a Line parallel to the Horizon, AB makes with it the oblique Angle ABC, and represents the inclin'd Plane. From the upper End of the Plane is let fall AC, a Line perpendicular to the Horizon. Plate XIII. Fig. 4.

Definition 2.

AB *is call'd the Length of the Plane.* 339.

Definition 3.

The Line AC *is call'd the Height of the Plane.* 340.

The Body P being set upon the Plane A B, endeavours to descend upon the Plane in the Direction AB; let us suppose this Body to be retained by the Thread *e* D parallel to this Plane. Let the Thread be continued along *ef*; the Point *e* is moveable in the Line D *ef*, and it is drawn downwards by the Weight of the Body along *ei*. If this Line represents that Weight, the Action in the Line D*f*, whereby the Body is drawn along it by its Weight, will be had by drawing *if* perpendicular to D*f*, and *ef* will represent the Force requir'd *. * 311.

If *eg* be perpendicular to the Surface of the Plane, and *ig* be drawn perpendicular to *eg*, *eg* † or *if* will express the Action of the Weight on the Plane itself. † 34. El. I.

The Angle *ief* is equal to the Angle BAC, for each of them is equal to the Angle *io* B ‡, therefore the Triangles *ief*, BAC, are equiangular; and *ef* is to *ei*, as A C to A B ||; and *the Force by which a Body endeavours to descend upon an inclin'd Plane, is to the Weight of the Body, as the Height of the Plane to its Length.* ‡ 29. El. I. || 4. El. VI.

342. *A* Machine, *whereby the Properties of the inclin'd Plane are ſhewn.*

Plate XIII. Fig. 5. An Iron Ruler or Bar BC, well work'd and poliſh'd, whoſe Thickneſs ſcarce exceeds a Quarter of an Inch, and whoſe other Dimenſions are eaſily determin'd by the Figure, which is reduc'd to a ſixth Part, is ſuſtain'd by a wooden Pillar EF. In the Head of this Pillar F, about a Point as a Center, the Bar (which is broader in that Place) turns in ſuch a manner that it may be inclin'd at pleaſure, and be made faſt by the help of the crooked Screw DC going thro' the Pillar. The Nuts *m* and *n* preſs it to the Pillar on either ſide.

This Pillar has a Tail which goes through the Table and turns round in it, which Tail is faſten'd by a Nut under the Table.

The Iron Bar above mention'd goes thro' the Braſs Cylinder G, whoſe Weight is 12 Ounces, but may be alter'd by putting a Ring round it. In theſe Experiments our Ring is of 4 Ounces, which makes the whole Weight one Pound. At the End of the Bar is join'd with it a wooden Cylinder A, which receives at its End the Bar, let in about one Inch. Into this laſt Cylinder is alſo
* 161. thruſt the Tail of the Pulley T *, about which goes a running Rope, which is faſten'd to the Cylinder G, and is parallel to the Bar; ſo that the Weight F applied to the Rope, ſhall ſuſtain the Force whereby the Cylinder G endeavours to deſcend along the Bar.

The Cylinder G is made uſe of in other Experiments alſo, eſpecially in thoſe relating to central Forces, which I ſhall conſider in the following Part of this Book; and where we ſhall more accurately deſcribe this Cylinder.

343. The flat Inſtrument L is uſed for determining the Inclination of the Bar BC. For various rectangular Triangles are drawn upon its Surface, which have one common Hypotenuſe, in one of whoſe Ends the Plumb-Line hangs. This Hypotenuſe contains 16 Parts, which ſerve to meaſure the Sides of the Triangles; but only their Lengths are ſet down, which are all terminated at the End of this common Hypotenuſe, as the Figure ſhews.

Experiment 9.

344. By the help of the little Board L, the Ruler BC is ſo diſpoſed, that the Length of the Plane is to its Height, as 16 to 6. And the Weight P of 6 Ounces will ſuſtain the Weight G of one Pound; which

which may be at reſt in any Part of the Ruler or Bar, and riſes or falls with the leaſt Impulſe.

When a Body laid on an inclin'd Plane, is drawn in a Direction different from the Plain's Inclination, we alſo determine the Force from what has been before demonſtrated *. 345.

* 317.

Experiment 10.

A little Board L, rectangular at *f*, on which is drawn the right-angled Triangle *a b c*, whoſe Sides are parallel to the Sides of the Board, is uſed here. 346. Plate XIII. Fig. 6.

The Bar B F is ſo inclin'd *, that the Length of the Plane is to its Height, as the Side *c b* to the Side which is parallel to the Bar, being *a c* of the Triangle *a c b*; that is, in the preſent Caſe, as 16 to 8.

* 343, 344.

The Rope faſten'd to the Cylinder G, goes round the Pulley T, join'd to the Pillar C *; a Weight of 9 Ounces is hung on; and the Cylinder can reſt but in one Place of the Bar. In that Situation, the Direction of the Rope agrees with *c e* drawn upon the Board, and whoſe Length contains 9 of thoſe of which *c a* contains 8, and *c b* 16.

* 162, 169.

The Triangle *a b c* is ſimilar to the Triangle *a o c*, in which *a o* is vertical, and *c o* horizontal; that follows from the Situation of the Bar B F. Therefore the Angle *b c a* is equal to the Angle *o a c*; and *b c* is parallel to *a o* *, and vertical. 347.

* 27 El. I.

The Cylinder G is drawn by three Powers. 1. By its own Weight acting vertically, in a Direction parallel to the Line *b c*. 2. It is ſuſtain'd by the Bar B F; that is, it is preſs'd in a Direction perpendicular to that Bar, and parallel to the Line *a b*. 3. Laſtly, it is drawn by the Rope *c e*. Theſe three Powers are as the Sides of the Triangle *e c b* *. Therefore when the Weight of the Cylinder is 16, the Weight of P is 9.

* 317.

The Force with which the Cylinder preſſes the Bar B F is as *e b*; if the Rope was parallel to the Bar, that Preſſure would be as *a b*; but it is diminiſh'd now becauſe the Cylinder is drawn more upwards. 348.

In our Experiment the Cylinder runs upon two little Rollers, and the Rope which goes out of the Hole in the anterior Surface, and is directed upwards, diminiſhes only the Preſſure of the anterior, and muſt raiſe it a little, before it can change the Preſſure of the hinder little Roller.

349. If therefore $a\ b$ be divided into two equal Parts at r; the Part $a\ r$ ſhews the Preſſure of the anterior Roller, and $r\ b$ that of the hinder, when they are not diminiſh'd; but in the laſt Experiment, $e\ r$ ſhews the Preſſure of the anterior Roller.

Experiment 11.

350. Things remaining as in the former Experiment, change only P, and add two Ounces; now the Force which draws the Rope is equal to 11 Ounces. The Cylinder changes its Situation, and the Line $c\ d$, which is 11, ſhews its Inclination. Now there is no Preſſure of the anterior Roller on the Bar, it is ſeparated a little
* 345. from it, which agrees with what we have explain'd *.

I'll add but one thing concerning oblique Forces, and make an end of this Subject.

351. *Mariotte*, in the ſecond Part of his *Treatiſe of the Motion of Water*, demonſtrates a mechanical Paradox, whoſe Explication alſo is eaſily deduc'd from the Proportion ſo often mention'd, of a Point drawn by three Powers.

This Paradox we ſhall immediately ſet before you by a particular Machine.

Experiment 12.

352. Plate XIV. Fig. 2. The Leaver A C B is ſo made, that when left to itſelf, it acquires the Situation in which the Part A C is horizontal. A C and C B are equal; let A C be produc'd, and let B f be perpendicular to that Production. The Arms of the Leaver make ſuch an Angle, that A C is double C f.

Let the Weight of one Pound P be applied at B; at A the
* 235. Weight of half a Pound G Q, and you will have an Æquilibrium *: if the Ratio between A C and C f were different, different Weights muſt be uſed.

Now the Paradox is, that tranſpoſing the Weights the Æquilibrium will be kept.

That the Experiment may anſwer, we muſt have the following Conſtruction of the Weight G Q: The chief Part of it is the Box G, which contains 3 Braſs Rollers faſten'd to their ſmall Steel Axes; theſe Axes are plac'd in the ſame Line, and the Rollers are ſeparately moveable: the outward ones are of the ſame Bigneſs, but the middle one ſomething leſs. To that Box is join'd the Hook V, and now the whole Machine weighs 3 Ounces, (that is, our Machine:) add a Scale of one Ounce, in which you muſt put a

Weight

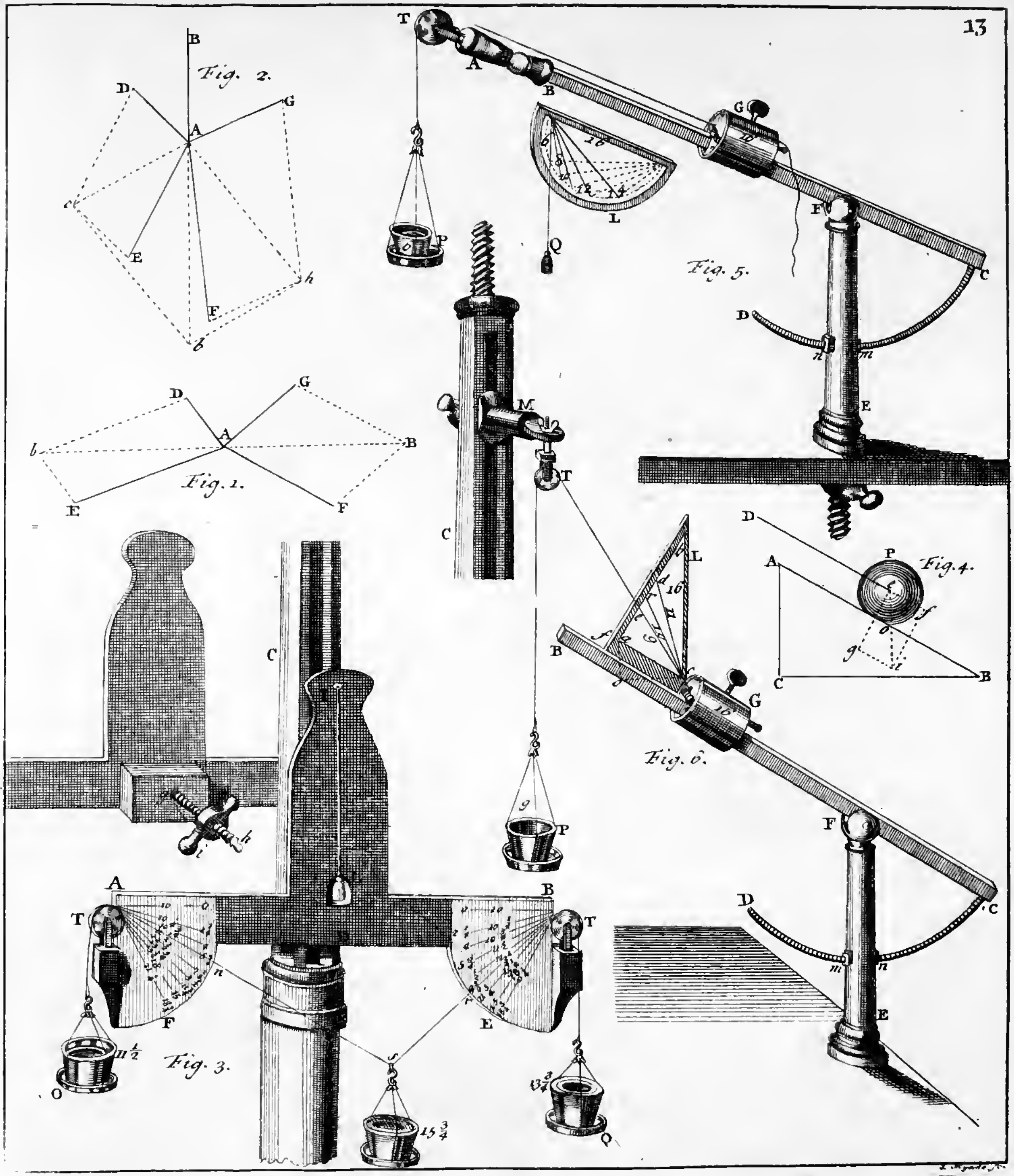
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Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.

Weight of four Ounces, and then we have the half Pound above-mention'd.

To the Pillar * ſo often made uſe of we join the vertical Board T, and make it faſt; in this Board is a Slit *m n*, in which the Leg of the Lever C B may move very freely, as the Lever is agitated about its Center. The Prop of the Lever is made faſt by putting on the Weights O O. The Weight G Q is now ſo applied, that whilſt it is ſuſtain'd obliquely by the Lever, it is alſo retain'd by the Board T; the two external Rollers touch the Board at the Outſides of the Slit *m n*, whilſt the middle one is on the Lever itſelf. The Pillar E muſt be ſet ſo far from the Board T, that the Leg A C being horizontal, and G being applied as we have ſaid, the Roller which bears on the Lever ſhall touch it in the Point *b*, which anſwers to B, drawing B *b* perpendicular to B C. * 162.

In this Situation G Q will ſuſtain the Weight P, (of *Fig.* 2.) which now hangs at A. The leaſt thing diſturbs this Æquilibrium; if G be depreſs'd a little, ſo that the Roller no longer touches the Lever at *b*, P will be rais'd; on the contrary, if G be ever ſo little rais'd, P will prevail.

This is the Demonſtration. In the rectangular Triangle C *f* B, let there be drawn from the right Angle *f d* perpendicular to C B; the Triangles *f d* C, C *f* B, will be ſimilar; after the ſame manner drawing *d h* perpendicular to C *f*, the Triangles *d h f* and C *d f* will be ſimilar; and therefore likewiſe *d h f* and C *f* B † will be ſimilar. 354. Plate XIV. Fig. 2. * 8 El. VI. † 21 El. VI.

The Weight P is to G Q, as A C, or C B, to C *f*; i. e. as *d f* to *d h*.

In *Fig.* 4. the Box is drawn by 3 Powers. 1. By its own Weight vertically downwards, to which Direction the Line *d h* (*Fig.* 2.) is parallel. 2. Horizontally by the Action of the Board T, which Direction *h f* ſhews. 3. Laſtly, the Lever itſelf acts in a Direction perpendicular to it, and that Direction is *d f*: therefore the 3 Powers are to one another as the Sides of the Triangle *d h f* *. * 316. Therefore the Weight G Q is to the Force, which preſſes the Lever at B, as *h d* to *d f*, that is, as (in *Fig.* 2.) C Q to P, therefore that Weight P is equivalent to the Preſſure whereby the Point B, or *b*, is preſs'd by the Roller applied to it; by which Preſſure is ſuſtain'd the Weight P, hanging at A, becauſe A C, B C, are equal *. * 143. Thus tho' you tranſpoſe the Weights the Æquilibrium will be kept, whatever be the Ratio between C *f* and C A; that is, *d h* and *d f*, if there be the ſame between the Weights G Q and P.

BOOK I.

Part III. Of Motions, alter'd by the Actions of Powers.

CHAP. XVII.

Of Sir Isaac Newton's *Laws of Nature.*

Hitherto we have consider'd Pressures, destroy'd by contrary Pressures. Now we shall examine Pressures acting on Bodies left to themselves, and persevering in Motion. Here, as in all Physics, we must reason from Phænomena, and deduce the Laws of Nature from them.

Sir *Isaac Newton* has given three, whereby we think we may explain what we know of Motion.

Law I.

355. *Every Body perseveres in its State of Rest, or of uniform Motion in a right Line, unless it is compell'd to change that State by Forces impress'd thereon.*

We see that a Body by its Nature is inactive, and incapable of moving itself, whence, unless it be mov'd by an external Cause, it must necessarily always remain at Rest.

For a Body once mov'd continues to move on in the same right Line, with the same Velocity, as appears fully by daily Experiments; for we see no Change in Motion, except from some Cause.

356. A Body is carried on by its *innate Force* *, and that Force, as follows from this Law, *is not chang'd, except by the Action of some external Cause.*

* 18.

Law II.

The Alteration of Motion is ever proportional to the moving Force impress'd, and is made in the Direction of the right Line in which that Force is impress'd.

358. We also deduce this Law from Phænomena; for in a Ship, a Body that is driven along moves in the same manner, whether the Ship

Ship be at reſt, or move with any Velocity equably. Which ſhews that *two Motions do not diſturb each other*, of which we have an Example in many Motions. When another Force is ſuperadded to a Body in motion in the ſame Direction, the Motion becomes ſwifter.

When the new Impreſſion is contrary to the Motion of the Body, the Motion is retarded.

If the new Impreſſion acts obliquely, the Body changes its Way.

Let there be a Body at A, mov'd along the Line AE, with a Celerity which we repreſent by that Line; let an Impreſſion act at A, along the Direction AD, which along that Direction communicates to the Body already in motion the Celerity AD. Now the Body is agitated by two Motions, by which the Lines AE and AD are gone through in the ſame Time; theſe two Motions do not diſturb one another *, but the Body is carried by a Motion compounded of both. 359. Plate XV. Fig. 1. * 358.

In order to determine this compound Motion, let us conceive the Line AD, whilſt the Body runs along it, to be carried by a parallel Motion with the Celerity with which the Body is mov'd in the Direction of the Line AE, which the Point A goes through in that Motion. That is, we conceive all the Points of the Line AD to run thro' Lines parallel to AE, with the Velocity AE; therefore the Body, wherever it is in that Line, has the ſame Motion with it; we ſuppoſe alſo that the Body by its proper Motion is carried along that Line, and ſo to be ſubject to a double Motion, as all this happens in a Ship mov'd uniformly.

Now let the Line be carried to *ad*, the Body will be in *b*, and AE will be to AD, as A*a* to *ab*; becauſe each Motion is equable. Having compleated the Parallelogram ADBE, and drawn the Diagonal AB, it appears clearly that the Point *b* is in that Diagonal, and that the Body is in B, when the Line AD is come to EB; therefore *the Body by a compound Motion goes through the Diagonal of a Parallelogram form'd by Lines, which by their Situation ſhew the Directions, and by their Lengths the Velocities; and the Diagonal expreſſes the Celerity of the compound Motion.* 360.

In what follows we ſhall ſee that this Law does alſo hold good in reſpect to the innate Force, that is, that the Force innate to the Body moving along the Diagonal, is equal to the Forces of which the firſt is communicated to the Body along AE, and the ſecond along AD. That is, if the ſecond Force does not in part act contrary

trary to the firſt; which happens when the Angle EAD is obtuſe, in which Caſe the new Impreſſion is ſpent in part in diminiſhing the firſt.

LAW III.

361. *To every Action there is always oppoſite and equal Reaction, or the mutual Actions of two Bodies upon each other, are always equal, and directed to contrary Parts.*

362. Every Action requires a Reſiſtance, take the one away and the other will vaniſh; for who can conceive an Action without an Obſtacle?

If the Action is greater than the Reſiſtance, it will in part act without an Obſtacle, which cannot be. If the Reſiſtance be ſuppoſed greater, as this is a contrary Action, we fall into the ſame Concluſion; and it is evident enough that contrary Actions are neceſſarily equal. But this will appear more evident by the following Examination.

363. Let there be a Force acting on an Obſtacle, if it does not yield, it is retain'd by ſome Force, and there is a Preſſure which acts contrary to the firſt, and deſtroys it, wherefore it is equal to it *. If I preſs a Stone fix'd in a Place with my Finger, my Finger is equally preſs'd by the Stone.

* 125.

364. If the Obſtacle yields, it reſiſts by its Inertia, or Inactivity *. Let a Body be drawn by a Rope, tho' it may be mov'd eaſily, yet the Rope will be ſtretch'd, and each way equally, which ſhews the Equality of the oppoſite Actions: but a Body yields tho' it reſiſts, with a Force equal to that by which it is drawn, becauſe it does not reſiſt as long as it is at reſt, but when it acquires Motion *.

* 19.

* 19.

A Carriage drawn, in the Beginning of the Motion reſiſts by its Inertia, afterwards by the Friction of the Wheels, and becauſe of Obſtacles, which grow leſs indeed but continually happen in the Way, and that Reſiſtance is overcome by the Horſe, when the Carr runs by its proper Motion *. In that Motion the Harneſs is equally ſtretch'd both ways, which ſhews the Action and Reaction to be equal; yet the Carriage follows the Horſe, becauſe it only reſiſts when it follows him; and it reſiſts becauſe it follows.

* 355.

The Motion of an agitated Body is deſtroy'd, in the ſame manner as it is communicated to one at reſt; wherefore as in this Caſe the Reſiſtance is equal to the Action, ſo it is in that.

365. Laſtly, We ſee that alſo in thoſe Motions which are referr'd to Attraction *, the Law of which we ſpeak has place.

* 73.

The

The Loadſtone draws Iron to it, and is equally drawn by Iron.

EXPERIMENT.

The Loadſtone M hangs on the Hook of the Scale of a Balance, and the Weight P keeps it in Æquilibrio. It may now move very easily, and bringing Iron near it, at a certain Diſtance the Loadſtone comes to the Iron; and drawing the Iron away before the Loadſtone is come to it, the Loadſtone follows the Iron, in the ſame manner, when the Iron hangs up and the Loadſtone is brought near. But when the Iron F is ſuſpended, taking away the Loadſtone, the Weight P is to be kept, and the Æquilibrium muſt be reſtored, putting a Weight in the Scale L, that in each Caſe the Balance may be equally loaded, and the ſame Quantity of Matter be put into motion. 366. Plate XIV. Fig. 5.

Let a Man ſitting in a Boat draw with a Rope another Boat equal and equally laden; both Boats will move equally, and meet in the middle of their firſt Diſtance: If one Boat be bigger than the other, or more heavily laden, the Celerities will be different according to the different Quantity of Matter in the Boats.

When the Boats are come together, their Reſt ſhews that their Motion was equal; for altho' they preſs each other, and they can eaſily yield, neither of them removes the oppoſite one from its Place. 367.

If before the Congreſs, an Obſtacle be interpos'd, which hinders the Concourſe not the mutual Action, that is at reſt with the Bodies, though it be retain'd by no Force; and it is manifeſt, that the Obſtacle is equally preſſed on each ſide as the Bodies tend to one another.

CHAP. XVIII.

Of the Acceleration and Retardation of heavy Bodies.

DEFINITION I.

AN accelerated Motion is that, whoſe Celerity becomes greater every Moment. 368.

DEFINITION 2.

369. *A retarded Motion, is that whose Celerity is diminish'd every Moment.*

The Force of Gravity acts continually in all Bodies according to
* 159. their Quantity of Matter *, and whatever they are, they are mov'd in the same manner by Gravity. When a Body falls freely, the Impression of the first Moment, is not destroy'd in the second Moment; therefore the Impression of the second Moment is superadded to it, and so of the rest: *the Motion* therefore *of a Body falling freely is accelerated*, and it is plain from Phænomena, that Motion is accelerated *equally in equal Times*. For the Consequences which we shall hereafter deduce from this Principle, that the Acceleration is equable, would not be true, if there was no such Principle. But that they agree with Experiments, shall be shewn in the next Chapter.

371. From this Principle we conclude, that *Gravity acts in the same manner in a Body in motion as in a Body at rest*; as it communicates equal Celerities to a Body in equal Moments.

372. *The Celerity* therefore *acquir'd in falling, is as the Time in which the Body falls.* The Velocity, for Example, acquir'd in a certain Time, will be double, if the Time is double; triple, if the Time is triple, &c.

373. Let the Time be express'd by the Line A B, and let the Begin-
Plate A.V. Fig. 2. ning of the Time be A. In the Triangle ABE, the Lines 1 *f*, 2 *g*, 3 *h*, which are parallel to the Base, are drawn through the Points 1, 2, 3, and are to one another as their Distance from A, A 1, A 2, A 3; that is, as the Times, which are mark'd by those Distances; and denote the Velocities of a Body falling freely
* 372. after those Times *. If instead of mathematical Lines, others are made use of having a very small Breadth, equal in every one, their
† 1 El. VI. Proportion is not chang'd by that means †, and these very little Surfaces equally denote the aforesaid Velocities. In a very small Time the Velocity may be consider'd as equable, and therefore the Space,
* 119. gone through in that Time, is proportional to the Velocity *; and the same very little Surfaces may denote the Spaces gone through in very small, but equal Times: therefore in every very little Space above mention'd, if the Breadth of the Surface may be taken for the Time, the Surface itself will denote the Space gone through. The whole Time A B, consists of such small Times; and the Area of the Triangle ABE is form'd of the Sum of all those small Surfaces,

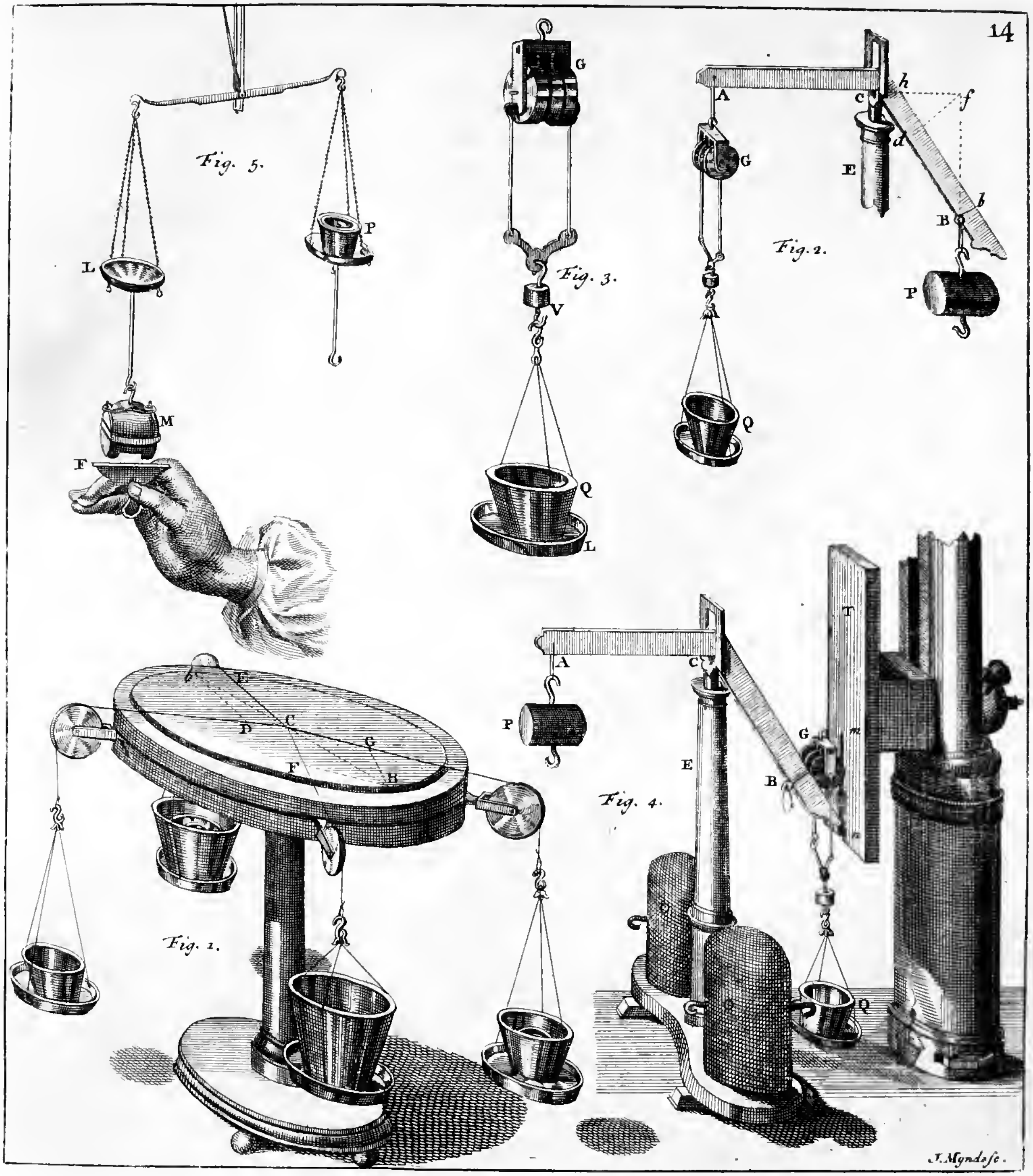
Fig. 5.
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P
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Fig. 3.
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Fig. 2.
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Q
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C
D
G
F
B
Fig. 1.
A
C
T
P
G
m
E
B
Fig. 4.
Q
J. Mynde sc.

faces, answering to those small Times: therefore that Area expresses the Space run through in the Time AB. After the same manner, the Area of the Triangle A 1 *f*, represents the Space gone through in the Time A 1; those Triangles are similar, and their Areas are as the Squares of the Sides AB, A 1 *; that is, *the Spaces gone through from the Beginning of the Fall, are to one another, as the Squares of the Times, during which the Body fell; or as the Squares of the Velocities acquir'd in falling.* * 19 El. VI. 374.

The Time AB being divided into equal Parts, A 1, 1. 2, 2. 3, 3. B; let Lines parallel to the Base be drawn through the Divisions; *the Spaces gone thro'* in those Parts, that is, *in the first, second, third,* &c. *Moment, supposing the Moments equal,* are to one another as the Areas A 1 *f*, 1 *f g* 2, 2 *g h* 3, 3 *h* E B; which Areas, as appears by a Sight of the Figure, *are as the odd Numbers* 1. 3. 5. 7. 9, &c. 375.

If a Body, after it has fallen during the Time AB, be no longer accelerated, but continues uniformly in motion with the Celerity BE acquir'd by that Fall, during the Time BC, equal to the Time of the Fall, the Space gone through by that Motion is denoted by the Area BEDC, double the Area of the Triangle ABE *; and therefore, * 41 El. I.

A Body falling freely from any Height, will be able to go through a Space double that Height, with the Velocity that it has acquir'd in falling, in a Time equal to the Time of the Fall with an equable Motion. 376.

The Motion of a Body thrown upwards, is retarded in the same manner, as the Motion of a falling Body is accelerated, as follows from Law 2 *; in this case the Force of Gravity conspires with the acquir'd Motion; and in that it acts contrarily. * 357.

But as the Force of Gravity in every equal Moment communicates to the Body equal Celerities *; *the Celerity of a Body thrown upwards is equally diminished or retarded in equal Times.* 377. * 370.

This same Force of Gravity generates a Motion in a falling Body, and destroys it in a rising Body, and always acts in the same manner upon a Body in motion as it does upon a Body at rest *. Therefore the same Celerities are generated and destroy'd in the same Times. *A Body thrown up* rises till it has lost all its Motion; *it ascends* therefore *during the Time that a Body in falling can acquire the Velocity with which it is thrown up.* * 371. 378.

If BA represents the Time, in which a Body ascends, and BE the Celerity with which it is thrown up; the Ascent ceases, when 379.

 the

the Celerity of the Body is nothing; therefore the Lines parallel to the Base in the Triangle ABE represent the Celerities in the Moments
* 377. of Time to which they answer *, and the Area of the Triangle denotes the Space gone through in the Ascent, as may be deduced from
† 373. the Demonstration given about falling Bodies †. For since B is the Velocity, that a Body may acquire in falling through the Time
* 378. AB *, that Triangle ABE is the same which represents the Space gone through by falling, whilst the Body in falling acquires that
† 373. very Celerity BE †.

380. Whence it follows, that *a Body thrown upwards rises to the same Height, from which falling it may acquire the Velocity with which it is thrown up.*

381. And *the Heights, to which Bodies, projected with different Velocities, can ascend, are to one another as the Squares of those Ve-*
* 374. *locities* *.

CHAP. XIX.

Of the Descent of heavy Bodies upon an inclin'd Plane.

382. THE Force, with which a Body endeavours to descend upon an inclin'd Plane, arises from Gravity, and is of the same Nature as Gravity, or rather is Gravity itself diminish'd, because a Body is partly supported by the Plane; therefore that Force, every
* 150. Moment, and in every Part of the Plane, is equal *, and acts upon
† 371. a Body in motion in the same manner as upon a Body at rest †. For the same Cause the *Motion of a Body descending freely upon an inclin'd Plane*, is of the same nature as the Motion of a Body falling freely; and what has been said of the latter, may also be affirm'd of the former. *It is* therefore *a Motion equably accelerated in equal*
* 370. *Times* *; and the Propositions of Numb. 372, 374, 375, 376, 377,
383. 378, 380, and 381, if, instead of a direct Ascent and Descent, we suppose the Motion to be upon an inclin'd Plane, will also take
384. place here.

The Celerities, by which *two Bodies* descend, *of which one falls freely, and the other goes down upon an inclin'd Plane, if they begin*
* 370, 382. *to fall at the same Time*, have the same Ratio to one another every Moment that they had at the Beginning of the Fall *; therefore they
† 341, 133. *run thro' Spaces in the same Time, which are in a Ratio of the Length of the Plane to its Height* †. *And in that very Ratio are the Velocities acquir'd in descending through those Spaces.*

In

In the Plane AB *the Space* run thro' by a Body, whilst another falls freely along the Height of the Plane AC, *is determin'd by drawing* AB *perpendicular to* GC; for then *the Length of the Plane* AB, *is to its Height* AC, *as* AC *to* AG. 385. Plate XV. Fig. 3. 8. 4. El. VI.

If a Circle be describ'd AC being the Diameter, the Point G will be in the Periphery of the Circle; because an Angle in a Semicircle, as AGC, is always a right one *. Therefore a Point as G, for a Plane any way inclin'd, is always in that same Periphery: Whence it follows, that all the Chords, as AG, are to one another as the Forces by which the Bodies endeavour to descend upon them †; and that they are run thro' by Bodies let down, in the Time in which a Body falling, can go thro' the Diameter AC *; wherefore the Times of the falling down along these Chords are equal. The Velocities also at the Ends of the Descents are as the Chords. 386. * 31 El. III. † 341. 385. * 384.

To every Chord as AG, drawn thro' A, another Line may be drawn parallel thro' C, which will be equal to it and equally inclin'd; therefore *in a Semicircle*, as AHC, *the Forces with which Bodies endeavour to descend along Chords terminated in a Point at bottom, as also the Velocities, acquir'd in falling along them, are to one another as these Chords*; *and when a Body is left to itself, it will come in the same Time to the lowest Point of the Semicircle, whether it falls freely along the Diameter, or descends along any Chord as* HC. 387. 388.

The Time of the running down, along the whole Plane AB, *may be compar'd with the Time of the Descent along the Height of the Plane* AC: for that Time is equal to the Time of the running down along AG; and the Square of *the Times are to one another* as AB to AG *: but AB is to AC, as AC to AG †; therefore the Squares of the Lines AB and AC, are also to one another, as AB to AG *; and therefore those Lines AB and AC are to one another, as the times of the Descents along AB and AC, or AC; that is, the Times in that Case are *as the Spaces run thro'*. 389. * 374, 383. † 385. * 20 El. VI.

In the same Case the Velocities at the End of the Descent are equal; for after equal Times, when Bodies are at G and C, the Velocities are in the same Ratio as in the Beginning of the Fall; that is, as AC to AB *. When the Body descends from G to B, the Velocity increases as the Time †; and the Velocity at G, is to the Velocity at B, as AC to AB *. Therefore the Velocities at B and C have the same Ratio to the Velocity at G, and are equal †. 390. * 384, 385. † 382. * 389. † 9 El. V.

If

391. If ſeveral Bodies deſcend along right Lines, drawn from A, and terminated at the horizontal Line CB, the Velocities of all thoſe
* 390. Bodies at the End of the Deſcent, will be equal *; and the Times of the Deſcent as the Spaces run thro': but if the Lines were terminated in the Periphery of the Circle, the Times would be equal,
* 389. and the Velocities as the Spaces run thro' *.

392. Again, let ſeveral Bodies be given, which deſcend along different Planes, and go through equal Spaces, as AC, AF; it is requir'd to find their Velocities at C and F. The Square of the Ve-
* 387, 388. locity at F, is to the Square of the Velocity at B or C *, as AE to AC, or AB; namely, as the Height of the Plane to its Length, that is, *the Squares of the Velocities, are as the Forces by which the*
* 341. *Bodies are driven* *.

393. We have ſhewn that *a Body acquires the ſame Velocity, in falling from a certain Height, whether it falls directly or comes down along*
* 390. *an inclin'd Plane* *. *But a Body may alſo run down along ſeveral Planes differently inclin'd, and even along a Curve* (which may be conſider'd as made up of an infinite Number of Planes differently inclin'd), *and the Celerity will be the ſame when the Height is equal.* For it is of no conſequence whether the Body deſcends along AB
Plate V. Fig 4. * 390. or EB, the Celerity will be the ſame at B †, and the Body will move in the ſame manner along BC; therefore it will have at C the Velocity which it might have acquir'd in coming down along EC, and at D the ſame Velocity that it would have had in deſcending along FD, or falling along GD.

394. Plate V. Fig. 5. But we muſt obſerve, that the paſſing from one Plane to another muſt be without a Shock, for by it the Velocity of the Body would be diminiſh'd, as will be ſhew'd in its proper Place, therefore the ſeveral Planes muſt be join'd by Curves.

A Machine,

395. *To compare the Aſcent of Bodies with their Deſcent.*

Plate XV. Fig 6. * 162. † 163. The firſt Part of this Machine is a Pillar before explain'd *, C, on which a leſs Pillar G is fix'd †, with which is join'd the Arm A *. Thro' one of the Holes *t* (*Plate* 4. *Fig.* 9.) mention'd in
* 173. † 175. * 178. the Deſcription of the Arm †, (we take that Hole which is fartheſt remov'd from the Pillar,) we put a Thread, to which is faſten'd a leaden Ball O. The other End of the Thread is faſtened to the Pin *x* †, and the Ball O is rais'd or let down by turning
† 243. of the Pin.

To the Pillar C is applied the wooden Ruler DE, as we have already explain'd it.

Another Part of this Machine is the Board FH, which ſtands on a Foot, part of which you ſee at L, and it has a Screw at I.

The Side B is placed horizontally, by help of the Plumb-Line P, and the Screw I.

The little Plates *m*, *n*, are horizontal, and ſo moveable about the Side BB, as to remain in the ſame horizontal Plane; and they are faſten'd by the Screws *s*, *s*, with Braſs Plates between. The ſmall Braſs Cylinders *p* and *q* are thruſt into the Holes of the Pillars; and as there are ſeveral ſuch Holes, they may be made faſt at different Heights. Beſides theſe there are alſo uſed two other Braſs Cylinders *l* and *r*, to which the Plates are faſten'd, like the Tails of the Pulleys *; * 243.
theſe Plates are thruſt into the Groove or Slit of the Ruler DE, that the Cylinders may be fix'd in any Place of the Slit, as we have ſaid before of the Pulleys †. † 243.

EXPERIMENT I.

Remove the Cylinders *p*, *q*, *l*, *r*.

The Thread being extended, the Ball O is rais'd; the Length of 396.
the Thread is ſo determin'd, and the Plate *m* ſo fix'd, that the Ball may be applied to its lower Surface. The other Plate *n* is fix'd in the ſame manner.

The Ball is applied to one Plate and left to itſelf, ſo it deſcends by Gravity, and with the Motion it has acquir'd, aſcends to the other Plate.

If the Thread be a little more ſtretch'd, it will ſtrike againſt the oppoſite Plate.

Now put on the Cylinders *p* and *l*; by which, according to their 397.
different Situation, the Way in which the Body deſcends is varied, becauſe of the different bending of the Thread; let the Plate *m* be fix'd in ſuch a Situation, that the Ball may reach it: If now the Globe be let fall from the Plate in the ſame manner as the firſt time, it will aſcend to *n* in the ſame manner, and along the ſame Curve as it did the firſt time.

This Experiment confirms what follows from what has been ſaid 398.
before; namely, that *a Body with the Celerity, which it has acquir'd by falling along any Surface, whether plain or curve, will riſe to the ſame Height along any other ſimilar Surface* *: *and that the* * 380, 383, 393.
Time of the Aſcent is equal to the Time of the Deſcent, is alſo manifeſt †. † 378.

A

399. *A Body with the Celerity which it has acquir'd in falling from a certain Height, can aſcend to the ſame Height in any Curve whatever; but in this Caſe the Times are not equal.*

Experiment 2.

400. This Experiment differs from the foregoing in this alone *; that now you muſt alſo join the Cylinders *q* and *r* to the Machine, in order to vary alſo the Way of the Aſcent. Now having four Cylinders, and the Plates *m* and *n*, as they are ſhewn in the Figure, let the Experiment be tried as before, and it will ſucceed in the ſame manner.

* 396, 397. Plate XV. Fig. 6.

401. From what has been demonſtrated in this Chapter *, we deduce the Method of confirming by Experiments, what has been before demonſtrated of the Velocity of falling Bodies †.

* 393. † 374.

A Machine,

By which the Velocities of falling Bodies are compar'd.

402. This Machine is made of Wood, whoſe Thickneſs A B is 2 Inches, and Height A D about 9; the Wood is hollow'd in the Portion of a Cycloid from the upper Part as far as F, where the Curve is terminated in its Vertex; the Wood is continu'd from F to G, along the Tangent of the Curve in the Vertex F, whoſe Diſtance from G is one Foot. It is requir'd that this Wood be very exactly work'd, and has its Surface very ſmooth. The Formation of the Cycloid we explain in the ſecond Scholium of the following Chapter.

Plate XV. Fig. 7.

This Wood muſt be encompaſſed by the Wooden Rulers H H, H I, I I; and the Space contained by them muſt be as it were divided into two Channels by the Ruler or Partition *m m*, which is $\frac{1}{4}$ Inch high.

In each Channel is mov'd a Braſs Ball of half an Inch Diameter, and in each of them there is alſo a ſtopping Piece O: theſe ſtopping Pieces have a ſide Screw each to fix them in any Place, the End of the Screw being arm'd with a ſmall Plate of Braſs that they may not damage the Wood.

The Machine is ſuſtain'd by three Braſs Screws, two of which are ſeen at C C, whoſe Uſe is to place the Surface F G horizontal, which is ſhewn by the Plumb Line P.

The

The Ruler *m m* is divided, from F to G into equal Parts; but from F upwards into unequal Parts; but they ſhew the equal Diſtances between the Heights.

The Property of this Machine is as follows: The Balls being let fall from any Heights, however unequal, come to F in equal Times; which will eaſily appear, if the Stops O, O, be fix'd at F, and the Balls at the ſame time be let fall from unequal Heights.

Thoſe that want a Geometrical Demonſtration of this Property may find it in the next Chapter; it being ſufficient here to obſerve it by the Machine.

Experiment 3.

The Machine being fix'd as before, join the Stops, the one to the fourth Diviſion from F, and the other to the ſixth. Now letting 403.
fall the Balls at the ſame Moment, from Heights, which are as 4 to 9; that is, letting fall that Ball from the leaſt Height, whoſe Stop is neareſt to F, and they will come to the Stops exactly at the ſame Moment.

The Balls are at the ſame Moment at F, therefore in the ſame Time they run thro' Lines, which are as 4 to 6; that is, as 2 to 3, in which Ratio are the Velocities of thoſe Balls *; the Squares of * 119.
thoſe Numbers are 4 and 9, which are in the Ratio of the Heights, from which the Bodies by falling have acquir'd their Velocities; which was to be confirm'd by Experiment. The Stops are to be ſo fix'd, that the Balls being applied, their Centers may anſwer to the Diviſions of the Line FG; and in letting fall the Balls you muſt obſerve exactly where their Centers are.

CHAP. XX.

Of the Oſcillation of Pendulums.

Definition I.

A Heavy Body ſuſpended by a very ſmall Thread, and moveable with the Thread about a Point to which the Thread is fix'd, is 404.
call'd a Pendulum.

The Motion of a Pendulum is vibratory, or oſcillatory.

When the Weight is rais'd, the Thread being extended; it deſcends by Gravity, and by the Celerity acquir'd riſes to the ſame Height

* 398. on the oppoſite Side *; then it returns again by Gravity, and continues its Vibrations.

Here we ſuppoſe the Rotation quite free about the Point of Suſpenſion, and that there is no Reſiſtance of the Air; which is very ſmall in large Pendulums.

405. Plate XVII. Fig. 1. Let C P be a Pendulum ſuſpended at C; the Body P in its Motion deſcribes P B *p* Part of a Circle; *if* inſtead of that Motion *the Body* ſhould deſcend along the Chord P B; and again aſcend along the Chord B *p*, and *performs its Vibrations in Chords*; the Deſcent will be made in the Time, in which a Body falling may run thro' the Diameter A B; that is, twice the Length of the Pendulum: * 398. in an equal Time it will aſcend thro' the Chord B *p* *, therefore *in the Time of one whole Vibration*, which is double the Time of the † 374. Deſcent, *the Body in falling might run thro'* 4 Diameters †; that is, *eight times the Length of the Pendulum.*

* 388. And as the Deſcent and Aſcent in any Chord is perform'd in an equal Time *, all the Vibrations in Chords, whether great or ſmall, are iſochronal; that is, of equal Duration.

406. *The Durations in ſmall Vibrations, while the Body moves in a Circle, have a conſtant Ratio to the Durations of Vibrations in Chords; namely, that which is given between the fourth Part of the Periphery of a Circle, and its Diameter*; of near 11 to 14.

407. Therefore *the ſmall Vibrations of the ſame Pendulum, tho' really unequal, are iſochronal as to Senſe.*

Experiment I.

408. * 173. Plate XVI. Fig. 2. † 175. 178. Put the Arm A on the Pillar C *. Hang on two Pendulums, P E, *p e*, whoſe Threads go thro' two of the three Holes in the Plate G H (*Plate* IV. *Fig.* 9. †) and are faſten'd with Pins, by turning of which they may be reduc'd to the ſame Length.

If theſe Pendulums are let fall at the ſame Time, from the Points P and *p*, they will come at the ſame Time to F and *f*; and ſo continue their Motion in the Arcs P B F and *p b f*, always in the ſame Time.

But we muſt explain this Equality more fully; and ſhew why the Vibrations in a Circle have the Ratio above-mention'd to Vibrations in Chords *. * 406.

409. Let the Circle F E B roll along the Line A D, till the Point B is come to A on that Line; by that Motion the Point B deſcribes B P A the Part of a Curve; in the ſame manner is deſcrib'd the Portion

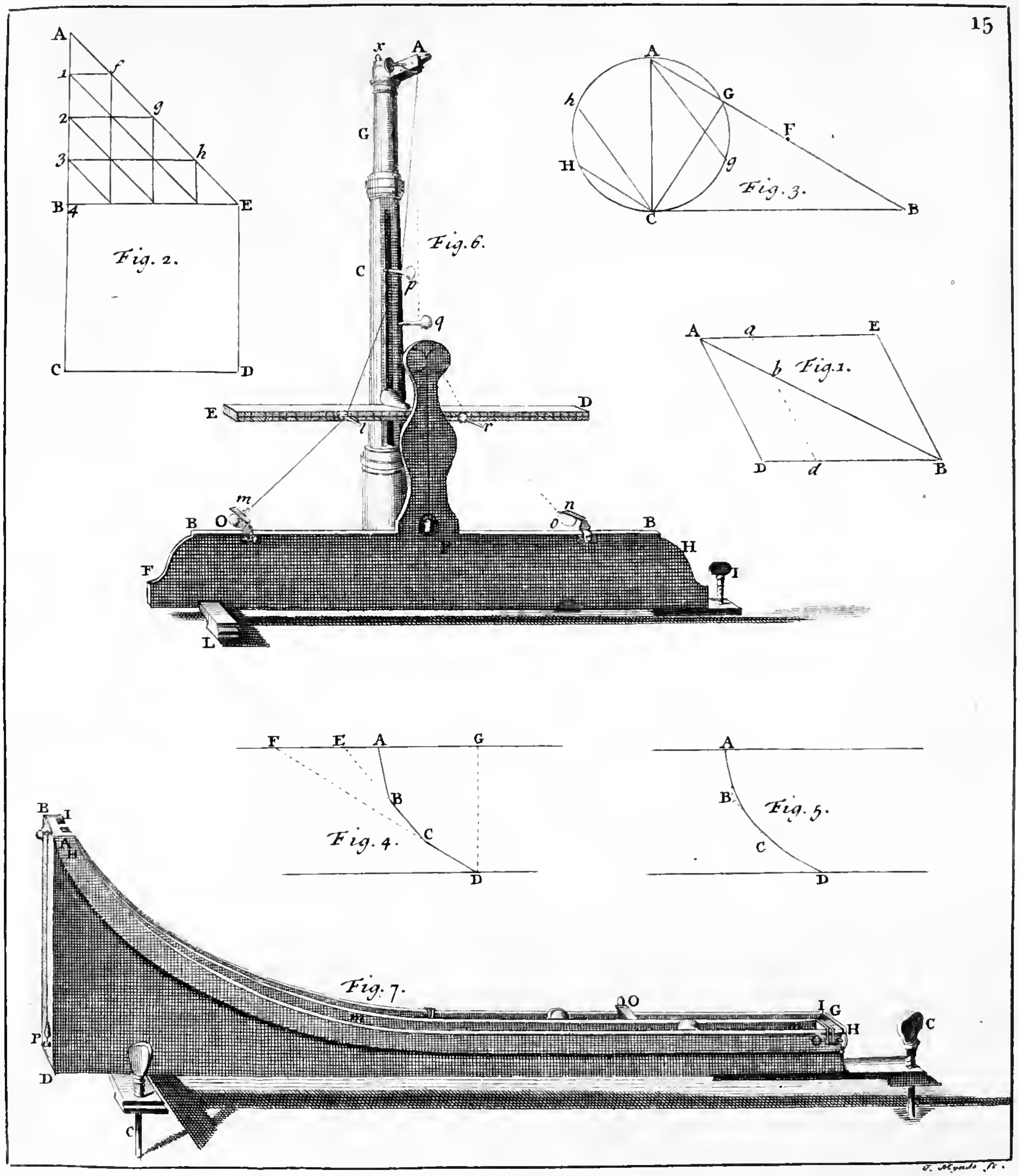


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.

Portion B D of a like Curve, and the whole Curve A B D is call'd a *Cycloid*, and the Circle F E B the *generating Circle*.

Let the Curve be divided into two equal Parts at B, and the Parts B A and B D be so dispos'd, that the Points A and D may join at C; and the Point B may coincide with the Points A and D of the Line A D. Let metalline Plates, or Cheeks, be so bent along the Curvature of those Parts of the Cycloid, that the Thread of the Pendulum suspended at C, in its vibratory Motion may apply itself to those Cheeks on either side, and put on the same Curvature. Now supposing the Length of the Pendulum to be C B, the Body P in its Vibrations will describe the Cycloid A B D, as we shall demonstrate in the next Scholium III. so that the Thread of the Length B C be equal to the Curve C A; therefore *the whole Curve* A B D *is* double the Line C B; and *quadruple the Axis* F B. 410. 411.

We demonstrate in the same Scholium, That *the Tangent to the Curve in a Point, as* P, *is parallel to the Chord* E B, *in the Circle* F B E *drawn from the Point* E *to the lowest Point* B, *in which the Circle is cut by a Line* P E, *parallel to the Base* A D, *and passing thro'* P; *as also that the Portion* P B *of the Curve is equal to double the Chord* E B. 412. 413.

Now as in each Point of the Curve the Body descends in the Direction of a Tangent to the Curve, it follows, that *the Body in any Point of the Curve, as* P, *endeavours to descend with a Force, which is proportional to* a correspondent Curve in the Circle, as E B *, which as itself is half *the Arc of the Curve, intercepted between that Point* P, *and the lowest Point of the Curve* †, follows the same Ratio with that Arc ‖.

* 412. 387.
† 413.
‖ 15 El. V.

Whence it appears, that if two Pendulums, as C P, are let fall in the same Moment from different Heights, the Celerities, with which they begin to fall, are to one another, as the Spaces which are to be run thro', before they come to B: if therefore they were agitated by these Celerities alone, with a Motion not accelerated, they would come to B at the same Time *. In the same manner, by the Velocities acquir'd the second Moment, they come also to B at the same Moment; and the same Reasoning will serve for the following Moments: therefore the half Vibrations, however unequal, as well as the whole Vibrations, are perform'd in equal Times.

* 119.

We demonstrate besides, in the third Scholium, That *the Time of each Vibration is to the Time of a vertical Fall, along the half* 415.

Length of the Pendulum, as the Periphery of a Circle to its Diameter.

416. In a Cycloid the lower Part, as to Senſe, coincides with a ſmall Arc of a Circle; and that is the true Reaſon why in a Circle the Time of very ſmall Vibrations, however unequal, is the ſame; and for that Cauſe, in a Circle alſo, if the Vibration be ſmall, its Duration is to the Time of its Fall thro' the half Length of the Pendulum, as the ſaid Ratio of the Circumference of a Circle to its
* 415. Diameter *. But the Time of the Fall thro' the half Length of the Pendulum, is the fourth Part of the Time of the Fall thro' eight
† 374. times the Length of the Pendulum †; which Time is equal to the
|| 405. Duration of the Vibration along the Chords ||. Therefore the Duration of a Vibration along an Arc, is to the Duration of a Vibration along Chords, as the Periphery of a Circle to four Diameters,
* 406. as we have already ſhewn *; that is, nearly as 14 to 11. And *the*
417. *Pendulum will perform its Vibrations faſter along the Arcs, than along the Chords.*

418. The Durations of *Vibrations of unequal Pendulums* are compar'd. When the Arcs are ſimilar, the Deviations in reſpect of the Chords are alſo ſimilar, and the Times of the Vibrations thro' the Arcs are as the Times of the Vibrations along the Chords; but theſe are the Times of the Deſcents thro' Lengths that are eight times that of
* 405. the Pendulums *, *the Squares of* whoſe *Durations are* as thoſe oc-
† 375. tuple Lengths †, or *as the Lengths of the Pendulums themſelves* ||.
|| 15 El. V.

Experiment 2.

419. Two Pendulums E P, *e p*, whoſe Lengths are as 9 to 4, are let fall at the ſame time from the Points P, *p*, ſo as they may deſcribe ſimilar Arcs in their Vibrations; the longer Pendulum makes two Vibrations, while the ſhorter makes three, as may be obſerv'd by their Concourſe. The Squares of the Durations of the Vibrations are as 9 to 4; namely, as the Lengths of the Pendulums.

420. When the Vibrations are ſmall, this Ratio does alſo take place,
* 407. altho' the Pendulums do not move in ſimilar Arcs *.

421. Concerning all that has been ſaid hitherto of Pendulums, we muſt obſerve, *that it does not ſignify how much a Body weighs that you make a Pendulum of, or whether the Bodies of different Pendulums be of unequal Weight or not, or form'd of different Matter.* Becauſe ſince the Force of Gravity is proportionable to the Quantity
* 156. of Matter in all Bodies *, all Bodies in the ſame Circumſtances are mov'd

mov'd equally swift by Gravity; which is also confirm'd by the following Experiment.

Experiment 3.

Take two equal or unequal Balls, the one of Lead, and the other of Ivory, hang them up by Threads, that they may make Pendulums of equal Lengths; let them vibrate, and their equal Vibrations (and all their unequal ones, provided they be small Vibrations) are perform'd in the same Time. 422.

Oftentimes instead of a Thread, a small, but stiff Iron-Rod is made use of, and sometimes also two or more Weights are fix'd to it. 423.

Definition 2.

Such a Rod suspended, and moveable about a Point, is call'd a compound Pendulum; as C Q P. 424. Plate XVII. Fig. 1.

In this Case the Rules above-mention'd are not applicable; but those Pendulums are reduc'd to simple ones, by determining in them such a Point, that, if all the Weights were united in it, the Vibrations would be of the same Duration as those of the compound Pendulum.

Definition 3.

This Point is call'd the Center of Oscillation; as O.

In Scholium IV. I have explain'd the Method of determining it. 425.

A Body of any Figure may be suspended, and vibrate about a Point, or rather an Axis: and in such a Body one may also determine the Center of Oscillation. 426.

When a right Line, such as is an iron Wire, vibrates about one End, the Center of Oscillation is distant from the Point of Suspension two third Parts of the Length of the Wire: as I demonstrate in the same Scholium. 427.

Experiment 4.

The brass Cylinder A B, two Foot and an half long, is suspended so, as to vibrate about its End A; it turns about an Axis at A join'd to it, which is like the Axis of a Balance, that there may be the less Friction. It is suspended by help of the Plate M L, (*Plate* IV. *Fig*. 9.) which how it is done, the Holes in the parallel Plates, join'd to the greater Plate M L at M, sufficiently shew. But the Plate should not be fix'd, before the Cylinder is suspended *; 428. Plate XVI. Fig. 5.

* 177.

that

that it may acquire a vertical Poſition, before it is fix'd, by the Weight of the Cylinder. The ſimple Pendulum *e p*, two thirds of the Length of A B, is let go at the ſame time with the Cylinder; and the Vibrations of the Pendulum, and Cylinder, are made in the ſame Time.

429. The Vibrations of Pendulums, (as I ſaid) tho' unequal, are
* 407. made in the ſame Time *; and this Property of Pendulums is of great uſe in the Conſtruction of a Clock, to which an equable Motion is communicated, by a Pendulum join'd to it.

430. Clocks being carry'd into different Places, it appear'd that the Force of Gravity is not equal every where, from hence, becauſe the Durations of the Vibrations of the ſame Pendulum, were found unequal, in different Countries; and this Difference of Gravity is meaſur'd by Pendulums.

431. *Let there be two Pendulums,* C P, *c p, whoſe Lengths are to one*
Plate XVII. Fig. 2. *another, as the Forces of Gravity by which they are mov'd*; if they run out into ſimilar Arcs, in correſponding Points the Gravities will always have the ſame Ratio to one another, upon account of the equal Inclinations, and this will be the Ratio of the Arcs to be run thro', (becauſe ſimilar Arcs are as the Lengths of the Pen-
* 119. dulums) which therefore will be run thro' in equal Times *; that is, *the Vibrations will be perform'd in the ſame Time.*

If they be reduc'd to the ſame Length by changing the Pendulum *c p* into *c q*, equal to C P; the Square of the Time of the Vibration of the Pendulum *c q* is to the Square of the Time of the Vibration of the Pendulum *c p*, or C P, as the Length *c q*, or C P, is to
* 418. *c p* *; that is, as the Gravity, which acts upon the Pendulum C P,
432. is to the Gravity, which moves the Pendulum *c q*. Therefore, *the Times of the Vibrations of equal Pendulums are, in a ſubduplicate, inverſe Ratio, of the Gravities acting upon the Pendulums.*

433. And in general, *the Squares of the Times of the Vibrations of*
* 418. *Pendulums are directly, as the Lengths of the Pendulums* *, *and in-*
† 432. *verſely as the Gravities whereby they are mov'd* †.

434. *The Gravities themſelves are directly, as the Lengths of the Pen-*
* 431. *dulums* *, *and inverſely as the Squares of the Times of the Vibra-*
† 432. *tions* †.

435. Many Phænomena of Nature depend upon Motions analogous to the Motions of Pendulums, and what is demonſtrated of Pendulums is of ſervice to explain them, and this is the chief Uſe of the laſt Propoſitions.

Pendulums

Pendulums are alſo of peculiar Uſe in the Experiments, which are made of Bodies in motion, and the Action of innate Forces; but in theſe Caſes the Velocities of Pendulums are to be compar'd, and we may conſider them two ways. 436.

DEFINITION 4.

We call that the Velocity of a Pendulum, by which the ſuſpended Body is mov'd with, when it comes to the loweſt Part of the Arc which it runs thro'. 437.

As for the compound Pendulum, inſtead of the ſuſpended Body, the Center of Oſcillation muſt be conſider'd.

DEFINITION 5.

The angular Velocity of a Pendulum is that, whereby it is turn'd about the Point of Suſpenſion, when it comes to a vertical Poſition. 438.
I muſt now ſpeak of theſe two Velocities.

Let there be two Pendulums C P, *c p*, which make their Vibrations in the Arcs P F, *p f*; with the Centers C, *c*, and the ſame Radius, let the Arcs L O, *l o*, be deſcrib'd; and let there be infinitely ſmall Parts B D, *b d*, in the loweſt Places of the Arcs P F, *p f*, run thro' in the ſame Time. The Velocities of the Pendulums are as B D, *b d*; and their angular Velocities as M N, *m n*; that is, as the Angles B C D, *b c d*. 439. Plate XVI. Fig. 6.

DEFINITION 6.

We call that the Angle of a Pendulum, which the Pendulum deſcribes, in deſcending or aſcending. 440.

The Velocities of a Pendulum, in unequal Vibrations, are to one another as the Subtenſes of the Arcs, which the Body deſcribes in deſcending. 441.

The Velocity, in the Deſcent along the Arc D B, is to the Velocity, in the Deſcent along P D B, as the Chord D B is to the Chord P B *. Plate XVI. Fig. 1. * 387. 393.

In ſmall Vibrations, the Arcs are (as to Senſe) as the Chords; wherefore *the Velocities are* as the Arcs, or *as the Angles of the Pendulums* *. 442. * 440.

In theſe the Arc, or Angle, is ſmall, not exceeding 15 Degrees; for this Arc is to its Subtenſe, as 350 to 349.

In different Pendulums, if the Arcs are ſimilar, or *the Angles are equal*, Bodies deſcend thro' Spaces, which are *as the Lengths of the Pendulums*, in which ſame Ratio *are the Squares of the Velocities* *. 443. * 374. 393.

If

444. *If the Pendulums are equal, and their Angles equal, but Gravities different*, the Forces, acting upon the Bodies in correſponding Points, are as the Gravities; and, in the Beginning of the Deſcent, by running thro' ſmall equal Spaces, *the Bodies acquire Velocities,*
* 392. *whoſe Squares are* as the Forces preſſing *; that is, *as the Gravities:* the Accelerations acquir'd in running thro' the following ſmall and equal Spaces follow the ſame Law, which, as it obtains every where in correſponding Points, upon account of the Forces being in a conſtant Ratio, and the ſmall and equal Spaces, the whole Velocities follow this Ratio alſo.

445. By joining together the three laſt Propoſitions we have this univerſal Rule; viz. *In ſmaller Vibrations the Square of the Velocity of a Pendulum is in a Ratio compounded of the Ratio of the Square of the*
* 442. *Angle* *, *the Ratio of the Length* †, *and the Ratio of the Gravity*
† 443. *acting upon the Pendulum* ‖.
‖ 444.

446. It is manifeſt, that *the angular Velocity*, if the Pendulums are
Plate XVI. equal, *follows the Ratio of the Velocity itſelf.* Let B D be a very
Fig. 6. ſmall Arc run thro' in a determin'd Time; this is as the Velocity of
* 437. the Pendulum *, and is the Meaſure of the Angle B C D; if the Velocity is kept, that is, if B D remains, and the Length of the Pendulum is chang'd, the Angle B C D is diminiſh'd, which determines the angular Velocity, as the Length of the Pendulum is increas'd, *and* this Angle follows *the inverſe Ratio of the Length.*

Therefore we ſhall have the Ratio of the Square of the angular Velocity, if we join the inverſe Ratio of the Square of the Length
* 445. with the three Ratio's above-mention'd *; but by joining the inverſe Ratio of the Square of the Length, with its direct Ratio, which is the mean Ratio of thoſe three, we have the inverſe Ra-
447. tio of the Length; and *the Ratio, which the angular Velocity* itſelf *follows, is made up of the Ratio of the Angle, and the ſubduplicate Ratio of the Gravity, and the inverſe, ſubduplicate Ratio of the Length of the Pendulum.*

448. *The Velocity of a Point in the Pendulum is as the angular Velocity, and as the Diſtance of the Point from the Center of Suſpenſion*; that is, this laſt Ratio muſt be added to the three Ratio's lately men-
* 447. tion'd *.

449. *If the Angles are equal, or the Arcs, deſcrib'd in the Deſcent, ſi-*
* 443. *milar*, the Squares of the Velocities are as the Lengths *, *and the*
† 445. Gravities †. Therefore, if theſe *Velocities* are *equal*, the Products of the Lengths by the Gravities are equal; and by how much that is ſmaller, by ſo much this is greater; that is, the Lengths are inverſely

versely as the Gravities; and the direct Ratio of the Lengths may be taken, instead of the inverse Ratio of the Gravities: which if we substitute in N° 433, we discover in the Case which we examine, that *the Times of the Vibrations* are as the Lengths, which, by reason of the similar Arcs, *are as the Spaces run thro', in the Descent, or Ascent.*

We reason in a like manner, *with respect to the same Gravity*; then the Squares of the angular Velocities, are as the Squares of the Angles directly, and as *the* Lengths inversely *. Therefore, these *angular Velocities being put equal, the Squares of the Angles are* in an inverse Ratio of the inverse Ratio of the Lengths, that is, *as the Lengths.* 450. * 448.

It plainly appears, *in the compound Pendulum, that the Distance between the Centers of Suspension and Oscillation, determines the Length of the Pendulum.* 451.

Where I speak of the Motion of the Pendulum I observe, that a Body descends swifter from one Point to another in an Arc than in a right Line *. To this I will add, that a Body can descend swifter than in an Arc of a Circle; and in the 5th Scholium following I will demonstrate 452. * 417.

That the Line of swiftest Descent, from one Point to another, more depress'd, and not plac'd in the same vertical with the first Point, is an inverted, vertical Cycloid, whose lower Point coincides with the upper Point, and which passes through the lower Point. 453.

A MACHINE,

Whereby the Descent along a Cycloid, is compar'd with the Descent along a right Line.

The Board AB, whose Thickness is three Quarters of an Inch, is made hollow in the shape of a Cycloid, and is so plac'd upon its Stand that the Plane of it may be vertical, and the Cycloid inverted, its Base being put parallel to the Horizon. By help of the three Screws passing through the Feet E, E, and F, the Machine is put into the Position above mention'd, which the Plumb-Line P shews. 454. Plate XVII. Fig. 3.

The Rulers *cc*, *dd*, are applied to the Sides of the Board A B, whereby a Channel is made, in which a Brass Ball, of half an Inch Diameter, may be mov'd, by running along the Cycloid.

The long Piece G H is join'd to the Machine, which Piece is an Inch thick; this is made hollow, and has a Channel of the same

Breadth with the Channel *c d d c*, that a Ball may be mov'd alſo in it. This Piece may be inclin'd as you pleaſe, becauſe it turns upon the Screw *i l*, which paſſes through the Piece and the Board A, and by means of this Screw, whoſe Head *i* is pretty broad, this Piece is applied to the Board, the Point O ſupporting the Piece. The Situation of this Point may be varied, by reaſon of the different Holes *r*, *r*, *r*, &c. by which the Inclination of the Ruler is determin'd.

The Machine is to be placed towards the Edge of a Table, that the Motion of the Ruler mayn't be hinder'd, when its Inclination is diminiſh'd. That the Machine may be ſo diſpos'd, the Foot F is applied to it at right Angles in the Midway between E, E, on the oppoſite Side to that to which the Piece G H is applied. This Foot F is made heavier by the Addition of Lead; or, when the Machine is to be fix'd, a Weight is put upon the Foot.

It is requiſite that both of the Channels be made pretty ſmooth, that the Deſcent of the Balls may not be hinder'd; in one alſo there is a Stop as *m* and *n*, fix'd at pleaſure.

Experiment 5.

455. Let the Ruler G H be inclin'd at pleaſure; the Stops *m*, and *n*, are to be fix'd in ſuch manner, that the Balls, if they be applied to them, may anſwer to each other. Now if, when the Balls are placed at *s* and *t*, to anſwer to one another, they are let go at the ſame time, the Ball that runs along the Cycloid, will come to the Stop firſt, which is diſcover'd by the Stroke; and this may be perceived by the Eye alſo in a greater Inclination. In the Inclination exhibited in the Figure, the Blow is heard at *m*, before the other Ball paſſes through a Quarter of the Length *s n*.

SCHOLIUM I.

Wherein ſome Properties of the Cycloid mention'd in this Chapter are demonſtrated.

THE Formation above mention'd of the Cycloid being laid down *;
456. let B E F be the generating Circle. Let us ſuppoſe it to have come
* 409. to the Point G of the Baſe; the Point F will be at *f*, the Arch G *f* being
Plate XVII. laid down equal to the Line G F; the Point deſcribing will be at *b*, and
Fig. 4. this will be a Point of the Cycloid.

Let

Let the Diameter G c H be drawn thro' the Point of Contact, this will be perpendicular to the Base *, and parallel to the Diameter B F. b L being now drawn thro' the Point b of the Cycloid, parallel to the Base, cutting the Circle F E B at E, and the Line G H at I; it is manifest, because of the equal Lines G I and F L *, that in the equal Circles b I, E L, are equal †; I E being added to each b E, I L, will be equal, to which G F is equal *. * 18 El. III. * 34 El. I. † 3. 14 El. III * 34 El. I.

It also plainly appears, that the Arches Gf, b H, E B, are equal to one another and to the Line G F; and therefore to the Line b E. 457.

From which I deduce this Property of the Curve; *If, from any Point of a Cycloid, there be drawn a Line parallel to the Base, which cuts a Semicircle describ'd upon the Axis towards the Curve*, such as is the Line b E L here, *a Portion* of it, *intercepted between the Cycloid and Semicircle, will be equal to an Arch of the Semicircle intercepted between the said Line and the Vertex:* that is, b E is equal to the Arch E B.

Let there be a Semi-cycloid A D B, the Vertex B, the Base A F, the Axis B F, which is the Diameter of the Semicircle F M B. 458. Plate XVII.

Any infinitely small Portion D d of the Cycloid being taken, this may be look'd upon as a right Line; and continued will form a Tangent in the Point D or d. Let D L, $d\,l$, be drawn parallel to the Base cutting the Semicircle at E, e; and B E and B e, being drawn, let this last be continued till it cuts the Line D L at b; let B O be also parallel to the Base, and a Tangent to the Circle at B, and which is cut at O by the Line e O, which is a Continuation of the Line E e. Fig. 5.

The Triangles b E e and e O B are similar, by reason of the Parallels B O and b E. But the Sides E O and O B are equal *; therefore e E, b E, are also equal; e E is the Difference of the Arches B e, B E, or the Lines $d\,e$, D E †; which Difference therefore is also b E, wherefore the Parallels D b, $d\,e$ are equal; therefore the Parallels D d, $b\,e$ are also equal *, that is, the Tangent in d is parallel to the Chord e B, which Property of the Cycloid I have mention'd above in N°. 412. * 36 El. III. † 457. * 33 El. I.

The same Things being given, let F E i be drawn; this will be perpendicular to B E or Bb, by reason of the infinitely small Angle e B E *, and will divide the Base of the Isosceles Triangle b E e into two equal Parts in such manner, that $e\,i$ will be half of $e\,b$ or d D. But $e\,i$ is the Difference between the Chords B E, B e; for if a Circle be describ'd with the Center B, and Radius B E, it will co-incide with E i, which is infinitely small; and D d is the Difference of the Arches D B, d B of the Cycloid. 459. * 31 El. III.

Now let us conceive a Line to be mov'd parallel to the Base of the Cycloid A F from B to F, and another Line in the mean time to be so turn'd about B, that it may continually pass through the Intersection of the first with the Semicircle. When the first, for Example, comes to d E, the second will be at B e; the first being carry'd to D L, the second is mov'd so as to be at B E. In this Motion they have a common Beginning, and the Arch D B of the Cycloid, and the Chord E B are continually increas'd; but the Increase of the former is always double that of the latter, wherefore the whole Arc alſo, which is the Sum of all the Increaſes, will be double the whole Chord, which alſo is equal to the Sum of its Increaſes. Therefore we have the Demonſtration of the Propoſition mention'd in N°. 413.

It remains that we demonſtrate, what was ſaid of the Evolution of the Cycloid in N°. 410.

460. Let there be given again the ſame Cycloid ADB, whoſe Baſe is AF, Axis FB, Semicircle FEB. Let BF be produc'd to C ſo, that BF and FC may be equal; and the Parallelogram *Af*CF being form'd, let there be a Semicircle A*mf*, equal to the Semicircle FEB; and a Semi-cycloid A*g*C, having A*f* for its Axis, and which is equal to the Semi-cycloid ADB. Let us alſo ſuppoſe a Thread fix'd at C, and applied to the Cycloid C*q*A, to be unfolded.

Let us ſuppoſe the Thread to be in ſuch a Situation, as to agree with the Cycloid only from C to *q*, and to be extended along the Tangent to the Curve at *q*: if the Line *q*R is equal to the Arch *q*A, to which the Thread, which is now ſtretch'd, has applied, R will be the End of the Thread.

Let *qp* be drawn parallel to the Baſe, cutting the Semicircle A*mf* at *m*, from which Point let the Line *m*A be drawn to A, *m*A and *q*N are parallel * and equal †; but *q*A, and therefore *q*R is double *m*A, or *q*N *; therefore N*q*, NR are equal; therefore if RP be given parallel to AF and *pq* thro' R, *p*F and A*p* will be equal; therefore the Arches FM, A*m* will be alſo equal; as the Angles MFA, *m*AF alſo *; and FM is parallel to A*m* †, and R*q*; whence it follows, that FMRN is a Parallelogram, and FN, RM are equal; and *qm*, AN, in the Parallelogram *m*AN*q* are alſo equal.

* 412, 458.
† 34 El. I.
* 413, 459
* 32. 27 El. III.
† 27 El. I.

The Line *mq*, or AN, is equal to the Arch A*m* *, or to the Arc FM; AF is equal to the Semicircle FMB †; therefore NF, or RM, is equal to the Arch MEB, and the Point R, that is, the End of the Thread, is given in the Cycloid ADB *, which this End of the Thread will run thro' whilſt the whole Thread is unfolding.

* 457.
† 409. 457.
* 457.

SCHOLIUM II.

Of the Deſcription of the Cycloid.

THE Generation of the Cycloid ſufficiently ſhews, that it may be deſcrib'd mechanically. But it is better to delineate it by Points, which may be done to a very great Exactneſs.

461. Plate XVIII. Fig. 1. Let BF be the Axis of the Curve; B the Vertex; FA the Baſe perpendicular to the Axis. Let the Axis be divided into two hundred equal Parts, all which Diviſions it is not neceſſary to mark; the Semi-baſe FA contains 314,2 of theſe Parts, which Length being given, A is determin'd. After the ſame manner the reſt of the Points are determin'd, by help of the following Table. In the firſt Column of the ſaid Table, the Diſtances are mark'd from B to F. Thro' the Points mark'd, are drawn Ordinates

Ordinates to the Axis, parallel to the Base, and their Lengths are given in the second Column.

A TABLE,
Of the Dimensions of the Cycloid.

462.

Absc.	*Ordin.*	*Absc.*	*Ordin.*
6.	68,9.	70.	222,0.
8.	79,5.	80.	234,9.
10.	88,7.	90.	246,5.
12.	97,0.	100.	257,1.
14.	104,6.	110.	266,6.
17.	114,9.	120.	275,2.
20.	124,3.	130.	282,9.
23.	133,0.	140.	290,0.
26.	141,0.	150.	296,0.
30.	150,9.	160.	301,4.
35.	162,3.	170.	306,0.
40.	172,7.	180.	309,8.
50.	191,3.	190.	312,7.
60.	207,6.	200.	314,2.

The Points near the Vertex are not set down in this Table, because they can't well be mark'd after this manner: but this Portion of the Curve is had, if an Arc of a Circle be describ'd thro' B, whose Center is in the Line BF continued, and whose Radius is double the Axis of the Curve *. If we would continue the Description by Points, the Line BO must be drawn, parallel to the Base; and in this we should mark those Distances from B, towards O, which are had in the first Column of the following Table; and Perpendiculars to BO being rais'd, their Lengths are determin'd in the second Column. 463.

* 410. 460.

A Second TABLE,
Of the Dimensions of the Cycloid.

464.

Absc.	*Ordin.*	*Abs.*	*Ordin.*
5.	0,00.	35.	1,53.
10.	0,12.	40.	2,00.
15.	0,28.	45.	2,53.
20.	0,50.	50.	3,12.
25.	0,78.	55.	3,78.
30.	1,12.	60.	4,50.

This is the Construction of the first Table. The Ordinate is determin'd from the Absciss given. 465.

Let

Plate XVII. Fig. 4. Let the Abſciſs be B L, whoſe Length is 60 of thoſe Parts of which the Radius contains an hundred, and B F two hundred: B L is the vers'd Sine of the Arch B E, which the Table of theſe Sines ſhews to be 66 Degrees, 25′, 20″, whoſe right Sine is 91,65 of the Parts mention'd; and this is the Length of the Line E L.

* 457. The Length of the Arch E B, to which the Line E *b* is equal *, is had by this Proportion; as 180 Degrees, *viz.* a Semicircle, are to 66 Degrees, 25′, 20″, ſo is 314,2, the Number of Parts in the Semi-circle, to 115,94, the Length E B, or E *b*; E L being added, which contains 91,65 Parts, we have *b* L containing 207,59 Parts; that is, the laſt Character of the Fraction 207,6, being neglected, as has been ſet down in the Table.

466. The Computation of the ſecond Table depends upon this Foundation, that the Points ſought for are given in an Arch of a Circle, whoſe Radius is

* 410. 460. equal to four hundred *.

SCHOLIUM III.

Of Motion in a Cycloid.

467. Plate XVI. Fig. 7. LET us ſuppoſe a Portion of a Cycloid, or a whole Cycloid, to be extended in the right Line A B D, and that a Body is mov'd in this right Line according to the Law of a Pendulum vibrating in a Cycloid; that is, that there is given a Preſſure acting upon the Body, which is in the

* 414. Ratio of the Diſtance of the Body from the middle Point B *, and which

† 371. acts upon the Body in Motion as upon a quieſcent Body †; with the Center B, and Radius B A, let the Semicircle A L D be deſcrib'd, which repreſents the Time, wherein the Body is mov'd from A to D; the Times in which any Portions of the Line A D are deſcrib'd, are determin'd, by raiſing Perpendiculars to it; the Arch H I denotes the Time in which F G is paſs'd thro', and the Arc A H that in which A F is paſs'd thro': But the Celerities in the Points F and G are proportional to the Perpendiculars F H, G I.

468. Which, that it may be demonſtrated, we muſt ſuppoſe a Body, which is mov'd in the Line A D, in ſuch a manner that in Times, which are as the Arches A H, H I, it may run thro' the Portions A F, F G, and ſo of the reſt: ſo that the whole Time may be repreſented by the Semicircle A L D. Let us further ſuppoſe the Semicircle divided into very ſmall and equal Parts, denoting very ſmall and equal Parts of Time, ſuch as H *h* and I *i*. Therefore, *f h* and *g i* being alſo put perpendicular to the Line A D, the Lines F *f* and G *g* are paſs'd thro' in equal Times; which being ſmall, are paſs'd thro' with an equable Motion, for the Parts of Time may be conceiv'd ſo ſmall, that the Acceleration, or Retardation, will be inſenſible;

* 119. therefore the Celerities, in the Points F and G, are as F *f* and G *g* *, which I demonſtrate to be to one another, as F H is to G I.

The

The Lines H *l* and I *m* being drawn parallel to the Line A D, the Triangles H B F, H *b l* will be similar; for they are rectangular, and the Angle F H B is equal to the Angle *l* H *b*, both of which have the same Complement to a right Angle, *viz.* B *b l*; after the same manner the Triangles B I G, *m* I *i*, are demonstrated to be similar. Therefore F H : H B = I B : : *l* H : H *b* = I *i*; and I B, G I : : I *i*, *m* I: and by Equality F H : G I : : *l* H : *m* I *. * 22 El. V.

The Increments of the Celerities, in the very small and equal Moments, in the Points F and G; that is, the Pressures acting in those Points *, are * 133. 371. as *l b* and *m i*; for the Differences of the Celerities are in the Points F, *f*, and G, *g*. But, by reason of the similar Triangles above-mention'd, *l b* and *m i* are to one another, as F B is to G B; therefore the Pressures, acting upon the Body in the Points F and G, are to one another, as the Distances from the middle Point B.

What is demonstrated of the Increments of the Celerities in the Part A B, of the Line A D, is demonstrated after the same manner of the Decrements in the Part B D. Therefore the Body is mov'd according to the Law of a Body vibrating in a Cycloid *. * 414.

Let a Body be given running thro' the Semicircle A L D with an equable Motion, in the Time of one Vibration in a Cycloid, that is, in the Time, in which a Body moving in the right Line A D, (as I have explain'd it) passes thro' it. From what has been said, it appears that H *b*, F *f*, and I *i*, G *g*, are pass'd thro' in equal Times; whence it follows, since the Directions are parallel in L and B, that the Celerities in these Points are equal.

Therefore a Body, with the Celerity which a pendulous Body has in B, in the Time of one Vibration, describes a Semicircle, whose Diameter is an Arch of the Cycloid run thro' by the Body. 469.

If the Body runs thro' a whole Cycloid, as A B D, the Diameter, which is equal to the Arch run thro', will be quadruple the Diameter F B *; and the Velocity in B will be that, which the Body acquires, by falling from the Height F B †; with which Celerity, the Body can run thro' a Line double F B, with an equable Motion, in the Time of the Fall *, and in the Time of one Vibration, it can run thro' a Semicircle, whose Diameter is quadruple F B †. But the Spaces pass'd thro', with equal Velocities, are as the Times ||; therefore the Time of the Fall, thro' half the Length of the Pendulum, is to the Time of one Vibration, thro' the whole Cycloid, or any Arch *, as the double of F B, is to the Circumference of the said Semicircle, or to the whole Circumference of the Circle, whose Diameter is also double F B; therefore, in general, as the Diameter of the Circle to its Circumference; as has been said in N° 415. 470. Plate XVI. Fig. 3. * 411. † 303. * 376. † 469. || 120. 407.

SCHO-

SCHOLIUM IV.

Of determining the Center of Oscillation.

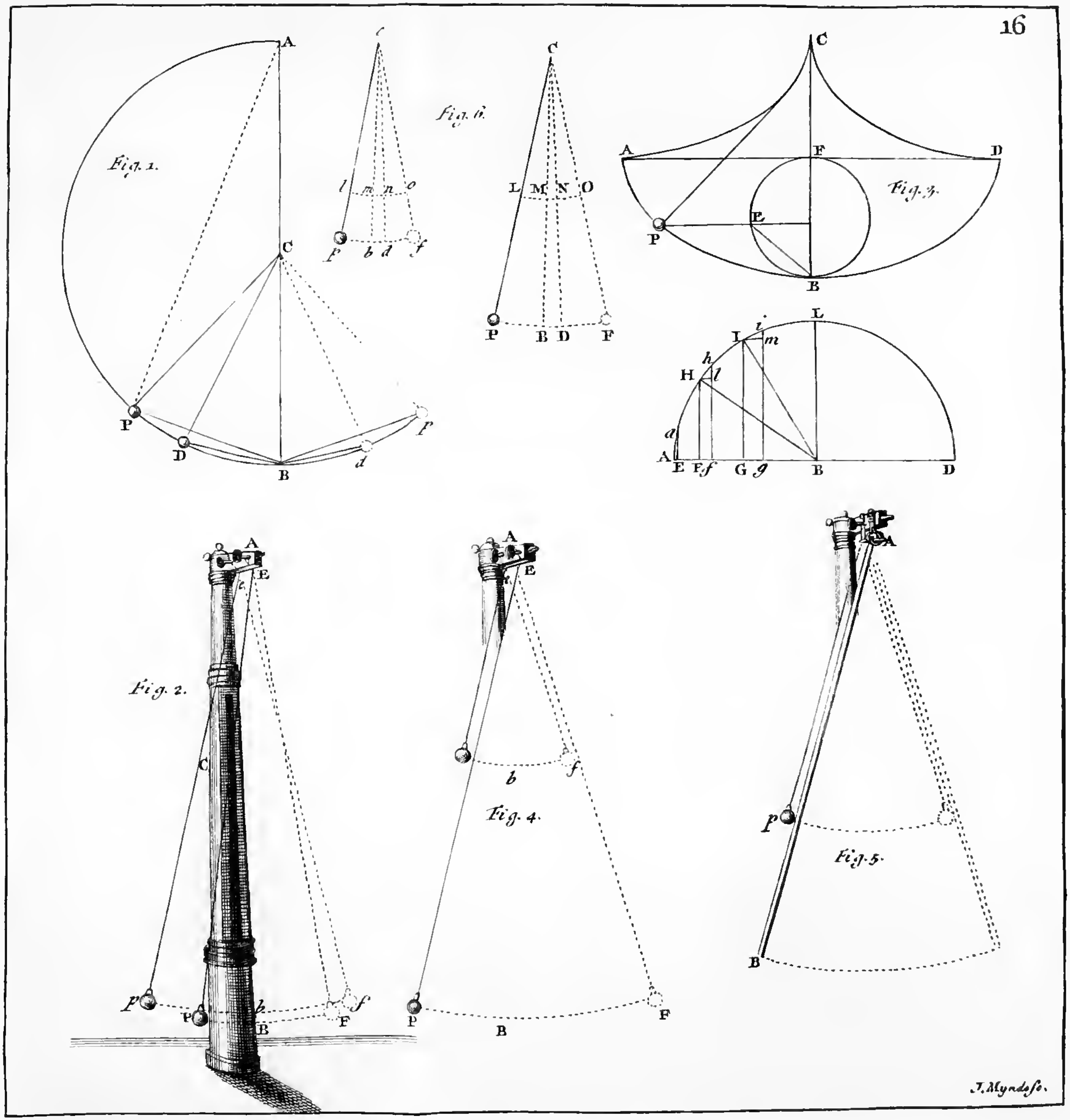
471. Plate XVII. Fig. 1. LET C A *be a compound Pendulum; the Weights* P *and* Q; *between these the Center of Oscillation* O *is given, which has this Property,* supposing A C to be a stiff Rod, without Weight, *as the Weight* Q, *multiplied by* B C, *is to the Weight* P, *multiplied by* A C, *so is* A O *to* O Q. Which that we may demonstrate, we must consider that the Weights Q and A are mov'd in Directions parallel to each other, that is, equally inclin'd to the Horizon; therefore they are continually agitated by the Impressions of Gravity, which, if the Bodies were not join'd by a stiff Rod; would communicate
* 152, 384. equal Celerities to them *. But the Celerities of the Weights join'd are necessarily unequal, and the Celerity of the Body P, is increas'd by the Action of the Weight Q, whilst this is retarded by the Action of the other; which
* 361. contrary Actions are equal *. In the mean time some intermediate Point O, namely, the Center of Oscillation, is mov'd with a Celerity arising from the Action of Gravity.

Let B*b*, O *o*, or A*a*, (for we put these Lines equal) be the Space run through from the Action of Gravity acting according to any Inclination in any very small time. When the Point O passes through this Space, Q is mov'd only through B E; and the Power, which acts upon Q, is diminished by the Quantity, whereby this Body would run through E *b* in the same
* 134. time, and which is express'd by Q × E *b* *. But the Power which acts upon P, is increas'd by the Quantity, whereby P is mov'd through *a* D in the
* 134. same time, and which is express'd by P × *a* D *; for we suppose B *b*, O *o*, A *a*, to be parallel; therefore the Intensity of the Power which retards the Motion of the Body Q, is to the Intensity of the Power, which accelerates the Motion of the Body P, as Q × E *b* is to P × *a* D: But these Powers are applied to a Lever, whose Fulcrum is C; therefore their Actions, which I have demonstrated to be equal, are as the Intensities multiplied by the Dis-
* 235. tances from the Fulcrum, that is, C B × E *b* × Q to C A × *a* D × P *. Therefore C B × Q is to C A × P, as *a* D is to E *b*, or A O to O B. *Q. E. D.* It also appears, that in such a compound Pendulum the Products would be equal, if every Weight were multiplied by its Distance from the Centers of Suspension and Oscillation.

472. *If many Weights were given, and each were multiplied by its Distance from the Centers of Suspension and Oscillation, the Sums of the Products on each side of the Center of Oscillation are equal.* This is evinc'd by a like Demonstration.

Whence I deduce a Method of determining the Center of Oscillation by Computation.

473. Let there be any Bodies A, B, C, D, E, whose Distances from the Center of Suspension are expressed by the Letters *a*, *b*, *c*, *d*, *e*, respectively; let the


16
Fig. 1.
Fig. 6.
Fig. 3.
Fig. 2.
Fig. 4.
Fig. 5.
J. Mynde sc.

the Distance of the Center of Oscillation from the Center of Suspension be x. Let us suppose a, b, c, smaller than x, but d and e greater.

The Distances of the Bodies A, B, C, from the Center of Oscillation are $x-a$, $x-b$, $x-c$, and the Distances of the rest of the Bodies from the same Center are $d-x$, $e-x$, by multiplying every one of the Bodies by their Distances from both Centers, we have $Aax - Aaa + Bbx - Bbb + Ccx - Ccc = Ddd - Ddx + Eee - Eex$ *, whence I deduce * 472.

$$x = \frac{Aaa + Bbb + Ccc + Ddd + Eee}{Aa + Bb + Cc + Dd + Ee},$$ which same Equation we have which soever of these Distances a, b, c, d, e, exceed x; wherefore we discover this general Rule:

If all the Bodies be multiplied by the Squares of their Distances from the Center of Suspension, and the Sum of the Products be divided by the Sum of the Products of all the Bodies, multiplied by their Distances from the same Center of Suspension, the Quotient of the Division will give the Distance between the Centers of Suspension and Oscillation. 474.

If, the Pendulum being continued beyond the Center of Suspension, certain Bodies be applied above the Points of Suspension, their Distances will be negative; if, for Example, such Bodies were A and B, instead of $+a$ and $+b$ the Computation should be made with $-a$, $-b$, whose Squares being also $+aa$ and $+bb$, the Distance x in this Case will be 475.

$$\frac{Aaa + Bbb + Ccc + Ddd + Eee}{-Aa - Bb + Cc + Dd + Ee}.$$

In this Determination the Divisor is equal to the Distance between the Center of Suspension and the Center of Gravity, multiplied by the Sum of all the Bodies *; and by thus expressing the Divisor the Rule is more universal, and may be applied to any Body whatsoever. But the Demonstration must be alter'd, and from this Principle is easily deduc'd; *If Bodies join'd together acquire different Velocities by descending, whilst they revolve at different Distances about the same Center, or the same Axis, and afterwards ascend separately, with the Velocities acquir'd; the common Center of Gravity will ascend to that Height from which it descended* *.

* 223.

476.

* 212, 399.

Let there be two Bodies P and Q, moveable about the Point C, with which they cohere by means of the Lines B C, A C, which make an Angle, that is not alter'd by the Motion of the Bodies. Let D be the common Center of Gravity, which the Bodies being at rest, is given in the vertical Line, drawn thro' C *, in which same Line the Center of Oscillation O is given. Further, let E F be horizontal, drawn through C, and B E, A F, perpendicular to it. 477. Plate XVIII. Fig. 2.

* 206.

I suppose $CA = a$, $CB = b$, $CD = d$, $BE = e$, $AF = f$, and lastly, $CO = x$.

The Bodies being rais'd, and then left to themselves, when the Center of Gravity returns to D, its Velocity is greatest, and then it ascends. The Velocities of the Points A, B, D and O, are to one another, as a, b, d and x, and at that Moment may be express'd by these Letters.

If A and B be left to themſelves at that Moment, that they may aſcend
* 381. ſeparately, they will come to Heights, which will be as aa, bb *, and which may be expreſs'd by theſe Squares. The Height to which the Center of
* 217. Gravity then aſcends is $\frac{Aaa+Bbb}{A+B}$, which * is equal to the Height from which it deſcends.

This Height being given, I determine the Deſcent of the Center of Oſcillation; for this is to the Deſcent of the Center of Gravity as x to d, and is equal to $\frac{Aaax+Bbbx}{Ad+Bd}$. But the Center of Oſcillation is mov'd, as
* 425. a Body, mov'd by Gravity only *; therefore this is the Height to which
† 399. the Body can aſcend with the Velocity x †; which is alſo equal to xx: for we ſuppos'd the Heights, to which the Bodies can aſcend, which are proportional to the Squares of the Velocities, to be expreſs'd by the Squares themſelves; therefore $\frac{Aaax+Bbbx}{Ad+Bd}=xx$, or $\frac{Aaa+Bbb}{Ad+Bd}=x$. Q. E. D.

478. I ſuppoſe d to be given, but if this Diſtance is to be determin'd,
* 217. we diſcover $\frac{Af+Be}{A+B}=d$ *; and a Subſtitution being made, we have $\frac{Aaa+Bbb}{Af+Be}=x$.

In the Numerator we multiply every Body by the Square of its Diſtance from the Center of Suſpenſion; becauſe, in the Motion of a Pendulum, the Velocities of Bodies are in a Ratio of theſe Diſtances: Thence it follows,
479. *If Bodies, or Parts of the ſame Body, be turn'd not about the ſame Center, but about an Axis, the Weight of every Point of the Body, or Bodies, muſt be multiplied by the Square of its Diſtance from the Axis, and the Sum of the Products muſt be divided by the Diſtance of the Center of Gravity of the Body, or Bodies, from the ſame Axis, or from an horizontal Plane paſſing thro' the Axis, multiplied into the Weight of the Body, or the Sum of the Weights of all the Bodies.* Which Diſtance of the Center of Gravity from the ſaid ho-
* 218. rizontal Plane, how we determine I have ſaid in its Place *.

480. That we may apply this Rule to the Line *, whoſe Extremity is the
* 479. Center of Suſpenſion, the Weights of all the Points, or rather ſmall Parts, muſt be multiplied by the Squares of their Diſtances from the Extremity, but each of the Particles is proportional to its proper Weight; therefore we ſuppoſe theſe alſo to expreſs the Weights: then the Sum of theſe Products is a Pyramid, whoſe Baſe is the Square of the Line, and Height the
* 7 El. II. Line itſelf. If the Line is call'd a, this Pyramid is equal to $\frac{1}{3}a^3$ *. This muſt be divided by the Weight of the whole Line, which is equal to a, multiplied by the Diſtance of the Center of Gravity from the Extremity, that is, by $\frac{1}{2}a$, and the Diviſor is equal to $\frac{1}{2}aa$. But by dividing $\frac{1}{3}a^3$ by $\frac{1}{2}a^2$ the Quotient is $\frac{2}{3}a$, the Diſtance of the Center of Oſcillation from the
* 428. Center of Suſpenſion, as I have confirm'd by Experiment above *.

To

To this Example I will add another alſo, which will be of uſe in the following Chapter.

Let there be a round flat Piece *a*, every where of the ſame Thickneſs; I ſuppoſe this hung upon a Center, about which it turns; and the Weight P being join'd to it, there is made a compound Pendulum: for I ſuppoſe the Line C B to cohere with the circular Piece, together with which it is turn'd about the Extremity C. The Center of Oſcillation O is requir'd. 481. Plate XVIII. Fig. 3.

We ſhould ſuppoſe the circular Piece A *to be divided into innumerable ſmall Parts.* We ſuppoſe this Diviſion to be made by concentrick Circles, equally diſtant from each other, whoſe common Center is C. Theſe Circles, or rather ſmall Rings intercepted between them, are to one another as their *Weights*, and alſo as their Radii; wherefore the Radii of the ſmall Rings may be taken for their Weights, and each of them *ſhould be multiplied by the Squares of the Diſtances from the Center* *, that is, we ſhould ſeek for the Sum of the Cubes of all the Radii; and this is not difficult, if we make uſe of the Calculus of Infinites. This Sum, if a is the Radius of the circular Piece A, is $\frac{1}{4}a^4$. But the Weight of the whole circular Piece is expreſs'd by the Sum of the Radii of all Circles, which *Sum is equal to* a rectangular Triangle, whoſe Baſe is equal to a, and whoſe Height is equal to it; wherefore the Weight is equal to $\frac{1}{2}aa$ *. Whence it appears, that the Sum ſought for, namely, $\frac{1}{4}a^4$, is equal to *half of the Weight of the circular Piece* A, *multiplied by the Square of the Radius.* 482. * 479. * 41 El. I.

To this Product I add the Weight P, multiplied by the Square of the Diſtance C B; and I divide this Sum by the Product of the Weight, multiplied by the Diſtance C B; the Quotient of the Diviſion will give C O *. * 479.

SCHOLIUM V.

Of the Line of the ſwifteſt Deſcent.

WE have ſeen above *, That a Body which deſcends from one Point to another, when both the Points are not in the ſame Vertical, ſhould not go in a right Line, to go through its Way in the ſhorteſt Time. What Line it ought to follow we have mention'd *, which we will now demonſtrate here; becauſe what is demonſtrated in *Scholium* I. of the Cycloid, is of uſe to this purpoſe. * 417. * 453.

Let there be two Points A and B, ſeparated by the Line C D; let a Point be mov'd, and go from A towards B, but according to ſuch a Law, that before it comes to the Line C D, it may be carry'd with a Velocity which we call v; but when it has paſs'd thro' this Line, let it go with a greater Celerity, which we call c: let us ſuppoſe further, that the Point runs through ſtrait Lines with each of the Velocities; and therefore is mov'd thro' the right Line AB, or goes thro' the Lines A E, E B: We muſt determine, how it 483. Plate XVII. Fig. 6.

 ſhould

ſhould direct its Motion, that in the ſhorteſt Time it may go from A to B.

Let us ſuppoſe the Time in which the Body, with the Velocity v, runs
* 120. thro' any Line, to be repreſented by the Line itſelf which is run thro' *; the Time wherein the Line is run thro', with any greater Velocity, is ſo much ſhorter, as the Velocity is greater, and is diminiſh'd in the Ratio in which the Velocity is increas'd; therefore the Time in which any Line is paſs'd thro' with the Velocity c, will be repreſented by a Line, leſs than that run through, and which has the Ratio to the Line run thro, which is given between v and c.

If the Point goes through A E and E B, the Time of the Motion thro' AE, becauſe this Line is run thro' with the Velocity v, is repreſented by this Line; the Time in which E B is paſs'd thro', is repreſented by the Line E F, which is to E B, as v is to c. But the Point F is determin'd, if B D be drawn perpendicular to C D from B, and $c : v :: BD : LD$, and a Parallel be drawn to D C through L, this will cut BE in the Point F: for on account of the Parallels E D, FL, we have $BD : LD :: BE :$
* 2 El. VI. FE *,

From this Demonſtration it alſo follows, if a Point paſſes thro' other Lines A M, M B, the laſt of which cuts L F in N, that the Time of the Motion is repreſented by the Lines AM, MN, ſo that we muſt determine thro' what Point of the Line C D the moveable Point paſſes, when the Sum of ſuch Lines, repreſenting the Times, is the ſmalleſt of all; which that it may be done, we muſt attend to what follows.

That the Sums on both ſides, as we recede from the Point ſought, are continually increas'd; and therefore in this Caſe only the neighbouring Sums are equal, if the Lines on either ſide are diſtant but a little from this very Point: therefore if this Point be between E and e, whoſe Diſtance is infinite-
Plate XVII. ly ſmall, $AE + EF$ and $Ae + ef$ will be equal, from which Equality the
Fig. 7. Situation of the Point E or e muſt be deduc'd, which Points coincide with the Points ſought; for by reaſon of the infinitely ſmall Line E e, this ſmall Line may be look'd upon as the Point ſought itſelf.

With the Center A, and Radius A e, let an Arch eh of a Circle be deſcrib'd; with the Center B and Radii Bf, and B E, let the Arches E i, fg be deſcrib'd, and $Ah + Eg$ and $Ae + if$ will be equal; theſe equal Quantities being ſubſtracted from $AE + EF = Ae + ef$, there remains $hE + gF = ei$. Whence we deduce $hE = ei - gF$.

By reaſon of the ſimilar Triangles eiE, fgF, and Bfg, BiE, as alſo BFL, BED; $ei : gF :: Ei : fg, :: BC : Bg$ or BF (for the Difference is infinitely ſmall) $:: BD : BL$. By Converſion $ei, ei - gF = hE ::$ $BD, BD - BL = LD$; that is, as the Velocity below the Line to the Velocity above the Line.

With the Center E let there be deſcrib'd a Circle, cutting the Line E A, or eA, (which we ſee may be look'd upon as the ſame) in M, and E B in N; from which Points let MP, NO, be drawn perpendicular to C D.

The

The Triangles *e i* E, E N O, are similar; they are rectangular, and have a common Angle at E, or *e*. After the same manner *e h* E and *e* M P are similar; therefore

e i : E *e* : : E O : E N

E*e*: *h* E : : M *e*, or N : (which are taken for Radii of the same Circle,) *e* P, or E P.

By Equality *e i* : *h* E : : E O : E P *. But these Lines are *the Cosines of the Angles, which the Directions of the Motions make with the Line* C D, *which separates the Spaces, in which the Velocities differ:* which Cosines of the Directions are therefore to one another, as *e i* to *h* E, which we see *are to one another, as the Velocities in those Directions, when the Time is the shortest of all.* 484. * 22 El. V.

Let the Body be mov'd again from A and tend towards B, with this Condition, that while it passes thro' the Lines C D, I L, M N, O P, it may change its Velocity every time, it is requir'd to know according to what Law it should be mov'd, those Lines being suppos'd parallel, that it may go from A to B in the shortest Time. 485. Plate XVII. Fig. 8.

It is requir'd that the Body come from A to F in the shortest Time possible, and from E to G, from F to H. and from G to B; for otherwise, in the whole Motion a shorter Time may be given. Therefore *the Cosines of the Angles, which the Directions of the Motion* A E, E F, F G, G H, H B, *make with the Lines, parallel to one another, separating the Spaces in which the Velocity is different, are to one another respectively, as the Velocities in which each of these is pass'd through.* 486.

Let us now consider a Body descending by Gravity. Its Celerity is continually increas'd by its descending, and is always the same at the same Depth *, therefore the Spaces, in which the Celerity varies, are divided by innumerable horizontal Planes, their Distances from one another being infinitely small: therefore *that is the Line of swiftest Descent between two Points, whose Tangent makes an Angle every where with the Horizon, whose Cosine is proportional to* the Velocity acquir'd by falling *, that is, to *the Square-Root of the Height through which the Body has fallen* †. But I shall demonstrate this to be the Property of the Cycloid. * 393. 487. * 486. † 374, 393.

Let us suppose A D B to be an inverted Cycloid, whose Axis is vertical, and that a Body descends from A; we must demonstrate, that the Cosine of the Angle *d* D E, or B E L *, is proportional to the Square-Root of the Height F L †. The Angle B E L is equal to the Angle B F E *; whose Cosine, if F be the Center of the Circle, and F B the Radius, is the Chord F E; which takes place in all the Points of the Cycloid, the same Radius F B remaining: But this Chord F E is as the Square-Root of the Height F L. For F L, F E, F B are in a continued Proportion †; therefore F L × F B = F E*g* *, but by reason of F B being constant, the Rectangle F L × F B is in the Ratio of F L †; in which Ratio also the Square of the Chord F E is chang'd. 488. * 412, 458. † 487. Plate XVII. Fig. 5. * 8 El. VI. † 8.4. El. VI. * 17 El. VI † 1 El. VI.

Therefore, *the Line of the swiftest Descent, from one Point to another, is an inverted Cycloid, whose lower Point,* as A, *coincides with the upper Point, and which passes through the other Point,* as I said in N° 453. 489.

C H A P.

CHAP. XXI.

Of the Use of Machines.

490. IN the foregoing Part, I ſpoke of ſimple and compound Machines; we ſaw how a ſmall Power may overcome a great Reſiſtance, but we have determin'd the Caſe of the Æquilibrium only; and obſerv'd in general that the Reſiſtance is overcome, if the Action of the Power be ever ſo little increas'd *. But this general Obſervation is not ſufficient, if we would perform the greateſt Effect that we can by help of a Machine.

* 284.

In the Uſe of a Machine we muſt conſider the Time; for *the Effect*
491. *which,* cæteris paribus, *is perform'd in a leſs Time, is greater, if we conſider the whole Uſe of the Machine.*

492. For the Machine which, in the ſame time, with a double Intenſity of the Power, produces a double Effect, is equal to that, whoſe Effect is ſingle, the Intenſity of the Power being ſingle; which therefore alſo agrees with that which produces a ſingle Effect, in half the Time, the Intenſity of the Power being double: ſo that the Product of the Time by the Intenſity of the Power muſt be conſider'd; and as long as this Product follows the Ratio of the Effect produc'd, in which Caſe this is the ſame, as often as the ſame Reſiſtance is overcome in the ſame manner, the Uſe of the Machine may be look'd upon as the ſame. Three Men finiſh a Work, in one Day, which one would perform in three Days; theſe Things agree, their Capacities being ſuppos'd equal, and their Diligence the ſame; the ſame Work is perform'd by the ſame Action in the whole.

493. From this I infer, That *it is requiſite in the moſt perfect Uſe of a Machine, that ſuch a Power be applied to it, whoſe Intenſity, multiplied by the Time, in which it performs a requir'd and determin'd Effect, gives a Product the ſmalleſt of all*; then the whole Action, whereby the Effect is produc'd, is the leaſt of all.

494. In the Uſe of the Lever this Conſideration can ſeldom be of ſervice; yet, as the Demonſtrations in this Machine are very ſenſible, and many Things, which are to be ſaid of it, are of uſe in the other Machines, I will now ſpeak of the Lever, and that we may conſider the moſt ſimple Caſe, I will take a Line without Weight for the Lever *.

* 232.

Let

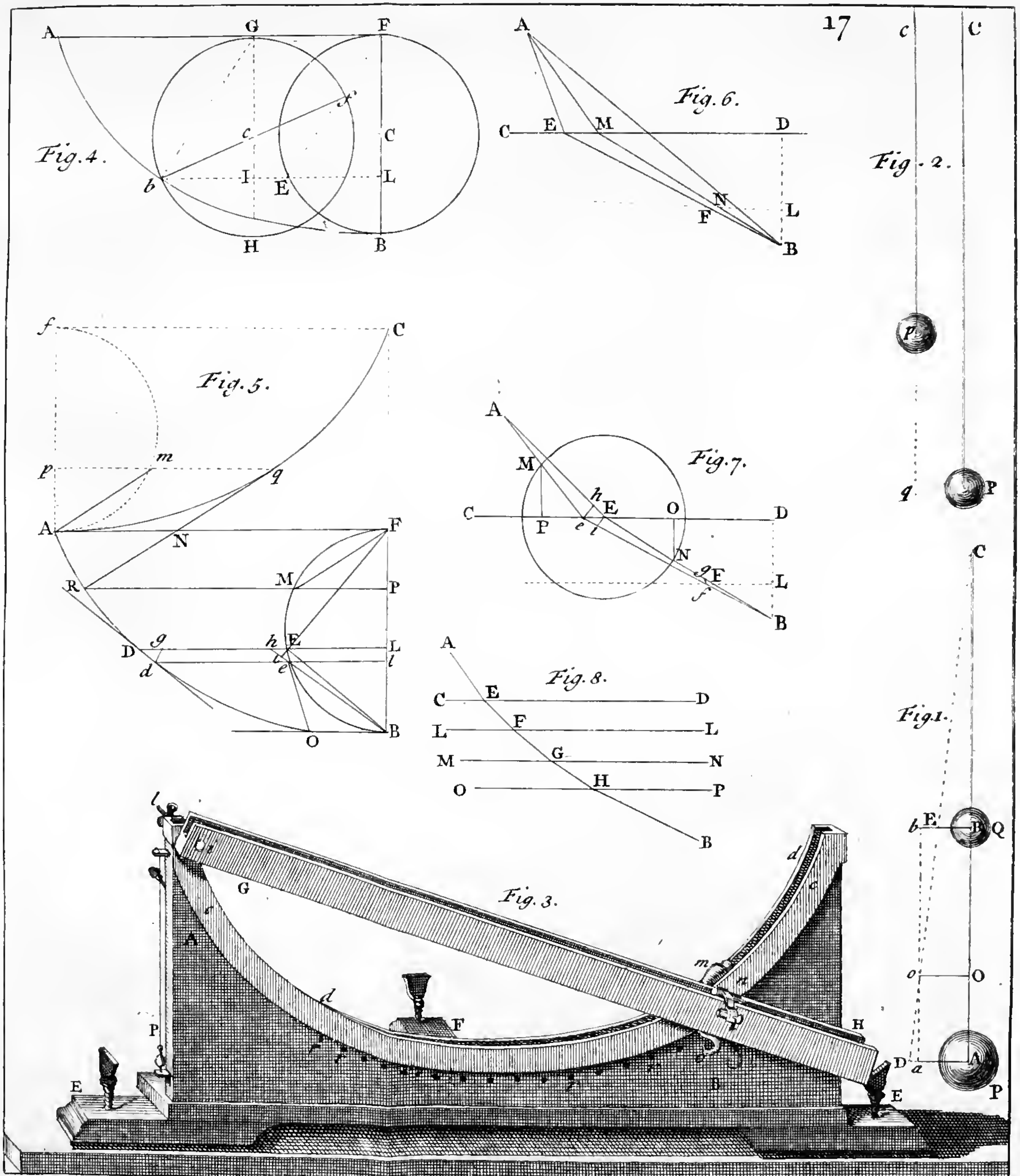


Fig. 4.
Fig. 6.
Fig. 2.
Fig. 5.
Fig. 7.
Fig. 8.
Fig. 1.
Fig. 3.

Let AB be the Lever, whose Brachia are to one another as one to ten; and let A be a Weight of an hundred Pounds, to be rais'd to a determin'd Height A *a*. 495. Plate XVIII. Fig. 4.

A Power equal to ten Pounds being applied, a Weight of an hundred Pounds will be sustain'd, but it can't be rais'd *: If I add one Pound, and the Weight B be eleven Pounds, A will be rais'd but slowly, and eleven Pounds are not sufficient that the Weight A may be rais'd, by the whole Action smallest of all: for another Pound being still added; that is, the Intensity of the Power being increas'd an eleventh Part, the Time is diminish'd almost three Parts in eleven; and the whole Actions, namely, the Products of the Times by the Intensities of the Powers, are to one another as 5 to 4, very nearly. * 235.

If the Intensity is increas'd more and more, we shall have, to a certain Limit, the whole Action diminish'd, which will be increas'd if the Intensity of the Power is increas'd beyond this Limit. The Table plac'd below makes this sensible, in which the Lever abovemention'd is consider'd, and in which 100,000 express the whole Action which is the smallest of all.

Powers.	*Whole Actions.*	*Powers.*	*Whole Actions.*	
10.	*Infin.*	15,16.	100000.	496.
11.	142360.	16.	100368.	
12.	114036.	17.	101611.	
13.	104677.	18.	103397.	
14.	101053.	19.	105575.	
15.	100016.	20.	108030.	

This Table demonstrates that the Power, which sustains the Weight to be rais'd, must be sufficiently increas'd in the Use of a Machine, but that it matters little whether it be increas'd a little more or less; for as much as this Intensity is increas'd, so much is the Time commonly diminish'd, and *the whole Action is a little chang'd within certain Limits.* In the Example which we consider, it does not matter which of these Powers we make use of, that of 14, 15, 16, or 17 Pounds *. 497. 498. * 496.

In the Use of the Lever this Reasoning is scarce of any use, as I have already said *; but in other Machines, the Axis in Peritrochio, Pulley, and Machines compounded of these, we should not neglect the Determination of the Power. * 494.

I will now ſhew how we muſt proceed in this Determination, and will give the Demonſtrations of the Operations in the following *Scholia*.

For the Axis in Peritrochio.

499. I collect theſe four Numbers into one Sum. 1. The Weight of the Limb of the Wheel. 2. The third Part of the Weight of the Radii. 3. The half of the Weight of the Axis, multiplied by the Square of its Diameter, and divided by the Square of the Diameter of the Wheel. 4. Laſtly, the Weight, whereby the Æquilibrium is had, multiplied by the Diameter of the Axis, and divided by the Diameter of the Wheel. I divide this Sum by the Weight, whereby the Æquilibrium is had; that is, which being applied to the Machine ſuſtains the Weight to be rais'd, and in the Quotient I ſhall have a Number, which I ſhall call *the Index of the Machine.*

500. We muſt have recourſe to the Table underneath with this Index *, which is of uſe in all Machines, and which contains the Indices in the firſt Column, and the Number in the ſecond Column, anſwering to the Index, will give the Increaſe to be added to the Power, whereby an Æquilibrium is had; which Increaſe is expreſs'd in hundredth Parts of this Power.

* 508.

501. *Example.* Let the Weight of the Limb be 100 Pounds; the Weight of the Radii 30 Pounds; the Weight of the Axis 80 Pounds; the Diameter of the Axis 1; the Diameter of the Wheel 10; the Weight to be rais'd 200 Pounds; therefore the Weight, which would give an Æquilibrium, ſhould be 20 Pounds.

I collect 100; 10; $\frac{2}{5}$, or 0,4; and 2 into one Sum: I divide the Sum 112,4. by 20. and diſcover the Index to be 5,62; which is a mean between 5 and 6. and the correſponding Number is nearly, 0,80. Therefore the Increaſe is equal to eighty hundredth Parts of 20 Pounds, and the Intenſity of the Power to be applied to the Machine will be 36 Pounds.

For the Pulley.

502. We ſuppoſe all the ſmall Wheels or Sheaves to weigh equally; and I multiply the Weight of one Sheave by the Product of the Number of little Wheels or Sheaves increas'd by one, and multiplied by double the ſame Number of Sheaves *plus* one; and I divide this Product by the Number of Sheaves twelve times taken. I add the Weight, whereby the Æquilibrium is had, divided by the

2 Number

Number of Sheaves; and divide the Sum by this Weight which gives the Æquilibrium, and *the Index* is given in the Quotient.

Example. Let the Weight of one Sheave be 3 Pounds; the Number of Sheaves 10; the Weight to be rais'd 200 Pounds; therefore the Weight, which gives the Æquilibrium, is 20 Pounds *. 503. * 260, 271.

I multiply 3 by 11, and the Product by 21; and I have 693; I divide this Number by 120; the Quotient is 5,775; I add 20, divided by 10, that is 2; and divide the Sum 7,775, by 20. and the Index is 0,389, less half; and the Increase, as the Table shews *, differs little from 57 hundredth Parts of the Power, which gives the Æquilibrium, wherefore the Power to be applied exceeds but a little 31 Pounds. * 508.

How small soever the Index becomes, yet it never vanishes entirely: In the Axis in Peritrochio, if the Machine had no Weight, and the Diameter of the Wheel were infinite, this would obtain: And in the Pulley also, if the Number of Sheaves were infinite, and had no Weight; therefore the Increase, of which I speak here, always exceeds the half of the Action, which can sustain the Weight to be rais'd; yet the Action is never to be doubled, since the Increase can never equal it; as appears by an Inspection of the Table *. 504. * 508.

What has been above observ'd of the Lever, may be also applied to all Machines *, whence I conclude, that this general Rule may be establish'd, without any sensible Error. 505. * 498.

The Power which is to sustain the Weight to be rais'd, must be increas'd by half, if the Weight is to be rais'd by help of many Sheaves; or by some other lighter Machine, or one whose heavier Parts are mov'd slowly; as in the Capstand: namely, an Axis round which a Rope is wound, whilst it is turn'd round by help of a Lever, or long Handle. 506.

In other Cases *when the Machine is heavier*, as in the Axis in Peritrochio, *the Power must be doubled, which is in Æquilibrio with the Weight to be rais'd.* 507.

The Table.

	Index.	Power.	Index.	Power.
508.	0.	0,50.	6.	0,81.
	0,5.	0,57.	7.	0,83.
	1.	0,62.	8.	0,85.
	2.	0,69.	9.	0,86.
	3.	0,73.	10.	0,87.
	4.	0,76.	15.	0,90.
	5.	0,79.	35.	0,95.
			Infin.	1,00.

509. Hitherto we have ſuppos'd the Machine determin'd in every reſpect, and have only treated of the chuſing of the Power; now
510. let us ſee, how we muſt proceed *if we have to do with Machines of the ſame kind*, and one is to be choſen.

Let us conſider the Lever again, but ſuch an one, the Ratio of whoſe Brachia is not determin'd, *and* let *the ſame Weight be to be rais'd to a determin'd Height.* That we may compare the whole Actions of different Levers, we ſhould attend to three Things, and theſe three Things only are to be conſider'd. For *the whole Action follows the Ratio*, 1. *Of the Intenſity of the Power acting.* 2. *Of the Time in which it acts* *. 3. *Of the Space run thro' by the Power.* For if the Weight of one Pound, while it is acting, deſcends two Feet, its Action is double that, which it would produce, if it deſcended but one Foot; for in the firſt Caſe the former State of the Weight can't be reſtor'd, unleſs it be twice rais'd, as in the ſecond Caſe it muſt be rais'd but once.

* 491.

511. Now if we determine the whole Actions, for different Lengths of the Lever, by applying for each of the Lengths a Power, which gives the ſmalleſt Action for that Length; we diſcover, by different Levers being compar'd, that the whole Action is leſs, if the Diſtance of the Power from the Fulcrum be leſs.

512. Let a Weight of an hundred Pound be to be rais'd to a determin'd Height; let a Lever with equal Brachia be made uſe of, that the Action may be ſmalleſt, a Power muſt be applied whoſe Intenſity is equal to 162 Pounds; if this Diſtance be reduc'd to half, the Intenſity of the Power will be 338 Pounds. But it deſcends only through half the Space, the Time alſo is diminiſh'd, and is to the firſt as 35 to 44 nearly, and the whole Actions are, as 162

x

$\times 1 \times 44$, is to $338 \times \frac{1}{2} \times 35$ *, that is, as 7128, to 5915, nearly * 510.
as 6 to 5.

Hence it follows that when greater Powers are applied the whole 513.
Actions are leſs, a juſt Ratio between the Brachia being laid down; but theſe have ſome Inconveniency, becauſe it is not eaſy to manage them, and Machines are made uſe of for raiſing Weights, that a greater Weight may be rais'd by a leſs Power.

If any one, when a determin'd Weight is to be rais'd by a de- 514.
termin'd Power to a determin'd Height, ſhould require the ſmalleſt Action, we muſt find the Dimenſions of a Machine, in which the Product of the Time by the Space run through by the Power would be ſmalleſt *; which cannot be done without difficulty; * 510.
becauſe the Weights of the Parts of a Machine, when it is chang'd, do not vary according to any determin'd Rule. A Solution of this Problem, which is ſolid in the moſt ſimple Caſe, would not be of much advantage in practice *. * 506, 507.

In the Lever of the third kind, the Power is always applied 515.
at a leſs Diſtance than the Weight to be rais'd *, and ſhould al- * 234.
ways overcome it †: wherefore in this Lever *the whole Action is* † 235.
leſs than in the ſecond, or common Uſe of the firſt *; *and* * 511.
in reſpect of this Action *the Lever of the third Kind ſurpaſſes the others.*

Many Authors, who treat of Mechanicks, compare ſeveral 516.
Machines with one another, conſidering only the Caſe of the Æquilibrium, and look upon this as the Foundation of the Uſe of Machines, *viz.* That the Time, in which an Effect is produc'd, is increas'd in the Ratio, in which the Intenſity of the Power is diminiſh'd. But what is demonſtrated in this Chapter proves, that this Propoſition muſt not be admitted.

In the Uſe of the Machines, treated of in this Chapter, when a 517.
conſtant Power is applied to them, the Weights are raiſed with an accelerated Motion, and we ſpoke of this Caſe only.

There are other Machines, ſuch as commonly are Hydraulick 518.
Machines, in which we don't conſider the determin'd Effect to be perform'd, but the ſucceſſive: in theſe, ſucceſſively different Water is rais'd with the ſame Velocity, by the continued Action of the ſame Machine. I ſhall afterwards ſpeak of the Uſe of ſuch Machines.

SCHOLIUM I.

In which is illustrated what was said of the Lever in the Beginning of this Chapter.

519. Plate XVIII. Fig. 5. LET A B be a Lever, C the Fulcrum, A C is equal to one, B C ten, there is at A an hundred Pound Weight to be rais'd. I apply ten Pounds at B, and have an Æquilibrium; then I successively use the Weight of eleven, twelve, thirteen Pounds, and A is rais'd after the same manner, in different Times. That we may compare these Times, we must look upon the Lever, loaded with Weights, as a compound Pendulum, and seek for the Center of Oscillation *. If the Power is equal to thirteen Pounds, this Center is D; if to fourteen, it is E; if to fifteen, F, &c.

* 479.

In these different Motions, when we consider the same Motion of the Pendulum, the Center of Oscillation passes thro' similar Spaces, and the Squares of the Times of the Descent are as the Spaces pass'd thro' *; which are as the Distances C D, C E, C F, &c. The Increase of the Power, when the Distance of the Center of Oscillation is great, sensibly diminishes this Distance; and this is the reason why the Time is more diminish'd, than the Intensity of the Power is increas'd, whereby the whole Action is diminish'd *. But when the Intensity of the Power being increas'd, the Center of Oscillation is less distant, as at H, I, L, &c. then, by increasing the Power more, the Center of Oscillation comes forward a little, the Time is diminish'd a little, and the whole Action is increas'd.

* 374. 383.

* 491.

520. In the Construction of the Table, N° 496. the Distances of the Centers of Oscillation were determin'd for different Powers, and the square Root of the Distance of every one of them was multiplied by its Power, and this Product, or rather the Numbers in the same Ratio with these, were put into the second Column. But the Power, which gives the whole Action smallest, was determin'd by the Method explain'd in Scholium III.

SCHOLIUM II.

Of the Indices of Machines.

521. WE call'd that Number the Index of a Machine *, by help of which we discover the Power, which produces a determin'd Effect, by the whole Action smallest of all.

* 500.

We discover this Number, by seeking the Center of Oscillation of the Machine.

Of the Index of the Lever.

Let a be the Weight to be rais'd; the Diſtance A C $= m$; C B $= n$; the Power whereby the Æquilibrium is had will be $\frac{ma}{n}$ *; and let x be the Increaſe of the Power, that Motion may be given. 522. Plate XVIII. Fig. 5. * 235.

The Diſtance of the Center of Oſcillation from C, the Lever being ſuppos'd to be without Weight, will be $\frac{mma + nma + nnx}{nx}$*. I ſuppoſe b ſo determin'd, that $bnma = mma + nma$; that is, I put $b = \frac{m}{n} + 1$. * 479.

The Diſtance of the Center of Oſcillation now is $\frac{bnma + nnx}{nx} = \frac{bma + nx}{x}$, but x muſt be determin'd, with relation to the Weight which gives the Æquilibrium; for the Increaſe, which is call'd x, was expreſs'd in the Caſe of the ſmalleſt Action, by hundredth Parts of this Weight, in different Machines *; that is, we ſhould put $\frac{am}{n}$, the Weight which gives the Æquilibrium, equal to Unity, then $am = n$, and *the Diſtance of the Center of Oſcillation*, will be $\frac{bn + nx}{x}$; which Diſtance *follows the Proportion* $\frac{b + x}{x}$. *And* b — 1 *is the Number, which we call'd the Index of the Machine*; by help of this I ſaid the Power was diſcover'd, which gives the whole Action ſmalleſt of all *. 523. * 500. 524. * 500.

In every Machine, by a like Expreſſion, we denote the Proportion of the Diſtance of the Center of Oſcillation; b only differs, but, this being given, the Problem, concerning the whole Action ſmalleſt, is ſolv'd in the ſame manner, as we ſhall ſee in the following Scholium. 525.

Of the Index of the Axis in Peritrochio.

Let there be a Wheel E, whoſe Radii are D, D, *&c.* Axis C; let p be call'd the Weight of the Limb, or Circumference of the Wheel; let r be the Weight of all the Radii taken together; and the Weight of the Axis q; the Semidiameter of the Axis is call'd m. The Semidiameter of the Wheel n, we ſet aſide the Breadth of the Limb; and then n expreſſes the Length of the Radii alſo. The Weight to be rais'd A, is call'd a; the Weight whereby the Æquilibrium is had will be $\frac{ma}{n}$ *; the Increaſe whereby Motion is communicated x; therefore B $= \frac{ma}{n} + x$. 526. Plate XVIII. Fig. 6. * 255.

In the Motion of this Machine a Point is given, which is moved, as if it was acted upon by Gravity only, and anſwers to the Center of Oſcillation in 527.

in a Pendulum; for we ſhall have a true Pendulum, if we ſuppoſe the Weights A fix'd at a, and B at b. We diſcover the Diſtance of this Center
* 479. from the Center of the Wheel by the Rule given *.

I multiply the Weight of the Limb by the Square of the Semidiameter of the Wheel, the Product is $n n p$. I multiply the Weights of each of the Points of the Radii by the Squares of the Diſtances from the Center, and
* 480. have $\frac{1}{3} n n r$ *, a like Product for the Axis is $\frac{1}{2} m m q$ †; for the Axis may
† 482. be look'd upon as a thicker Wheel. The reſt of the Products are $m m a$, and $B n n = m n a + n n x$, and the Diſtance of the Center of Oſcillation is
* 479. $\frac{n n p + \frac{1}{3} n n r + \frac{1}{2} m m q + m m a + m n a + n n x}{n x}$ *.

Let us put $n n p + \frac{1}{3} n m r + \frac{1}{2} m m q + m m a + m n a$, all which are known Quantities, $= b m n a$. Therefore the Diſtance of the Center of Oſcillation is equal to $\frac{b m n a + n n x}{n x} = \frac{b m a + n x}{x}$; but if we would expreſs x with an immediate relation to the Weight, whereby an Æquilibrium is
* 528. had, as we did with reſpect to the Lever *, we ought to take this Weight; *viz.* $\frac{m a}{n}$, for Unity. Then $m a = n$ and $\frac{b m a + n x}{x} = \frac{b n + n x}{x}$, which Quantity follows the Proportion of this $\frac{b + x}{x}$, in the ſame manner as was
* 524. ſaid of the Lever *. The Index of the Machine is $b - 1$, whoſe Value we have, if we divide the Equation, in which b was aſſum'd, by $m n a$, and we ſhall have $\frac{n p}{m a} + \frac{n r}{3 m a} + \frac{m q}{2 n a} + \frac{m}{n} = b - 1$, which Determination
* 499. of the Index agrees with that which we gave above *. For we ſaid that theſe four Numbers muſt be collected into one Sum, $p + \frac{1}{3} r + \frac{m m q}{2 n n} + \frac{m m a}{n n}$, which Sum I ſaid muſt be divided by the Weight, whereby an Æquilibrium is had, $\frac{m a}{n}$, and is chang'd into this $\frac{n p}{m a} + \frac{n r}{3 m a} + \frac{m q}{2 n a} + \frac{m}{n}$.

528. We muſt proceed in the ſame manner in all the Machines; we muſt ſeek the Number which is call'd the *Index*, which has this Property, that being increas'd by Unity, if it be call'd b, the Diſtance of the Center of Oſcilla-
* 524. tion ſhall follow the Proportion $\frac{b + x}{x}$ *.

Of the Index of the Pulley.

529. In this alſo, as in all other Machines, which, ſetting Friction aſide, are mov'd by Gravity only, there is a Point which anſwers to the Center of Oſcillation in a Pendulum; that is, which is mov'd with ſuch a Velocity, as it would acquire, if it were mov'd by its own Gravity only.

When

When all the Parts of the Machine are mov'd about the same Center, or the same Axis, the Velocities of each of the Points are proportional to the Distances from the Center, or Axis; and for this Reason, that the Center of Oscillation may be determin'd, every Weight is multiplied by the Square of its Distance from that Center, or Axis *: for the same reason, when the Motion is not such, as in the Pulley, we should multiply every heavy Point by the Square of its Velocity; that is, *in the Determination of the Center of Oscillation we must act, as if all the Points turn'd about the same Axis, keeping the Velocity which they really have.* 530. * 479. 531.

Let there be equal Sheaves, weighing equally. For the first Sheave I multiply each of the Points by the Squares of their Distances from the Center, and have half of the Weight of the Sheave, if we suppose its Semidiameter to be denoted by Unity *. The Velocity of the second Sheave is double; that is, every Point is mov'd with a Velocity double of that, which the correspondent Point has in the first Sheave; and therefore the Sum of the Products, for the second Sheave, is quadruple of that, which is determin'd for the first Sheave. In the same manner the Product is nine times as much for the third Sheave, and sixteen times as much for the fourth, &c. If n be the Number of Sheaves, for the last Sheave the Product will be half of the Weight of one Sheave by nn, and the Sum of the Products will be equal to the Product of the half of the Weight of one Sheave by the Sum of the Squares of the natural Numbers from Unity to n; which Sum is easily discover'd, as will be presently shewn: Let this last Product be nnp. 532. * 482.

Let f be the Weight whereby an Æquilibrium is had, and nf will be the Weight to be rais'd *; x the Increase of f, that Motion may be communicated to the Machine. * 262, 271.

The Weight to be rais'd nf should be multiplied by the Square of the Semi-diameter of the first Sheave, that is, by Unity, and we have nf. Lastly, $f+x$ must be multiplied by nn, and we shall have the Distance of the Center of Oscillation $\frac{nnp+nf+nnf+nnx}{nx}=\frac{np+f+nf+nx}{x}$ *. * 531, 474. 475.

We suppose b to be so determin'd that $np+f+nf=bnf$, and the Distance of the Center of Oscillation is equal to $\frac{bnf+nx}{x}$. Now if, as in the foregoing Machines, x must be express'd with relation to f, we put $f=1$; then $\frac{bnf+nx}{x}=\frac{bn+nx}{x}$; and it appears that the Distance of the Center of Oscillation follows the Proportion $\frac{b+x}{x}$.

The Index is $b-1=\frac{p}{f}+\frac{1}{n}$; and I will now demonstrate, that this was exactly determin'd above.

We put nnp equal to the Product of the half of the Weight of one Sheave by the Sum of the Squares of the natural Numbers from Unity to n.

I will say something about discovering this Sum; the Problem is well known, and it is a singular Case of a Problem, which is itself particular in respect of another more universal. To take this Method in the Demonstration would be tedious; I will take a shorter Method, and give a Demonstration, which relates to that Case only of which we are speaking.

534. Plate XVIII. Fig 7. For Unity we have a small Cube, as Z. Of such Cubes I suppose Squares to be form'd of the natural Numbers, which together make the Solid X; whose Magnitude, the Unity Z being applied, expresses the Sum sought.

I suppose this Solid X, to be inscrib'd in the Pyramid ABDC, whose Base is the Square of the Sides $n + 1$, and Height is also $n + 1$; this Pyramid is equal to $\frac{1}{3} n^3 + nn + n + \frac{1}{3}$ * (* 7 El. XII.); but it exceeds the Solid X, and the Excess for each of the Squares or Strata, consists of a Pyramid as HDLGI, which is equal to $\frac{1}{3}$; and moreover of two Prisms, as HBEGIF, and GNCLIM, which together make a Parallelopiped, which contains so many Cubes, like A, as there are Unities in the Side of the Square EN.

We have the whole Excess of the Pyramid, above the solid X, 1. By multiplying the Value of the smaller Pyramid $\frac{1}{3}$ by the Number of such Pyramids $n + 1$, and the Product is $\frac{1}{3} n + \frac{1}{3}$; and 2. By seeking the Sum of all the Parallelopipeds, which make the Arithmetical Progression 1. 2. 3. .... n; which Sum is equal to $\frac{1}{2} an + \frac{1}{2} n$. Therefore the whole Excess is $\frac{1}{2} nn + \frac{5}{6} n + \frac{1}{3}$: this being subtracted from the Value of the Pyramid, I have the Sum sought of the Squares $\frac{1}{3} n^3 + \frac{1}{2} nn + \frac{1}{6} n = \frac{2 n^3 + 3 nn + n}{6}$.

535. If q be the Weight of one Sheave, it will be $\frac{2 n^3 q + 3 nn q + n q}{12}$

* 533. $= nnp$ *, and $p = \frac{2 nn q + 3 n q + q}{12 n} = \frac{\overline{n + 1} \times \overline{2 n + 1} \times q}{12 n}$.

* 532. We saw that the Index $b - 1$ was equal to $\frac{p}{f} + \frac{1}{n}$ *. Therefore $b - 1$

$= \frac{\overline{n + 1} \times \overline{2 n + 1} \times q}{12 n f} + \frac{1}{n}$, which is the above-determin'd Value of the

* 502. Index *.

SCHOLIUM III.

Of determining the smallest whole Action.

536. WE have seen how, the Index of a Machine being given, by help of the Table N°. 508. we discover the Power, which gives the whole

* 500. Action smallest of all *; we must now speak of the Construction of this Table.

In a Machine propos'd, we consider the raising the same Weight to the same Height; therefore of the same Motion of the Machine, so that the Space,

Space, run thro' by the Center of Oscillation, follows the Proportion of this Distance from the Center of Motion. The Way run through by the Center of Oscillation in this Motion, is always similar to itself, and the Square of the Time is as the Space run through *; therefore it is as the Distance of the Center of Oscillation, that is as $\frac{b+x}{x}$, of which Expression this Distance always follows the Ratio, as I have demonstrated in the foregoing *Scholium*. The whole Action is had by multiplying the Time by the Power *, which in this Case is equal to $1+x$; and the Square of the Action follows the Proportion of the Square of the Time by the Square of the Power, that is $\frac{b+x}{x}$ by $\overline{1+x}^2$. And this Action will be the smallest when x is so determin'd, that this Product is the smallest of all.

* 519.

* 492.

537. Plate XVIII. Fig. 8.

BA, BD being drawn, which make a right Angle at B, let BI be $=b$; and IA $=x$. Thro' I I draw IC $=1$, parallel to BD, and put IH also equal to Unity.

By reason of the similar Triangles ACI, ABD, AI (x) : AB $(b+x)$ :: CI $(1.)$: BD $\left(\frac{b+x}{x}\right)$*. Therefore the Distance of the Center of Oscillation follows the Proportion of the Line BD, howsoever x be chang'd.

* 16 El. VI.

If from $\frac{b+x}{x} =$ BD Unity be substracted on both Sides there is given $\frac{b}{x} =$ ED.

The Intensity of the Power is HA $= 1+x$; therefore the Product, which expresses the Square of the whole Action, is $BD \times HA^q$, and we seek IA, when this Product is the smallest of all similar Products.

If in this Case x be a little increas'd, or diminish'd, the Products become greater, and these may be so taken, as to be equal; let us suppose such to be $BD \times HA^q = Bd \times Ha^q$; the Value sought of x is a mean between IA and Ia.

But the Points A and a, may be so mov'd mutually towards each other, that the Distance will be infinitely small; in which Case Aa is had for the Point, and IA is the Value sought of x.

This Æquation $BD \times HA^q = Bd \times Ha^q$ is resolv'd into this Proportion *; $Bd : BD :: HA^q : \overline{HA + Aa}^q = HA^q + 2HA \times Aa + Aa^q :: HA : HA + 2Aa$; for Aa^q is infinitely small in respect of the other Quantities, and may be neglected, and by dividing the Terms of the last Ratio but one by HA, we light upon the last. Therefore $Bd : BD :: HA : HA + 2Aa$; whence by Conversion and Alternation we deduce $Bd : HA :: Dd : 2Aa$, or $Bd : \frac{1}{2}HA :: Dd : Aa$.

* 16 El. VI.

With the Center C let the Arches AG, dF of a Circle be describ'd thro' A and d, which may be look'd upon as Right Lines, because they are suppos'd to be infinitely small.

 The

The Ratio of D d to A a, is made up of these three Ratios of D d to dF, dF to AG, and AG to A a.

By reason of the similar Rectangular Triangles DCE, D dF, A aG, and CAI, these Ratios are reduc'd to these different ones; D d : dF :: CA : AI; dF : AG :: DE : CI; AG : A a :: CI : CA.

Therefore D d is to A a in a Ratio made up of these three Ratios of DE to CI, CI to CA, CA to AI; the Ratio of which compound Ratio is that of DE to AI, in which Ratio therefore is D d to A a, or B d to $\frac{1}{2}$ HA, that is, DE $\left(\frac{b}{x}\right)$, AI (x) :: B d $\left(\frac{b+x}{x}\right)$, $\frac{1}{2}$ HA

* 16 El. VI. $(\frac{1}{2}x+\frac{1}{2})$, and $b+x=\frac{1}{2}b+\frac{b}{2x}$*; whence we deduce the Æquation $xx+\frac{1}{2}bx-\frac{1}{2}b=0$, whose positive Root is $x=\frac{1}{4}\sqrt{8b+bb}-\frac{1}{4}b$; and we have what is sought.

538. For the Index of the Machine being given, we add Unity, and we have the Value of b; we have substituted this for b in the Æquation, and x is given.

539. In the Construction of the Table N°. 508. we have successively put $b=1$, $b=2$, $b=3$, &c. that is, we put the Value of the Index 0, afterwards 1, then 2, &c. and the correspondent Values of x being found, were put into the Table.

CHAP. XXII.

Of the Projection of heavy Bodies.

540. Plate XIX. Fig. 1. * 357. IF a Power acts upon a Body in motion, the Motion is alter'd*; *if a Body be projected* along AB, in the Time, in which *it* can run thro' AB, by the Force of Gravity, it is carry'd towards the Center of the Earth along BF; and thus, is mov'd along AF,

* 360. with a Motion compounded of these two*; and with this Motion it would run thro' FC, equal to AF, in the second Moment, except in the second Moment, it were carried thro' CG by the same Force of Gravity, so that the Motion in the second Moment be thro' FG. In the same manner, the Motion of the third Moment is thro' GH, and of the fourth Moment thro' HI; but as the Force of Gravity acts continually, those Moments of Time are very small, and there will be always a Motion otherwise compounded; that is, an Inflection of the Direction; in that Case therefore the Body *is mov'd in a Curve Line.*

541. *This Motion of a Body from Projection* may be consider'd more simply, in all Projections, which may be caus'd by us; because all the

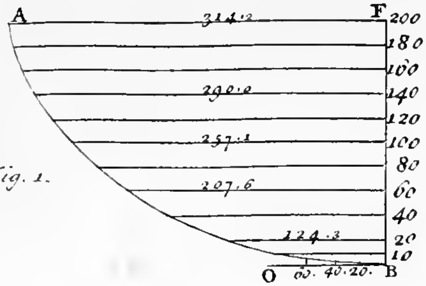
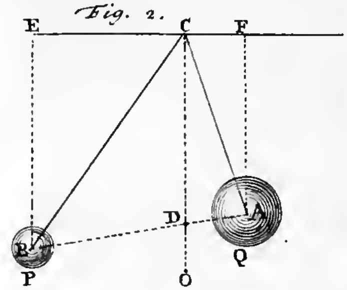


Fig. 1.

Fig. 2.



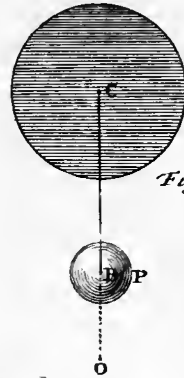


Fig. 3.

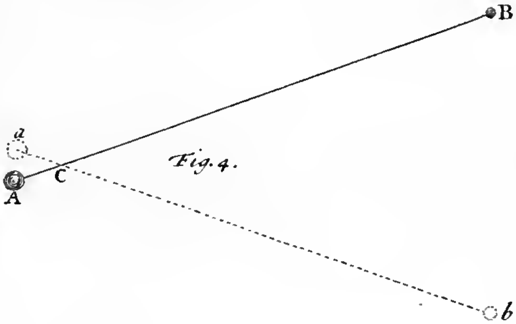


Fig. 4.

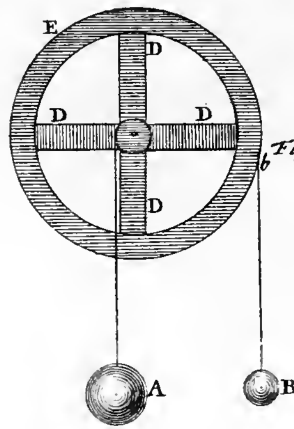


Fig. 6.

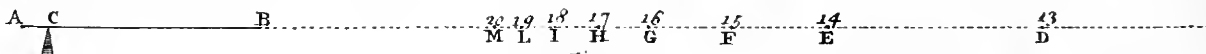


Fig. 5.

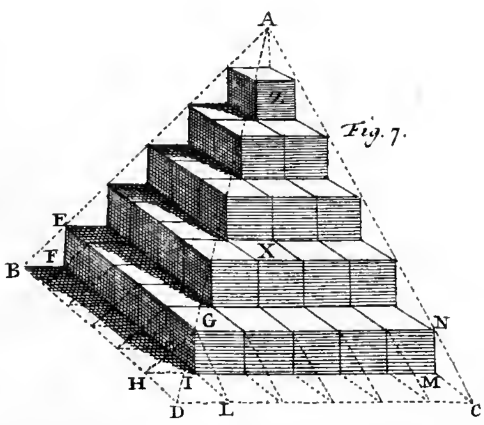
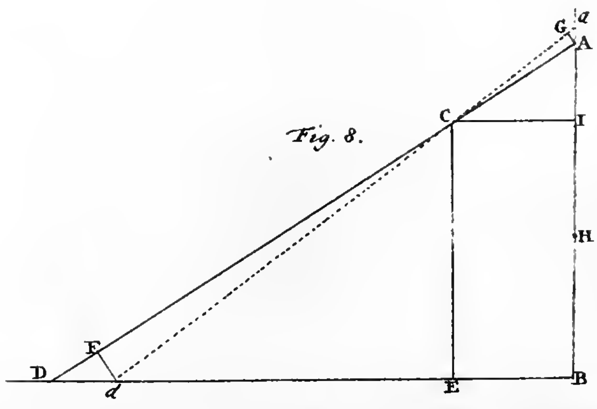


Fig. 7.

Fig. 8.



J. Mynde sc.

the Lines, which tend towards the Center of the Earth, in the Space, thro' which the Body passes, may be look'd upon as parallel; wherefore the Direction of the Motion is not chang'd by reason of Gravity; whence the Motion from Projection *is made up of two Motions* only, *the first equable along the Line of Projection* *, * 355.
the second accelerated towards the Earth †; *which two Motions don't* † 370.
disturb one another *. * 358.

542. Plate XIX. Fig. 2. Let a Body be projected along the Line A E, parallel to the Horizon; in equal Times, with this Motion, it will run thro' the equal Parts, A B, B C, C D, D E: by Gravity it is carried with a Motion perpendicular to the Horizon, in the Direction B F, C G, D H, or E I, which Lines are taken for parallel; this Motion is accelerated, and therefore, if after the first Moment the Body be at F, after the second it will be at G, after the third at H, after the fourth at I; so indeed that B F being put one, C G will be four, D H nine, and E I sixteen *. The Body will run thro' * 374.
a Curve passing thro' all the Points, which may be determin'd in the same manner as F, G, H, I; and it is call'd a *Parabola*.

A Machine, *whereby what has been demonstrated of the Projection of Bodies is confirm'd.*

543. Plate XIX. Fig 3. The chief Part of this Machine is a solid Piece of Wood A, one Foot high, and two Inches thick: this has half its Height from C to B made hollow circularly, or according to any other Curve; yet so, that a Ball may descend regularly from B to C; which, that it may do more freely, the Wood is cover'd with a Copper Plate, or an iron one tin'd, which is smooth and polish'd. A Marble Ball is made use of, whose Diameter somewhat exceeds half an Inch; and the Curve B C is so plac'd, that the Motion of the Ball at C may be horizontal.

This solid Piece is put upon a Board D E, to which also it is join'd, and which is supported by three Screws, as G, G, for the third can't be seen; by help of a Plumb-Line, applied to the back Part of the Machine, and whose Thread is fasten'd to the little Wedge N, the solid Piece A is put into a vertical Situation, and the Board D E into an horizontal one.

There is join'd to the Side of the Machine the Board M, which may be remov'd, and at pleasure applied to either Side.

When it is applied to the anterior Side, it is put in between the Board H, join'd to the Machine, and the Surface of the Solid A, being retain'd by the Ruler I also.

This Board is faſten'd in the Situation here repreſented, after this manner.

The Ball is plac'd at B, which is let down, that it may deſcend freely along B C, and the Diſtance F, to which it falls, is obſerv'd; which is always the ſame, if the Ball be often let down; becauſe it deſcends every time from the ſame Height, and therefore is projected horizontally from C, with the ſame Velocity.

There is an hollow in the Wood F, which is fill'd with Cotton, and the Point *l*, anſwering to F, is mark'd on the Board M.

Thro' *l* the vertical Line *lf* is drawn; the Ball is plac'd at the End C of the Curve B C, and the Point *a*, anſwering to its Center, is alſo mark'd on the Board M, and the horizontal Line *af* is drawn, which here makes the Extremity of the Board M.

af is divided into five equal Parts at *b*, *c*, *d*, *e*, and the vertical Lines *bn*, *cn*, *dn*, *en*, are drawn; whoſe Lengths are thus determin'd; *fl* is divided into 25 equal Parts, of which *bn* contains one, *cn* four, *dn* nine, *en* ſixteen: and thro' the Points *n*, *n*, &c. the Curve *annl* is drawn, which ſhews the Way, which a Body, horizontally projected from *a* in ſuch manner, as to fall at *b*,
* 542. paſſes thro' in its Motion *.

The four Copper Rings, O, O, O, O, are applied to the Board M; theſe have cylindric Tails, which are thruſt into Holes at *n*, *n*, *n*, *n*, in ſuch manner, that the Centers of the Rings are in the ſame Plane, parallel to the Board M, and paſſing thro' the middle of the Thickneſs of the Solid A.

The Apertures of the Rings are of one Inch Diameter, and their Planes are perpendicular to the Board M, and the Curve *al* drawn on the Board.

Experiment.

544. The Ball is let down from B, having roll'd to C, is there horizontally projected, and falls at F, and in the mean time paſſes thro' the Rings O, O, O, O.

545. What has been ſaid of the Curve run thro' by a Body horizontally projected, belongs alſo to any Projection whatever.

Plate X Let a Body be projected along A E; and let A B, B C, C D,
Fig. 4, 5. D E, be equal; the Body will paſs along the Curve A F G H I ſo, that the vertical Lines B F, C G, D H, E I, will be to one an-
* 542. other, as 1. 4. 9. and 16 *; in which Caſe alſo the Curve is call'd a *Parabola*.

De-

Definition.

Let A I *be a Plane passing thro'* A, *if the Curve mention'd cuts it in* I; A I *is call'd the Amplitude of the Cast.* 546.

The Motions of Bodies, which are projected with the same Celerity, according to Directions differently inclin'd, may be compar'd with one another.

And a Body may be projected to a given Distance, with a given Celerity, in a given Plane. 547.

Let the given Celerity be that, which a Body acquires in falling from the Height M A, which we suppose to be perpendicular to the Horizon A L, and let the Body be to be projected in the Plane A I to I. M N being drawn parallel to the Horizon, let A N be rais'd perpendicular to the Plane A I, cutting M N at N; with the Center O, the middle Point of the Line A N, let a Circle be describ'd thro' A, passing thro' M also. Let A R be a fourth Part of the Line A I; thro' R let there be drawn the Line R *b*, perpendicular to the Horizon; that is, parallel to the Line A M, which cuts the Circle in B and *b*; if the Body be projected along A B, or A *b*, it will fall at I. By which Method the Direction of the Cast is determin'd, whether the Point be in the horizontal Line, passing thro' A, in which Case M and N coincide, or in any Plane, inclin'd above or below this horizontal Line. 548.

Let us suppose the Direction to be well determin'd. The Body can run thro' A E, in the Time in which it falls thro' E I, with an equable Motion, and such a Celerity, as it had when the Projection was made *. But because the Body is projected with a Velocity, acquir'd in falling thro' M A, it can run thro' the double of M A with the same equable Motion, in the Time in which it falls from the Height M A *. The Spaces, pass'd thro' with the same, and an equable Velocity, are as the Times in which they are pass'd thro' †; therefore the Time of the Fall thro' M A is to the Time of the Fall thro' E I, as the double of M A to A E. Therefore 2 M A^q is to A E^q, as M A is to E I *. Which Proportion therefore, if we demonstrate it to be given in the foregoing Construction, it will appear that the Direction was well determin'd. 549.

* 541. * 376. † 126. * 374.

Let M B be drawn, and we have the Angle B A R, form'd by the Tangent A R *; for it is perpendicular to the Radius A O, and by the Line A B, cutting the Circle, equal to the Angle A M B in the opposite Segment †. The alternate Angles R B A, M A B are also equal *; therefore the Triangles A B R, A M B, are 550.

* 16 El. III. † 32 El. III. * 29. El. I.

* 32. El. I. † 4. El. VI. ‖ 20. El. VI. ‡ 15. El. V. are equi-angular *; and the Lines M A, A B, B R, are proportional †; therefore M A q is to A B q, as M A is to B R ‖; therefore 2 M A q is to 2 A B q, or A C q, as M A is to B R ‡; by multiplying the Consequents by four, we have 2 M A q is to A C q, multiplied by four; that is, 2 A C q, or A E q, as M A is to
* 4. El. V. 4 B R *, or E I, which was to be demonstrated.

551. It is a like Demonstration, if a Body be projected along A *b*. Whence it follows, that *a Body may be projected along two Directions, so as to fall on the same Point*; *but if the Distance be the greatest of all, to which the Body can be projected, with a given Velocity, in a given Plane, there is only one Direction, along which the Body must be projected*; the Points B and *b*, co-inciding in Q, the middle Point of the Arc M Q A, from which the Points B and *b* are always equally distant.

If the Celerity be chang'd, and the Body be projected according to the same Direction, the Amplitude in the same Plane, is chang'd
552. in the same Ratio with the Height A M; that is, *the Amplitude, when the same Direction remains, are* as the Heights, from which the Bodies by falling, can acquire the Velocities, with which they
* 374. are projected; therefore they are *as the Squares of the Celerities* *.

553. Plate XIX. Fig. 5. If A I is *horizontal*, the Arc A Q M, is a Semi-circle, and in this Case the *Amplitude*, the Celerity remaining, with which the Projection is made, *is greatest of all, when the Direction of the Projection makes an half Right Angle with the Horizon.*

554. Let M A be the Height again, from which a Body falling acquires a Velocity, with which it is projected along A B; the highest Point of the Way pass'd thro' is determin'd, if a Semicircle being describ'd, whose Diameter is A M, the horizontal Line T B G be drawn thro' the Point B, in which it is cut by the Direction of the Projection, and B G is equal to B T, the Point sought will be G.

555. The Demonstration of this will appear, if we attend to what follows; the horizontal Line A I being drawn, the Body projected as mention'd, will fall at I, A I being put quadruple of T B, or
* 548. A R *.

Whilst a Body is projected along A B, this Motion co-incides with a twofold Motion, one horizontal and equable, the other
* 366. vertical *. By the last Motion the Body ascends and descends, and the Time of its Ascent is equal to the Time of its Descent; therefore the Ascent is terminated, when the Body passes thro' the half of A I; that is, T G, with an horizontal Motion; therefore the highest Point is given in the vertical Line S C, which passes

thro'

thro' G. Let there be the Verticals I E and B R, the first of which cuts A B continued in E: because T G is double of T B; that is, A S is double of A R; C S also is double of B R, or G S; that is, C G is equal to G S: But A I is double of A S; therefore E I is double of C S, and quadruple of C G; A E also is double of A C. Whilst the Body runs thro' A E with a projectile Motion, it falls thro' E I; whilst it runs thro' A C, it falls thro' a fourth Part of E I; that is, goes thro' C G, in its Fall *; therefore in its Motion it passes thro' the Point G; but the highest Point is given in the Line C S, it is therefore the Point G. * 374.

If there be given a Curve, run thro' by a Body, the Velocity which the Body has in any Point, as F, *is that, which the Body can acquire by falling from an horizontal Line, drawn thro'* M, *to the Point* F. For the Body can ascend to this horizontal Line, thro' any Plane, from A, with the Velocity with which it is projected *; now if a Plane be given, agreeing with the way of the projected Body as far as F, but bent upwards in F, the Body will have at F that Velocity, whereby it can come to the horizontal Line mention'd, along this Plane; that is, which it can acquire by falling from that horizontal Line to F *. 556. * 399. * 399.

Let a Body be to be projected from A thro' the Point H to I, these three Points being suppos'd to be in the same vertical Plane, and the middle Point above the Line which joins the other two. Let A L be horizontal, and L E, N D, A M perpendicular to it thro' three given Points. Let there be drawn the Lines I A, I H, from I thro' the Points A and H, the last of which cuts A M in P; let G D be equal to A P, and A D the Direction of the Cast, or Projection, is had. The Celerity is discover'd, if A R being taken a fourth Part of A I, and R B being drawn vertical, which cuts A D in B, B M be drawn so, that the Angle A B M be equal to the Angle A R B, the Velocity sought, is that which the Body acquires in falling from M to A. 557. Plate XIX. Fig. 6.

The Body projected runs thro' A E and A D, with an equable Velocity, whilst it falls thro' E I and D H: therefore that we may demonstrate the Body to pass thro' the Points H and I, we must demonstrate A E ^{q} to be to A D ^{q}, or E I ^{q} to D G ^{q} *, as E I is to D H †. 558. * 22 El. VI.

In the similar Triangles I H G, I P A, A I is to A G, as A P, or D G, is to D G *minus* G H, that is H D. But in the similar Triangles A E I, A D G; A I is to A G, as E I is to D G; therefore E I is to D G, as G D is to H D; therefore E I ^{q} is to D G ^{q}, as † 545.

E I

* 20 El. VI. E I is to H D *. Which was to be demonſtrated. But it will appear, that the Velocity is rightly determin'd from comparing *Fig.* 6. with 4. if we attend to the Points B, M, which are mark'd with the ſame Letters in both Figures. In *Fig.* 4. we have demonſtrated, that a Body projected along A B, with a Velocity acquir'd in falling thro' M A, paſſes thro' I, and we deduc'd this
* 549, 550. from the Similitude of the Triangles A M B, B A R *. In *Fig.* 6. the ſame Triangles are alſo ſimilar, which follows from the Conſtruction; therefore the ſame Concluſion takes place here alſo.

CHAP. XXIII.

Of Central Forces.

559. A Body in motion continues its Motion in a right Line *, and
* 355. does not recede from it, unleſs a new Impulſe acts upon it; after ſuch an Impulſe the Motion is compound, and ſo from the
* 360. two there ariſes a third Motion in a right Line alſo *. If therefore a Body is mov'd in a Curve, it receives a new Impulſe every Moment; for a Curve cannot be reduc'd to right Lines, unleſs you conceive it divided into Parts infinitely ſmall. We have an Exam-
* 540. ple of that Motion in the Projection of heavy Bodies *; and another in all Motions round a Point as a Center.

560. *If a Body, that is continually driven towards a Center, be projected in a Line that does not go thro' that Center, it will deſcribe a*
561. *Curve; and, in all the Points of it, it endeavours to recede from that Curve, according to the Direction* of a Curvature; that is, *of a Tangent to the Curve*; ſo that if the Force driving towards the Center ſhould immediately ceaſe to act, the Body would continue its Motion in a right Line along the Tangent.

A Stone whirl'd round in a Sling deſcribes a Curve, becauſe the Sling does every Moment, as it were, draw it back towards the Hand; but, if you let the Stone go, it will fly out in the Tangent of the Curve.

Definition I.

562. *The Force with which a Body, in the Caſe above-mention'd, endeavours to fly from the Center,* ſuch as the Force by which the Sling in motion is ſtretch'd, *is call'd a centrifugal Force.*

De-

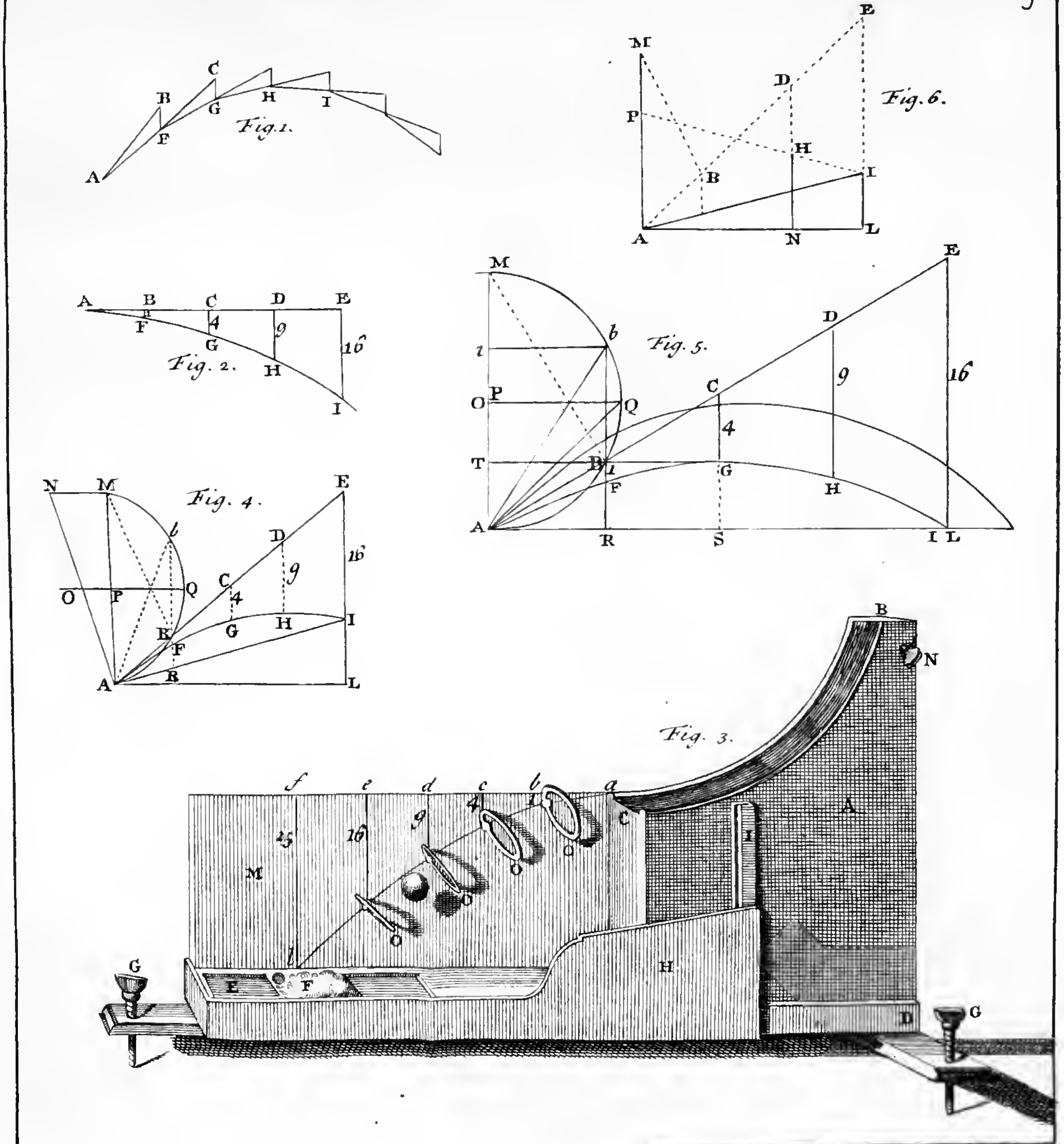


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.

DEFINITION 2.

But *the Force, by which a Body is drawn, or impell'd towards that Center, is call'd a centripetal Force.* 563.

DEFINITION 3.

Thefe Forces are by a common Name call'd *central Forces.* 564.

In all Cafes, the centrifugal and centripetal Forces are equal to one anthoer; for they act in contrary Directions, and deftroy one another. By the Centripetal Force a Body is retain'd in a Curve, and by the Centrifugal it endeavours to recede from it. The whirl'd Sling is equally ftretch'd both ways *, and the Stone endeavours to recede from the Hand with as much Force as it is drawn towards it. 565.

* 361. 364.

Central Forces are of great ufe in Natural Philofophy; for all the Planets move in Orbits, and moft of them, if not all, turn upon their Axes. 566.

I fhall chufe out the chief Propofitions relating to thefe Forces, and explain them, and confirm them by Experiments, and fhall demonftrate them in the Scholia, annex'd to this Chapter.

But fome things in general muft be premis'd concerning the Machines, with which thefe Experiments are perform'd.

A MACHINE, *whereby the Experiments of Central Forces are demonftrated.*

This Machine has a wooden Stand, confifting of three Parts, A B, C D, which are join'd by a third E F. 567. Plate XX.

This Stand is put upon four Rollers, two of which are reprefented at G, G. Thefe, befides their Motion round their own Axis, turn about a vertical Axis together with their Box, that the Machine may be eafily mov'd in any Direction. Such Rollers are commonly ufed at this Time. But when the Experiments are to be made, the Machine muft be fix'd, by raifing the Rollers a little, by help of the Screws H, H, H, H; by which the Machine is alfo plac'd in fuch a Situation, as is requir'd in the Experiments, which is fhewn by the plumb Line a b.

Upon this Stand are plac'd two Pillars I L, M N; which are join'd to one another, by the tranfverfe Piece Q R. 568.

I fhall now fpeak of I L only, and I exhibit a Section of it by itfelf, in which I have mark'd with greater Letters, what I have reprefented in the general Figure by the fame Letters fmaller. Plate XXI. Fig. 1.

569. To this Pillar is join'd an iron Axis A B, which ſtands upon the Support C; to which is applied the Steel Plate e, hollow'd a little, which receives the End of the Axis, that it may be turn'd about eaſily.

The upper Part of the Axis is retain'd by the Arm N O, which embraces the Collar of the Axis ſo, that the turning round of the Axis is not hinder'd; which how it is done, will appear by comparing *Fig.* 1, 2. of *Plate* 23.

570. To the Axis are join'd four wooden Wheels D, which the Axis paſſes thro'; but the Holes of the Wheels are ſquare, and the Axis fills them exactly; the Wheels are faſten'd by means of the Screw m.

The Diameters of the Wheels are meaſur'd from the Bottom of the Groove, but there is added to each the Diameter of a Rope, which is put round them, and of which I ſhall ſpeak in what follows; and the Diameters, thus determin'd, are, the firſt four Inches, the ſecond five Inches, the two loweſt are equal, and ſix Inches in Diameter. The Bottom of the Grooves is arm'd with ſmall iron Pins, that they may retain the Rope.

The Axis *a b* of the Pillar M N, (*Plate* 20.) differs from the Axis of the other Pillar only in the Largeneſs of its ſmaller Wheels, and is repreſented by itſelf in *Fig.* 2. *Plate* 23. The Diameters of the three upper Wheels, which are equal to one another, and to the Diameter of the upper Wheel of the other Axis, are of four Inches; the Diameter of the lower Wheel is of three Inches. Theſe eight Wheels are of the ſame Thickneſs.

571. Plate XXIII. Fig. 2. At the upper End C, of the Axis A B, there is faſten'd to it the copper Plate D D, which the Axis paſſes thro' at C, that it may be better faſten'd; but that all Inequality of the Plate D D, ariſing from the Iron paſſing thro' it, may be taken away, this is cover'd with a larger copper Plate I I, which is join'd to the firſt by four Screws *n*, *n*, *n*, *n*, whoſe Heads are not rais'd above the Surface of the Plate, but make the ſame Plane with it.

Plate XX. Such a Plate is applied to both of the Axes, and both in the general Figure are mark'd with the Letters *i i*, *i i*.

Both the Axes, *d* and *e*, being applied to the Machine, are turn'd about by a Rope carried round them, by help of the Wheel d; but many things are to be obſerv'd with reſpect to this Motion, which muſt be explain'd diſtinctly.

572. In the middle, between the Pillars I L, M N, there is a ſmaller Pillar O P; this anſwers to the ſquare Part S of the tranſverſe Piece

QR, and is join'd to this Part, by two thin Pieces, or wooden Plates e f, e f, in ſuch manner, that the Space, between the End P of the Pillar, and the Solid S, remains empty.

To the upper Surface of the Piece S there is applied a wooden Head T, whoſe Tail goes thro' the Solid S, that the Head may be faſten'd by help of the Wedge g, which is repreſented by itſelf at T, (*Plate* 21. *Fig.* 2.) This may be faſten'd at different Heights, by applying the wooden Rings V, V, V, thro' which the Tail *a b* goes, and all of which, or ſome of them are plac'd above, or below the Piece S, according as the Head is to be rais'd more, or leſs. The Thickneſs of the Rings is equal to the Thickneſs of the Wheels above-mention'd *. 573.

* 570.

To the ſame Head T are faſten'd four Pulleys h, i, m, n, the laſt of which does not appear in the general Figure; h and n are vertical, the other two are horizontal.

A Rope being put round the Wheel d, deſcends towards the Pulley h, and is bent by it, that it may be made horizontal, and come to i, whence it is carried to that Wheel *d*, which anſwers to the Pulleys, and thence goes to the correſponding Wheel at *e*, which it goes round, tending to the Pulley m, whence, paſſing over the Pulley n, it returns to the Wheel d, by turning round which both the Axes *a b*, *a b* are now mov'd. 574.

The Height of the Head being alter'd *, the Rope is put round other Wheels at *d* and *e*, which then anſwer to the Pulleys i and m; of which we muſt further obſerve, that the firſt is rais'd above the ſecond about an Inch, that there may be no Friction between the Parts of the Rope. * 573.

The Wheel d is eaſily turn'd round, for its Axis c, which is of Steel, well work'd, and poliſh'd, turns in copper Plates. The Pillar X Y ſupports this Wheel, which ſtands upon the Piece Z Z; which is faſten'd between the Parts A B, C D, of the Stand of the Machine, and may be turn'd about, that the Pillar may be inclin'd, and the Wheel d, remov'd from the Head T; whereby the Rope is ſtretch'd, when the Screw l o, which paſſes thro' the Pillar X Y, is turn'd round, that it may preſs the Pillar O P. All this appears diſtinctly, if we compare *Fig.* 3. *Plate* 21. with this Figure, which repreſents a Section of the Wheel d, and the Pillars X Y, and O P. 575.

According to the different Inclination of the Pillar *x y*, the Direction of the Preſſure of the Screw l o is different; upon this account a Piece of Wood G H is applied to the Side of the Pillar *o p*, 576.

Plate XXI. Fig 3.

 whoſe

whose Figure is such, that the Screw may always press its Surface perpendicularly.

The Surface of it G H is a Curve, and the Evolute of this Curve is a Circle; but in Practice it will be sufficient, if we determine this Curvature after the following Method.

Let E be the Point, about which the Axis of the Pillar *x y* turns; E N is a Portion of this Axis, which is terminated in the Axis of the Screw l o. Let E M be vertical; that is, parallel to the Axis of the Pillar *o p*, and equal to E N. With the Center E, let there be describ'd the Arc M N F of a Circle, thro' M and N, equal to the greatest Inclination of the Pillar *x y*; which Inclination is determin'd at pleasure. Let F H be a Tangent to this Arc at F; thro' M let M G be drawn horizontal, and let this be continued, 'till it cuts F H in L; with the Center L, and Radius L G, a Portion G H of a Circle is describ'd, which determines the Curvature sought for. The Distance, between G and the Surface e f, is taken at pleasure, and this determines the Thickness of the Wood in that Place.

577. Plate XX. The Wheel d is mov'd by a Handle, applied at c; but in many Cases the Motion, and chiefly the Acceleration in the Beginning of the Motion, is not regular enough by this Method; I then make use of another, the Handle being remov'd.

578. To the Wheel d another greater Wheel p is join'd, which turns upon the same Axis with the first; to the Wheel p is fasten'd the Rope q, one of whose Ends sticks in the Bottom of the Groove, whereby the Wheel was encompass'd; to the other End of the Rope is hang'd the Weight r of six Pounds.

The Weight by its Descent communicates Motion to the Wheel, which is accelerated regularly; but the Acceleration is greater, or less, according to the different Circumstances, but chiefly depends upon the stretching of the Rope, which moves the Wheel d, and every thing else. But to hinder all Action of the Weight r upon the Machine, when its Motion is finish'd, there is a third Rope t t, whose End is fix'd in some elevated Place, answering to the Machine, whilst the other End is also tied to the Weight r; this Rope retains the Weight, when it comes to a determin'd Depth.

This is the general Explanation of the Machine, in which I have said nothing of what is put upon the Plates *i i*, *i i*; this is different in different Experiments, and the Explanation of this will be better understood, when I speak of the Experiments.

When

When a Body laid upon a Plane, is turn'd round a common Center together with that Plane, in equal Time, and describes a Circle; if the centripetal Force, whereby the Body, every Moment, is drawn or driven towards the Center, ceases to act, and the Plane continues its Motion with the same Celerity; the Body begins to recede from the Center, with respect to the Plane, along a Line which passes thro' the Center. 579.

The Body does indeed endeavour to recede along the Tangent *, but the Point of the Plane, to which it answers, is mov'd with the same Velocity with the Body, and the Motion along the Tangent of a quiescent Circle, is, in the first Moment, a Motion along the Radius of a Circle, mov'd with the same Velocity with the Body. * 561.

Experiment 1.

The Machine describ'd above must be made use of *; but in that State, in which I have exhibited it: that is, we suppose it to have every thing remov'd from it, that is represented as plac'd upon the Plates *i i*, *i i*, in this Table. 580. Plate XX. * 567.

Upon one of these Plates, that for Example which is join'd to the Pillar M N, whose Head h may be remov'd, the circular Board A is plac'd, of about two Foot Diameter, and above half an Inch thick; this Board is fasten'd by two Screws, going thro' the Holes *m m*, *Plate* XXIII. *Fig.* 2. And, that it may be more firm, it has its lower Part made hollow, to receive the Plate, as is shewn in E, where the middle Part of the Board is represented in an inverted Situation, and less diminish'd. Plate XXII. Fig. 1.

Upon this Board is put the Ball B, tied to the String, one of whose Ends is fasten'd to the Pin C, plac'd in the Center.

Now let the Machine be mov'd by the Handle *, in the Beginning the Ball is mov'd slowly, but is continually accelerated, 'till it performs its revolution, in the same time with the circular Board, in respect of which it is then at rest. In this Situation, the Ball is retain'd by the String only, tied to the Board; therefore it suffers no Impression upon the Plane, except that by which the String is stretch'd; that is, whose Direction passes thro' the Center of the Board: therefore, if the Body be left to itself, it cannot in the first Moment be mov'd on this Plane according to any other Direction. * 577.

A Body projected, and acted upon by a Force, tending towards a Center, is mov'd in a Plane, passing thro' the Line, according to which the Body is projected, and thro' the Center of the Forces. 581.

When

582. *When a Body is mov'd 'round a Center, if, whilst it is moving, it draws nearer to it, its Motion is accelerated; on the contrary, it is retarded, if it recedes from the Center.*

In the first Case the Motion, arising from the central Force, conspires, in part at least, with the Motion already impress'd on the Body; in the second Case, these Motions are contrary.

Experiment 2.

583. Plate XXII. Fig. 2, 3. Let the circular Board, mention'd in the foregoing Experiment be taken away, and in the same Place let the iron Ruler A B be applied, which is fasten'd by the Screws *c*, *c*, going thro' the Holes *m*, *m*, *Plate* III. *Fig.* 2. as was said of the Board, in the foregoing Experiment.

This Ruler is broader in the Middle, and at its Ends the small copper Pillars E, E, are put upon it.

Upon this Ruler is put the wooden Box F F, to whose Ends the copper Plates L, L, are join'd, thro' whose Holes *e*, *e*, the Ends of the Pillars E, E, which are cut into a Screw, penetrate, that the Box may be fasten'd. The Bottom of it is almost an Inch thick, and is made hollow beneath, and receives the Ruler, as appears at G. Two Screws D, D, going thro' the Bottom of the Box, go into the Ruler in *d*, *d*, that the Box may be still better fasten'd.

In the middle of it there is a transverse wooden Piece H, perforated in the middle, to receive the wooden Cylinder, or rather truncated Cone I, which does not reach to the Bottom of the Box, and stands above H one Inch at least. A Glass Tube, about a quarter of an Inch in Diameter, passes thro' this Cylinder, and sticks in it. The Apertures of this Tube are so narrow'd, its Ends being melted by the Flame of a Lamp, that there remains only a small Hole in the middle of each End, which answers to the Center of the Motion of the Box, when the Machine is mov'd.

A Ball tied to a Thread, is put into the Box; the Thread is put thro' the Tube mention'd, so as to go out of the upper Aperture, and to be reach'd by the Hand, which holds the End of the Thread, whilst the Box is turn'd round by the Motion of the Machine.

In this Motion the Ball is applied to the Side of the Box, and is carry'd about so, as to be mov'd with equal Celerity with the Box. Let the Thread be drawn, that the Ball may come nearer to the Center, it will immediately run to the opposite Side of the Box, because it is mov'd faster than the Box. Now if the Hand be mov'd mov'd towards it, the Ball recedes from the Center, and returns to the firſt Side of the Box; becauſe it is carried ſlower than it.

Setting aſide the Acceleration and Retardation, which we undertook to demonſtrate by this Experiment, the ſtriking againſt the Sides of the Box, which was mention'd, will alſo take place; becauſe, when the Ball is mov'd towards the Center, it deſcribes a ſmaller Circle; and therefore, if it keeps its Velocity, when it anſwers to a Point of the Box mov'd ſlower, it is mov'd faſter than the Box. But in this Caſe, when the Breadth of the Box is four Inches, if the Diſtance of the Ball from the Center be one Foot, it muſt be drawn towards the Center almoſt two Inches, that it may run to the oppoſite Side of the Box, after an whole Revolution: but I obſerve in the Experiment, that the ſtriking is made in a ſmaller time, alſo in a ſmaller Approach of the Ball to the Center. 584.

I determine in the firſt Scholium following, the Acceleration in the Apprcach of the Body towards the Center, and the Retardation, as it recedes from it.

A Body, which is retain'd in a Curve, by a Force tending towards a Center, deſcribes Areas, about this Center, proportional to the Times. 585.

Let there be a Body running along the Curve A B D E, in which it is retain'd by a central Force, tending towards C; if Lines be drawn at pleaſure, as A C, B C, D C, E C, the Area of the mixt Triangle A C B will be to the Area D C E, as the Time, in which A B is run thro' by the Body, is to the Time, in which D E is run thro'. Plate XXII. Fig. 4.

I demonſtrate alſo the Inverſe of this Propoſition, That *a Body, which is mov'd in any curve Line in a Plane, and deſcribes Areas about a Point, proportional to the Times, is turn'd away from the right Line, and acted upon by a Force tending to the ſame Point.* 586.

I muſt now ſpeak of comparing central Forces one with another, which that it may be done we muſt conſider, That the centripetal Force is a Preſſure, acting upon a Body. When a Body in every Point is bent from a Right Line, in every Moment the bending from a right Line is the immediate Effect of the Preſſure, ſo that what is demonſtrated about the Actions of Powers, acting upon Obſtacles left to themſelves, may be applied here *. 587. * 128.

The greater the Quantity of Matter is in a Body, with the more Difficulty, cæteris paribus, by reaſon of the greater Inertia, *is it drawn towards the Center, and has the greater centrifugal Force* *. 588. * 565.

If

589. If Fluids, whose equal Bulks weigh unequally, be included in a determin'd Space, so that the heavier cannot recede from the Center, except the lighter come to it, and are dispos'd in such manner, that the heavier go towards the Center by their Weight, in their Motion round the Center the lighter are carried towards it, and the heavier fly from the Center.

If a Solid be included with a Fluid in a determinate Space, it goes to the Center, if it is lighter than the Fluid; if heavier, it recedes from it. All which things arise from the greater centrifugal Force in the heavier Body.

Experiment 3.

590. Plate XXI. Fig. 4. The Iron Ruler, made use of in the foregoing Experiment, being remov'd, instead of it the wooden Ruler A B must be applied to the Machine, and fasten'd, to which are join'd two other Rulers D E, D E, plac'd obliquely, and made hollow, that to each may be applied the Glass Tube F, G, about a Foot long, and of about an Inch Diameter. Many such Tubes are wanted; I have represented them in the *Fig.* hermetically seal'd, but we make use of others also, having one of their Ends shut by a Glass Stopple, which is cover'd with a Bladder or Piece of Leather, that the Stopple may be kept in: four such are sufficient, the first, as F, contains Mercury with Water; the second, Oil of Tartar *per deliquium*, and Water; the third, as G, Water with a Piece of Cork; lastly, in the fourth, there is Water with a Ball of Lead. The two first are applied to the oblique Rulers, and the Machine is turn'd round, the Mercury in the first, and the Oil of Tartar in the second, immediately take place in the highest End of the Tube.

If we make use of the third and fourth Tube; in the third the Cork applies itself to the lower Surface of the Water, rais'd by the Motion of the Machine, whilst in the fourth the Ball of Lead passes thro' the Water, and joins itself to the Glass.

In all of them, if they are not fill'd, the lower Part of the Tube is empty, in the Experiment.

What I have hitherto deliver'd is general, but central Forces must be examin'd more distinctly, and accurately measur'd, by comparing them one with another.

591. These Forces differ not only in respect of the Quantity of Matter, but the Distance from the Center causes an Alteration also, as the Celerity likewise, with which the Body goes round; besides these, there is nothing discover'd in those Forces, which can cause a Difference

Difference between them; and in comparing theſe, thoſe Things only are to be conſider'd.

DEFINITION 4.

The periodical Time, is the Time in which a Body, revolving about a Center, performs one entire Revolution; that is, if it deſcribes a Curve, which returns into itſelf, the Time paſs'd between its receding from a Point, and its Return to the ſame Point: if the Curve does not return into itſelf, a Line, paſſing thro' the Center, is to be taken for that Point. 592.

The periodical Time depends upon the Celerity of the Body; and therefore, in comparing central Forces, this Time may be conſider'd inſtead of the Celerity. 593.

Things to be added, *to the Machine explain'd in* N° 567. *for comparing central Forces with one another.*

Thoſe Things, which are to be added to the Machine, mention'd, are put upon the Plates *i i*, *i i*, *Plate* XX. and theſe I here exhibit ſeparately; but the Things applied to each of the Plates are alike. 594. Plate XXIII. Fig. 2. and Plate XXI. Fig. 1.

I repreſent one of them in I I, upon this are plac'd the ſmall copper Pillars F, G; whoſe lower Ends go thro' the Holes *m*, *m*, whilſt the Baſes are applied to the Plate, to which they are join'd faſt, by the Screws *f*, *f*.

Theſe Pillars are join'd together by the Plate Q R, which they ſupport, and which the Cylinders, or Screws, join'd to the upper Part of the Pillars, go thro'.

To this Plate another ſmaller Plate L is join'd underneath; this is moveable, and may be remov'd out of its Place, as appears more plainly in *Fig*. 3. The Plate Q R has a Hole in the middle of it, and there is another Hole in the Plate L anſwering to it; but when a Thread paſſes thro' both Holes, the Plate L is moveable, whilſt the Thread remains, by reaſon of a Slit in the Side of it *a*, (*Fig*. 3.)

To the Plate mention'd Q R there is join'd an iron Ruler S T; whoſe Part S *b* is broadeſt, and its Breadth is three quarters of an Inch, whilſt the Breadth of the other Part *b* T ſcarce exceeds a quarter of an Inch; but the Thickneſs of the Ruler is every where the ſame, and is equal to half an Inch. 595. Plate XXIII. Fig. 2. 4.

The upper Ends of the Pillars G and F, (*Fig.* 2.) go thro' this Ruler at *c* and *e*, riſing above Q R, and the Ruler is faſten'd by the Screws *d*, *d*.

Between the Holes *c*, *e*, there is a greater Aperture, which contains the little Wheel M, moveable about an Axis; and ſo plac'd, that a Thread, paſſing thro' the Holes of the Plates Q R and L, may touch the Wheel in ſuch manner, as to be bent towards *e*, when it is put round the Wheel.

596. This Thread is faſten'd to the Cylinder H, to which is join'd at its lower End, the ſmall Plate, or Nut *h*, which the Cylinder goes thro' in ſuch manner, that its lower End *i* paſſes thro' the Plate *h*, and remains below it.

This Cylinder is put upon the Plate I I, and ſtands upon the Plate *h*, whilſt its lower End *i* goes into the Hole *o*.

The upper Part *l* of the Cylinder is broadeſt, and it is not above a tenth Part of an Inch diſtant from the Plate L, that it may be rais'd but a little, leſt the Thread ſhould be ſeparated from the little Wheel, the upper Part of which is alſo retain'd, as I will now ſhew. The End P of the Pillar F, which goes thro' the Hole *e*, ſtands almoſt half an Inch above the applied Plate *d*; this Part is open by an Inciſion in ſuch manner, that the Thread, which is extended from M towards T, freely paſſes thro' this Inciſion; and, that it mayn't ſlip out, the Screw *p* paſſes tranſverſly thro' the upper Part of the Inciſion. The Knot N keeps the Thread extended, for it is applied to the Inciſion at P, when the Plate *h* is put upon the Plate I I, and ſo hinders the Thread from being ſeparated from the little Wheel M; but it does not hinder that ſmall Aſcent of the Cylinder H, which I mention'd.

597. This Cylinder H, with the Plate *h* added to it, weighs exactly two Ounces; but this Weight may be increas'd, and any how varied, by help of cylindric leaden Weights, of one, two, four, eight, ſixteen Ounces, (*Fig.* 5.) Theſe have a Hole in their Axis, and the Cylinder H exactly fits the Cavity of each of them, when it is put into it.

When a Weight is to be added to the Cylinder, the Plate L is remov'd out of its Place, then the Cylinder H may be rais'd, and taken out of its Place, that any Weight, when the Plate *h* is taken away, and many Weights if requir'd, may be added to it; then *h* is join'd again, and the Cylinder is put into its former Situation, and the Situation of the Plate L is reſtor'd.

Theſe

Theſe Things being thus order'd, if the Thread, often mention'd, be drawn towards T, the Cylinder H, with the Weight faſten'd to it, is rais'd, but it can be rais'd only a little. But in the Experiments we ought to determine exactly the Moment of its Riſe, which is perform'd by the following Method.

The copper Plate *q r* is applied to the Ruler S T, between the 598.
End S and Hole *c*, and faſten'd with Screws, upon which Plate two copper Supports ſtand, the firſt of which ſuſtains a ſmall Bell O, like thoſe that are uſed in Watches; the ſecond *r g* ſupports a Hammer *v*, which ſtrikes the Bell; the Tail of the Hammer turns upon a Pin at *g*, made of ſmall Braſs Wire. The Hammer falls by its own Weight, and ſtrikes the Bell, but is hinder'd from remaining upon it by the Steel Spring *s*.

The Arm *z t* is join'd to the Support *r g*; this ſuſtains a ſmall copper Lever b d, which is moveable about *t*, and which retains the Tail of the Hammer at b, which is then rais'd, but is let looſe by the ſmalleſt Action applied at *d*. But Care muſt be taken, that the Hammer be not let go by means of the Preſſure, acting at *v*, and tending downwards; then it will keep its Situation, even when the Machine turns very ſwiftly. A braſs Wire *x y*, which is ſmall, elaſtick, and bent, is join'd to the Supporter *r g*, and preſſes the Lever b d gently at *y*, when it retains the Hammer.

A ſmall Thread is tied to the Thread of the Cylinder H, at the Knot N; this paſſes thro' a Hole in the End d of the Lever, and thro' a Hole in the Head of the Pin alſo, about which the Lever turns at *t*, and is brought to a ſmall copper Wedge a, which it is ſo wound round, that the Length of the Thread may be alter'd; but this muſt by Tryals be ſo determin'd, that the Cylinder H may be rais'd a little, before the Hammer is let looſe; which nevertheleſs ought to be let go, if the Cylinder riſes a little higher, and indeed before its Surface *l* comes to the Plate L. See *Plate* XX.

The Thread of the Cylinder H is join'd to a greater Cylinder V, 599.
which the iron Ruler goes thro' *, upon which it is moveable be- * 595.
tween *b* and T; for this reaſon it is requiſite that the upper and ſide Surfaces of the Ruler be very regular, and ſmooth, the upper eſpecially muſt be well poliſh'd.

This Cylinder V is of Copper, and hollow, its Baſes are join'd to it by Screws, as is ſhewn in *Fig.* 6. in which the hind Baſe is repreſented by itſelf, the true Magnitude being kept.

The Aperture thro' which the Ruler paſſes is f, whoſe Height is ſuch, that the Ruler may paſs thro' it freely, which muſt be applied alſo to the Breadth of the Aperture, but ſo that it ſcarce exceeds the Breadth of the Ruler in the middle.

A Steel Ruler g, turning upon an Axis freely, which, together with another like it in the oppoſite Baſe, hinders the Friction when the Cylinder moves along the Ruler, anſwers to the upper Part of the Aperture f. The Roller is ſmaller in the middle, that it may touch the Ruler leſs, when applied to it.

600. There is in the Baſe above the Roller a Hole n, thro' which the Thread of the Cylinder H is put, as I ſhall preſently mention. Theſe Things are the ſame in both Baſes, which differ in this only; to the poſterior Baſe there is join'd a ſmall copper Plate i, but by one Pin m only, in ſuch manner, that the Thread paſſing thro' n, may be eaſily inſerted between the Surface of the Baſe and this Plate; and it is retain'd by a cloſer Application to the Plate, by help of the Screw h, which, paſſing thro' the Plate i, goes into the Baſe.

Fig. 4. The Cylinder V is mov'd along the Ruler very eaſily and freely, becauſe it is ſupported by the Rollers; but that, which we call'd the poſterior Baſe, reſpects the End T, to which the Head e having a Screw in it is join'd, leſt the Cylinder ſhould fall, or when the Thread is looſen'd, to which it is join'd, be caſt off by the Motion of the Machine.

This Thread is the ſame as we ſaw before, to which the Cylinder H is join'd; but it is put thro' both the Holes, as n, *(Fig. 6.)* of the Baſes, and is retain'd by the Plate i, as was ſaid before; whereby a Body is hinder'd from receding from the Center of its Revolution beyond a certain Diſtance.

601. This Diſtance is determin'd by Diviſions mark'd on the Line *b* p, which makes one of the lower Angles of the Ruler; but the Diviſions, when they anſwer to the anterior Face of the Cylinder, ſhew the Diſtances of the middle Point; that is, of the Center of Gravity, of the Cylinder from the Center of Revolution of the Ruler. In my Machine the Diſtance between two Diviſions is half an Inch, and the greateſt Diſtance is fourteen Inches.

602. The Weight of the Cylinder V is three quarters of a Pound; the Weight is communicated to it by two Pieces of Lead, which are join'd to the anterior Surface of the Cylinder, in the lower Part of it, a little towards the Sides.

The Weight of the Cylinder is increas'd by Rings, as *z*; theſe contain the Cylinder exactly: and when one of theſe is put round the Cylinder, the Screw q is taken away, and the Ring is turn'd ſo, that the Hole r may agree with the Hole of the Screw, whereby the Ring is then faſten'd.

There are three ſuch Rings, the ſmalleſt weighs a quarter of a Pound, the ſecond twice as much, the third three times as much. Upon the Cylinder V is inſcrib'd the N° 3. on the firſt Ring is inſcrib'd 4; the ſecond 5; the third 6; theſe Numbers expreſs the Weight of the Cylinder, whether it be uſed alone, or with a Ring join'd to it.

That there may be an Æquilibrium between the Parts of the Machine, when it is mov'd; there is join'd to the End S of the 603.
Ruler, by help of a Screw join'd to the Lever, a copper Tail X, which itſelf has a Screw; the Weight of this is ſuch, that the outward Screw Y being added, the Ruler may be in Æquilibrium about the Center of Motion. When the Cylinder V is applied to the Ruler, round Pieces of Lead, (as *z*, *z*, *z*, *Plate* XX.) are join'd to X, that the Æquilibrium about the ſame Center may be reſtor'd. In my Machine theſe are ſuch, that eight are requir'd, when the Cylinder is put at the greateſt Diſtance from the Center, and the heavieſt Ring is put round it, ſo that its Action is the greateſt of all *. * 189.

But it muſt be obſerv'd, that a ſenſible Effect does not follow in many Experiments from a want of this Æquilibrium; yet ſince, if it is entirely wanting in the more violent Motion of the Machine, it will thence follow, that there will be a tremulous Motion of the Pillars I L, M N, (*Plate* XX.) it appears that this is not entirely to be neglected; but it is ſufficient, in particular Caſes, to determine the number of round leaden Pieces, to be join'd to the iron Ruler, by a leſs perfect Computation; but I make uſe of this. I 604.
multiply the Weight of the Cylinder by half its Diſtance from the Center, and the firſt Character of the Product expreſſes the Number ſought for, which is increas'd by Unity, if the following Character exceeds five. For Example, let the Weight of the Cylinder be 5, its Diſtance 14, whoſe half is 7, the Product will be 35; three Pieces are to be applied. If the Weight were 4, the Diſtance 18, the Product would be 36, and the Number of Pieces would be four.

When the periodical Times are equal, and the Diſtances from the 605.
Center equal, the central Forces are as the Quantities of Matter in the Bodies revolving *. For in equal Times, Bodies are mov'd by * 132. 587.
central Forces, after the ſame manner.

Ex-

Experiment 4.

606. Plate XX. To the two Plates *i i*, *i i*, we join thoſe Things which we explain'd in N° 594. and the following. To each of the Rulers *s t*, let its own Cylinder *v* be applied alſo, the Thread being put thro' it, which is join'd to the Cylinder *b*; this is done thus; a Needle is requir'd, whoſe Length exceeds the Length of the Cylinder *v*, and thro' the Eye of it a Thread, that is ſmall, and doubled is put; the Needle is put into the Cylinder thro' the Hole in its anterior Baſe in ſuch manner, that the Point of it goes out thro' the Hole of the poſterior Baſe; then we put the End of the Thread, which we would have paſs thro' the Cylinder, into a little Loop in the Thread of the Needle; and by drawing the Point of the Needle, the Threads follow, and the Needle with its Thread is remov'd.

* 602. Round one of the Cylinders is put a Ring*; for Example, that which is mark'd 4; then the Weights of the Cylinders will be as 3 and 4: the Weights join'd to theſe are requir'd to be in the ſame Proportion; wherefore to one Cylinder *b* four Ounces are added, * 597. to the other ſix*.

The Cylinders *v*, *v*, are to be plac'd at equal Diſtances from the Center; let this Diſtance be, what is determin'd at pleaſure 24: to the Ruler *r t* a wooden Obſtacle A is join'd, (*Plate* XXII. *Fig.* 5.) by putting the Ruler into it; this Piece is plac'd in ſuch manner, that its Surface *b*, which is the uppermoſt in the Figure, is turn'd from the Center of Motion, and exactly agrees with the Diviſion * 601. 24*; the Piece is faſten'd by the copper Screw *c*, the copper Plate *d*, which is elaſtick, and a little bent, hindering the Screw from damaging the iron Ruler.

The anterior Surface of the Cylinder *v* is applied to the wooden * 601. Piece, then the Diſtance of its Center of Gravity is 24*: the Thread is ſtretch'd, which paſſes thro' the Cylinder, as much as * 600. may be, without raiſing the Weight *b*, and faſten'd*, and the Piece is taken away. As the Weight *b* determines this ſtretching of the Thread, therefore I ſaid that this Weight muſt firſt be applied.

Now the Bodies plac'd at equal Diſtances from their Centers of Motion, and whoſe Quantities of Matter are as 3 to 4, can't recede from the Center ever ſo little, unleſs the Weights are rais'd, which are in the ſame Ratio of three to four.

Theſe Bodies will alſo perform their Revolutions in equal Times, if the Head T be rais'd as much as it can, three Rings being put upon

upon the ſolid Piece S *; for then the ſuperior Wheels at *d*, and * 574. 570.
e, which are equal, anſwer to the Pulleys i and m.

Motion is communicated to the Machine by the Weight *r* *; * 578.
then each of the Bodies *v*, and *v*, is mov'd round the Center, and ſtretches the Thread by the centrifugal Force, and is retain'd by the Weight *h*; but, by the Deſcent of the Weight r, the Motion is ſo accelerated, that the Cylinders *v*, and *v*, raiſe the Weights faſten'd to them, and indeed exactly in the ſame Moment, as appears, by
the Hammers of the Bells being lifted up *: for theſe are looſen'd * 598.
in the ſame Moment, ſo that one Stroke only is perceiv'd; which ſhews that the Ratio of the Forces is well determin'd.

In the turning of the Screw l o, care muſt be taken, that the Acceleration be not too ſudden; for unleſs the String is ſtretch'd, which communicates Motion to the Axes *a b*, *a b*, their Accelerations will not agree; but the ſtretching of the String muſt be ſo order'd, that the Hammers may be let looſe; that is, that the Weights *h*, *h*, may be rais'd, before the Weight r comes to its greateſt Depth.

When the Quantities of Matter in revolving Bodies are equal, and 607.
their periodical Times equal, the central Forces are as the Diſtances
from the Center *. * 587. 133.

Experiment 5.

This Experiment differs from the foregoing * in a few Circum- 608.
ſtances only. The two Cylinders *v*, *v*, either without Rings, or Plate XX. * 606.
with equal Rings, are made uſe of. Theſe are plac'd at unequal Diſtances; for Example, one is put at the ſixteenth Diviſion, the other at the twenty-fourth. Now Weights muſt be rais'd, which are in the ſame Proportion of 16 to 24; therefore to the firſt Cy-
linder *h* two Ounces are join'd, to the other four *, and the * 597.
Weights are as four to ſix; that is, as 16 to 24: other Things remain as before, and the Experiment is made in the ſame manner, as the foregoing. But we have equal Bodies, turning round in equal Times, whoſe Forces, which are equal to the Weights rais'd, are to one another as their Diſtances from the Center.

When the periodical Times are equal, but the Diſtances from the 609.
Center, and the Quantities of Matter in the Bodies revolving differ, the central Forces are in a Ratio made up, of the Quantities of Matter, and the Diſtances; which follows from the two laſt Propoſitions. To determine this compound Ratio, the Quantity of Matter in

in each Body muſt be multiplied by its Diſtance from the Center,
* 23 El. VI. and the Products have to one another the Ratio ſought for *.

EXPERIMENT 6.

610. Plate XX. * 608. Thoſe Things remaining, which were explain'd in the fifth Experiment *, to the Cylinder *v*, whoſe Diſtance from the Center is 24, let there be added a Ring, whereby its Weight may become 5; the periodical Times remain equal, and the central Forces will be according to this Proportion, as 3×16 is to 5×24; that is, as 2 is to 5; therefore four Ounces remaining, which ſhould be rais'd by the Cylinder *v*, whoſe Weight is three; to the ſix Ounces, which are join'd to the other Body, let four be added, that there may be ten Ounces; and by the Motion of the Machine both Weights will be rais'd in the ſame Moment, as the Bells will ſhew again.

The Differences of central Forces, ariſing from the Differences of their Diſtances from the Center, and Quantities of Matter, may
611. mutually compenſate one another, and *the Quantities of Matter, in Bodies driven round, being put in an inverſe Ratio of their Diſtances from the Center, the central Forces will be equal*; as much as one Force is greater than the other, in reſpect of the Quantity of Matter, ſo much this exceeds that in its Diſtance.

EXPERIMENT 7.

612. Plate XX. * 610. The Bodies carried round remaining, which we made uſe of in the ſixth Experiment *, which are as three to five; let this be put at the diſtance fifteen, that at the diſtance twenty-five: the periodical Times remain equal, and the Weights at *h*, and *h*, will not riſe at the ſame time, unleſs they are equal.

613. We have a Caſe of this Propoſition, *when two Bodies, join'd by a Thread, turn round a common Center of Gravity.* For the Diſtances from this Center are in an inverſe Ratio of the Weights of the Bodies *; and therefore the central Forces are equal †. With the
* 202. 192. † 611. Force, with which one Body endeavours to recede from the Center, the other is drawn towards it; and, by reaſon of the Equality of their Forces, *they mutually retain one another, and continue their Motion*; if they revolve about any other Point, they cannot continue their Motion, and the Body, whoſe centrifugal Force overpowers, recedes from the Center, and carries the other Body along with it.

Ex-

EXPERIMENT 8.

Every thing is to be taken away, that is put upon the Plates *i i*, *i i*; which is done at once in each Plate by only loosening the Screws *f f*, (Plate XXIII. Fig. 2.). The Machine is afterwards to be put into that Situation, which we mention'd in the second Experiment *, before the wooden Box was put upon it. 614. Plate XX. * 583.

Then the Iron Ruler AB is applied to the Machine; upon this is put, and sustain'd by the Pillars E, E, another Iron Ruler H I, which is fasten'd by Screws, as was said of the wooden Box *. Plate XXII. Fig. 2. 6. * 583.

This Ruler is every where of the same Thickness, and Breadth, and its Surfaces are regular, and polish'd. This goes thro' two Copper Cylinders F, G, which, though they don't stand upon Rollers, are easily mov'd along the Ruler, especially if this is rubb'd over with Oil; which would hinder the Motion, if there were Rollers, as in the other Cylinders, which we used in the foregoing Experiments *. A Thread is put through both Cylinders †, and fasten'd so, that the Knot N, which is in the Thread, exactly answers to the common Center of Gravity of the Bodies F, G. The Thread is stretch'd, and the Knot plac'd, so as to answer to the Center of Motion, mark'd on the Ruler H I: the Machine is mov'd by the Handle *, and the Bodies keep the Place which they occupy in the Ruler H I. * 606. 608. 610. 612. † 606. * 577.

If the Knot N answers to some other Point, both Bodies are mov'd along the Ruler, often with a violent Motion.

But that the Machine mayn't be damag'd in this last Case, we make use of wooden Obstacles as L (Fig. 7.): we place one between H and F, another between I and G. These Obstacles are to be plac'd before the Ruler H I is fasten'd in its place. But when an Obstacle is put upon the Ruler A B, we fasten it in any place by the Wedge M, passing through the Hole *o* beneath the Ruler A B. The Obstacle has its upper Part, where the Body strikes it, cover'd with a thick Piece of Leather; the anterior Surfaces also, (which are turn'd towards each other) of the Cylinders F and G, are in the same manner cover'd with Leather. 615.

When the Quantities of Matter in Bodies revolving, and their Distances from the Center, are equal, the central Forces are in an inverse Ratio of the Squares of the periodical Times, that is, directly as the Squares of the Revolutions, perform'd in the same Time. 616.

Experiment 9.

617. Plate XX. * 606. 608. 610. 612. The Machine is to be reſtor'd to that Condition, in which it was in the Experiments * in which the central Forces were compar'd with one another, but the periodical Times muſt be varied.

When the String goes round the upper Wheels at *d* and *e*, the
* 570. 606. Times are equal, as we have ſeen *.

† 574. If, the Head T being let down †, the following Wheels are made uſe of, the Times of the Revolutions are, as four to five; if again the following, they will be as two to three; laſtly, if the loweſt as
* 570. one to two *.

We ſuppoſe the Head T to be plac'd as it is exhibited in the Figure, that is, that the periodical Times are to one another, as two to three; the Time is leſs at the Part *e*.

At this ſame Part we join ſeven Ounces to the Cylinder *h*, that the Weight may be nine Ounces; at the other Part we muſt join only two Ounces to *h*, that the whole Weight may be four Ounces.

Now if equal Bodies, at equal Diſtances, ſhall thus turn round,
* 606. there will be heard only one Stroke of the Bells as in the fourth *, and ſome other Experiments; whence it will appear that the Forces are, as nine to four, that is, inverſly as the Squares of the periodical Times, which are as two to three.

618. *Howſoever the central Forces differ from one another*, from what has been ſaid they may be compar'd with one another; for *they are in a Ratio compounded, of the Ratio of the Quantities of Matter in the Bodies revolving, and the Ratio of their Diſtances from the Center, as alſo the inverſe Ratio of the Squares of the periodical Times.* By multiplying the Quantity of Matter in each Body by its Diſtance from the Center, and dividing the Product by the Square of the periodical Time, the Quotients of the Diviſions will be in the ſaid compound Ratio, that is, as the central Forces.

Experiment 10.

619. Let the Bodies revolving be as three to five, and applied the firſt to the eighteenth Diviſion, the ſecond to the twenty ſeventh; let
* 617. the periodical Times be moreover as four to five *; Weights of five and eight Ounces will be rais'd at the ſame time.

I multiply 3 by 18, and divide the Product 54 by 16, and I have $3\frac{3}{8}$: I multiply 5 alſo by 27, and divide the Product 135 by 25, the he Quotient is $5\frac{2}{5}$. Therefore the Forces are as $3\frac{3}{8}$ to $5\frac{2}{5}$, that is, as 5 is to 8, as in the Experiment.

When the Quantities of Matter are equal, the Diſtances are divided by the Squares of the periodical Times, that the Ratio may be determin'd, which obtains between the central Forces. 620.

In this Caſe, *if the Squares of the periodical Times are to one another as the Cubes of the Diſtances*, the Quotients of the Diviſions will be *in an inverſe Ratio of the Squares of the Diſtances*; and in this Ratio *the central Forces will be* alſo. 621.

EXPERIMENT II.

Let the Bodies revolving be equal; the Diſtances from the Center $14\frac{1}{2}$ and 19; the periodical Times as 2 to 3. 622. Plate XX.

The Cubes of the Diſtances are $3048\frac{5}{8}$ and 6859; the Squares of the periodical Times are 4 and 9, which are as $3048\frac{4}{9}$ to 6859, nearly as the Cubes of the Diſtances; we don't take notice of a ſmall, and inſenſible Difference.

The Squares of the Diſtances are $210\frac{1}{4}$ and 361, that is, nearly as 7 to 12, inverſly as 12 to 7; Weights alſo of 12 and 7 Ounces, riſe at the ſame Time. Here alſo we neglect a ſmall Fraction.

If we determine the Forces mathematically *, they are as 261 to 152; the Squares of the Diſtances are inverſly as 261 to $152\frac{13}{144}$. This ſmall Difference proceeds from the ſmall Difference, which we ſaw there was between the Ratios of the Cubes of the Diſtances, and the Squares of the periodical Times. * 618.

If the Bodies are unequal, but central Forces act upon them, of the ſame Nature with Gravity, it matters not what are the Maſſes of the Bodies, or how they are mov'd, they are turn'd towards the Center, in equal Times, thro' Spaces, which are as the Forces themſelves *, *and the laſt Propoſition obtains alſo in unequal Bodies.* 623. * 133. 155. 587.

A Body may deſcribe different Curves, by the central Force 624.
Geometricians call an oval Line an Ellipſis, which is thus deſcrib'd. Let A *a* be a right Line; C its middle Point; F, *f*, Points equally diſtant from C; F G *f* a Thread, whoſe Ends are faſten'd at F and *f*, which is equal to the Line A *a*. The Thread being ſtretch'd by the Pin G, by the Motion of which upon a Plane, in which A *a* is, an Ellipſis is deſcrib'd. The Points F *f* are call'd the Foci; C the Center; A *a* its greater Axis; its leſs Axis paſſes thro' the Center, at right Angles to the greater, and is terminated both Ways by the Curve, as B *b*. 625. Plate XXIV. Fig. 1.

Let us ſuppoſe, as in the laſt Propoſition, *a Force, which acts upon Bodies in Motion as upon thoſe at reſt, which is equal at equal* 626.

Distances from the Center, and at unequal Distances decreases in an inverse Ratio of the Squares of the Distances from this Point, the Body will describe an Ellipse, one of whose Foci coincides with the Center of the Forces, so, that in every Revolution the Body comes to it once, and again recedes from it. In the Recess the Celerity
* 582. of the Body is diminish'd *, and indeed in such manner, that the central Force, tho' it be diminish'd, bends the Way of the Body enough, to make it draw nearer the Center: but by the Approach of the Body its Velocity is increas'd, the Inflection of the Way is diminish'd and it recedes from the Center again.

627. The Circle belongs to this kind of Curves, the Foci coinciding with the Center. And *a Body being given which*, as we said, *describes an Ellipse, another, with the same Force*, will be retain'd *in the Circle, about the same Center*, if it be projected with a just Velocity perpendicularly to a Line, passing thro' the Center. *If the Diameter* of the Circle *be equal to the greater Axis of the Ellipse*, the Body *must be mov'd with that Velocity which it has in an Ellipse, at that time, when it passes through one or the other Extremity of the less Axis; and both these Bodies will perform their Revolutions in equal Times.*

628. A Body may be projected with such a Celerity, that, in its Recess from the Center, the Force, which is diminish'd by the Increase of the Distance, can't bend the Way of it enough to make the Body return; in this Case the Body runs through some other Curve of the conic Sections, a Parabola, or Hyperbola.

629. *If the central Force decreases in any other Proportion in its Recess from the Center, the Body cannot describe a Line returning into itself, and little different from a Circle.*

630. But *if the Force decreases in a Proportion little differing from this, or the Curve does not differ much from a Circle, the Curve describ'd by the Body, may be referr'd to a moveable Ellipse*; namely, whose Axis is mov'd with an angular Motion in the Plane, in which the Body revolves, the Focus remaining in the Center of the
631. Forces. But *the Motion of the Axis is directed to the same Part with the Motion of the Body, if the Force decreases faster, the Distance being increas'd, than according to the inverse Ratio of* the Square of the Distance: but *if the Force decreases in its Recess from the Center more slowly*, that is less, *the Motion of the Ellipse is directed the contrary way.*

632. *The Body also describes an Ellipse, if the Central Force, in the Recess from the Center, increases, and is every where in the Ratio of the*

the Distance from the Center, which in this Case coincides with the Center of the Ellipse.

EXPERIMENT 12.

Let a leaden Ball be suspended by a long Thread; if it be drawn from the Point where it is at rest, it is always carried towards it by its Gravity; and every where with equal Force, if the Distance is equal. The Ball in its Motion from the said Point describes a Circle, which soever way it is drawn from it: if the Portions of the Circle are not very great, they coincide with a Cycloid *, and the Force with which the Ball, in whatsoever Point it is turn'd, tends towards the lowest Point, is as its Distance from this Point *; therefore that Force increases in the Ratio of the Distance. 634.

* 416.

* 414.

Let the Ball be drawn from the lowest Point, and projected obliquely, it will describe an oval Figure round this Point, which, when the Ball does not run through a great Space, scarce differs at all from an Ellipse, by reason of the Proportion of the Forces, and because in this Case the Ball is as to Sense mov'd in the same Plane.

The Center of the Ellipse is the Point where the Ball, when it is not projected, is at rest; in every Revolution the Ball comes to it twice, and recedes from it twice. If the Ball be suspended above a Table in such manner, that it almost touches the Table, when at rest, and the Point, which it then answers to, be mark'd on the Table, the Experiment becomes much more sensible; by following the Ball its way may be mark'd with Chalk upon the Table.

If the Force increases in another Ratio, the Curve does not return into itself; but it may often be referr'd to an Ellipse moveable on a Plane. 635.

EXPERIMENT 13.

The same Things being given, as in the foregoing Experiment, let the Ball be projected in such manner, that it may run out to a greater Distance; it will describe a Curve which may be referr'd to a moveable Oval; it will indeed in every Revolution come to the Center twice, and recede from it twice; but the Situation of the Points, in which it is at the least or greatest Distance, will be chang'd in each Revolution, and these Points will always be carried towards the same Part, and their Motion will conspire with the Motion of the Ball. 636.

From this last Proposition, if we attend to Number 629, it follows, That *by no central Force, acting equally at equal Distances,* 637.

a

a Curve can be describ'd that returns into itself, which is little different from a Circle, and excentric; that is, whose Center does not coincide with the Center of the Forces, *except an Ellipse, in one of whose Foci is the Center of the Forces; and that the central Force, in this Case, follows the inverse Ratio of the Square of the Distance.*

638. But it is manifest, *That a Circle, whose Center coincides with the Center of the Forces, may be describ'd by a Force increasing or decreasing, in any Ratio,* if it only acts equally at equal Distances.

SCHOLIUM I.

General Considerations of Central Forces.

639. Plate XXIV. Fig. 2. LET us conceive a Force to be given, whereby a Body, wherever it is, is driven towards the Center D; it is no matter in what manner this Force is varied in different Points: let us conceive this Force not to be continual, but to act upon the Body by Strokes, and the Moments of Time between the Strokes to be equal. Let us also suppose the Body projected along A B, to run thro' that Line in a certain Moment; the Body in the following Moment would continue in motion along B L, equal to A B, unless by receiving a Blow when at B, it was driven to C; let us suppose the Celerity arising from this Stroke in the Body already agitated, to be such, that by it the Body may, in the Interval of Time between two Strokes, go thro' the Line L D; if L D be parallel to B C, the Body acted upon by two Motions, goes thro' B D *, and is at D the Moment that by the next Stroke it is again driven towards the Center. If this Stroke was not given, in the following Moment it would run thro' D E, supposing D E and B D to be equal; but in the same Time that it is carried towards the Center, that is, it is driven along D C; if along that Direction it runs thro' a Line equal to the Line E F, in the Time that it would run thro' D E, the Body is carried by a compound Motion along D F, supposing E F and D C to be parallel. In the same manner we demonstrate, that in the following Moment the Body runs thro' F H, if G H be equal to the Space to be run thro' in that Moment from the Stroke towards C, supposing F G and D F equal, as also G H and F C parallel.

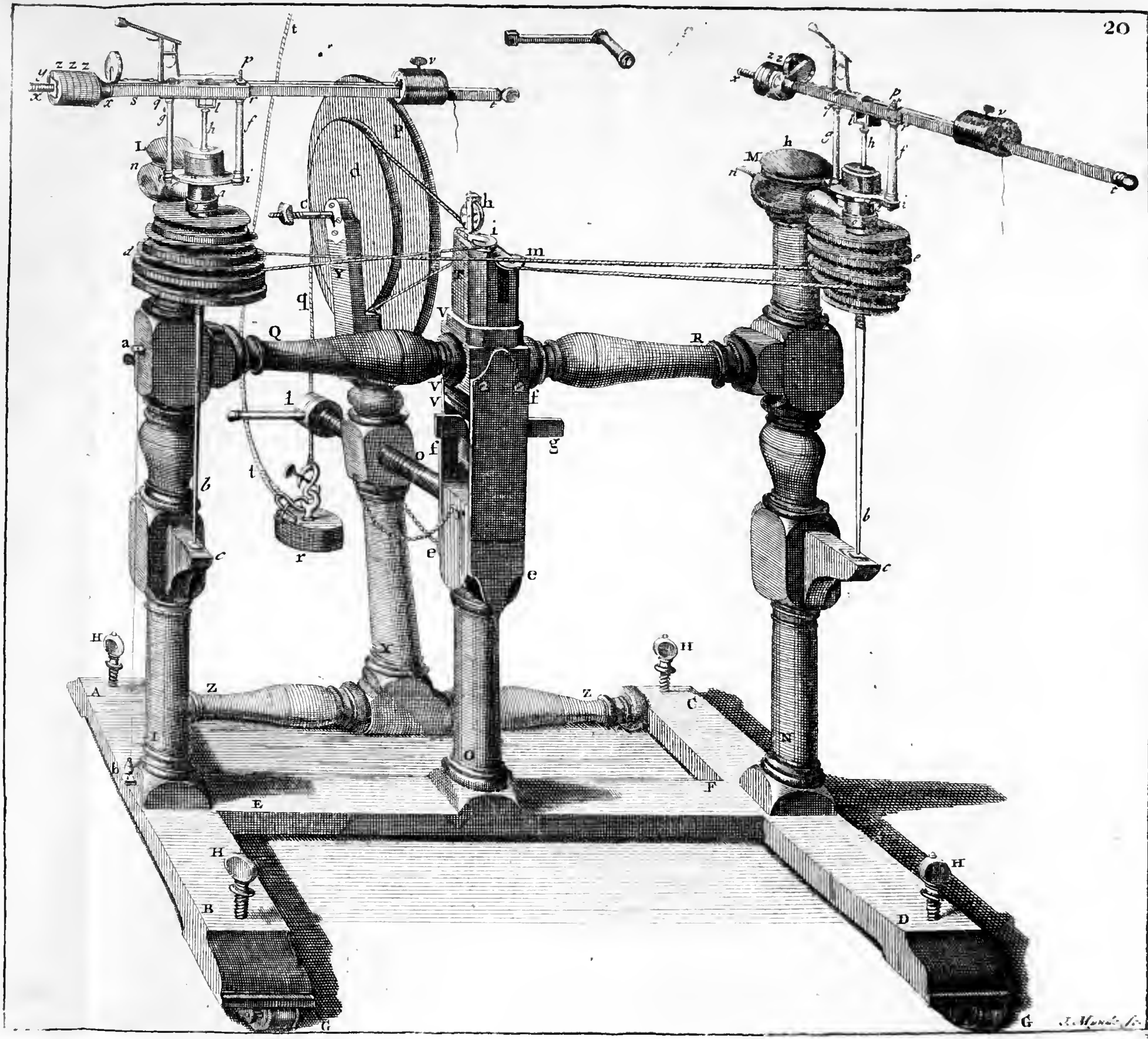
* 360.

The Triangles A B C, B L C, have equal Bases A B, B L, in the same Line, and a common Vertex C; therefore they are equal *. The Triangles B L C, B D C, have the common Base B C, and stand between the Parallels B C, L D, therefore they are equal †. For that reason also A B C, and B D C are equal. We demonstrate in the same manner the Triangles B D C, D F C, to be equal; and generally any Triangles to be equal, as A B C, B D C, D F C, F H C, whose Bases are run thro' in equal Moments, by the Body projected. These in their Access to the Center become larger, and the Body is mov'd swifter, as we said in N° 582.

* 38 El. I.

† 37 El. I.

It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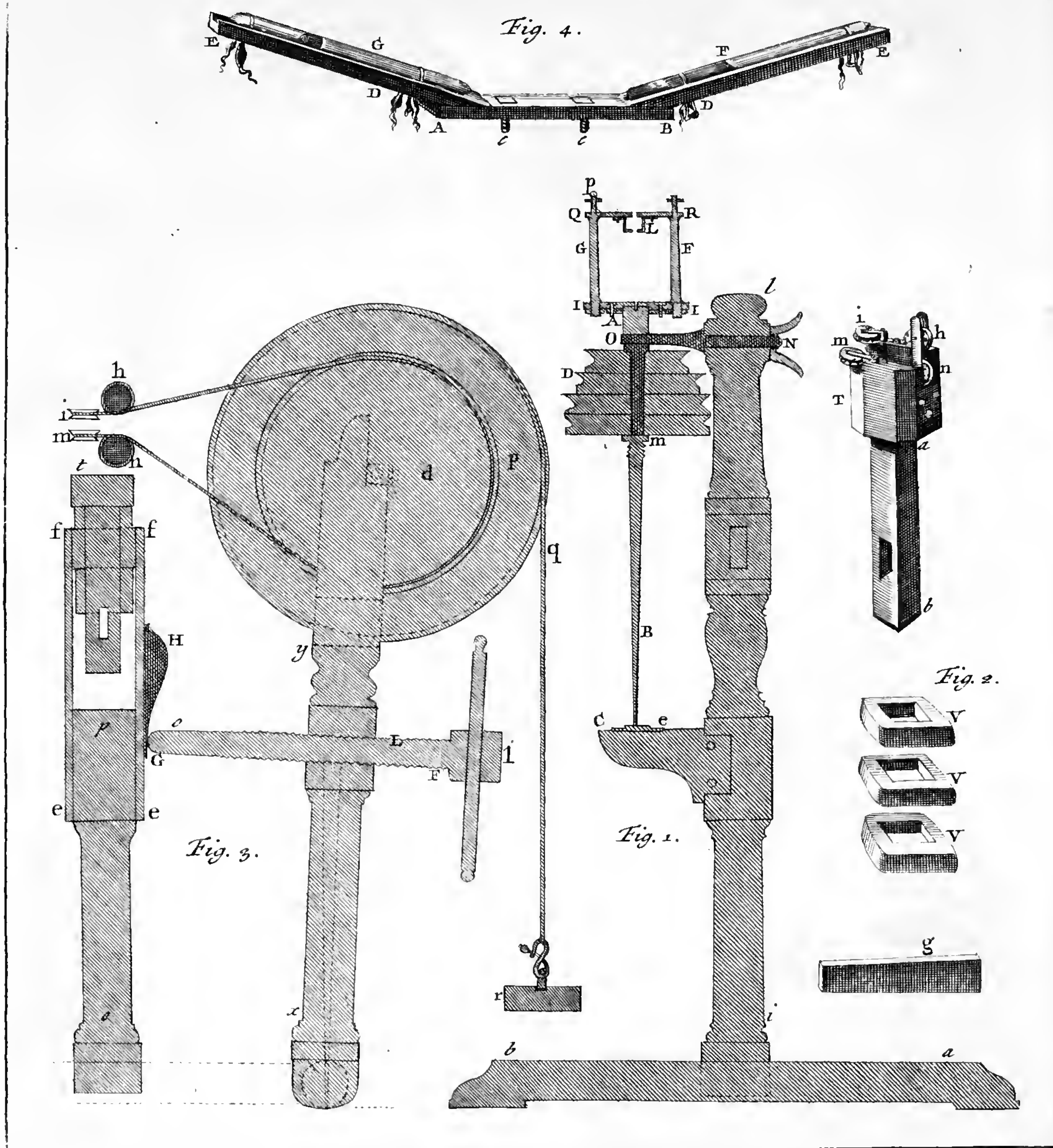

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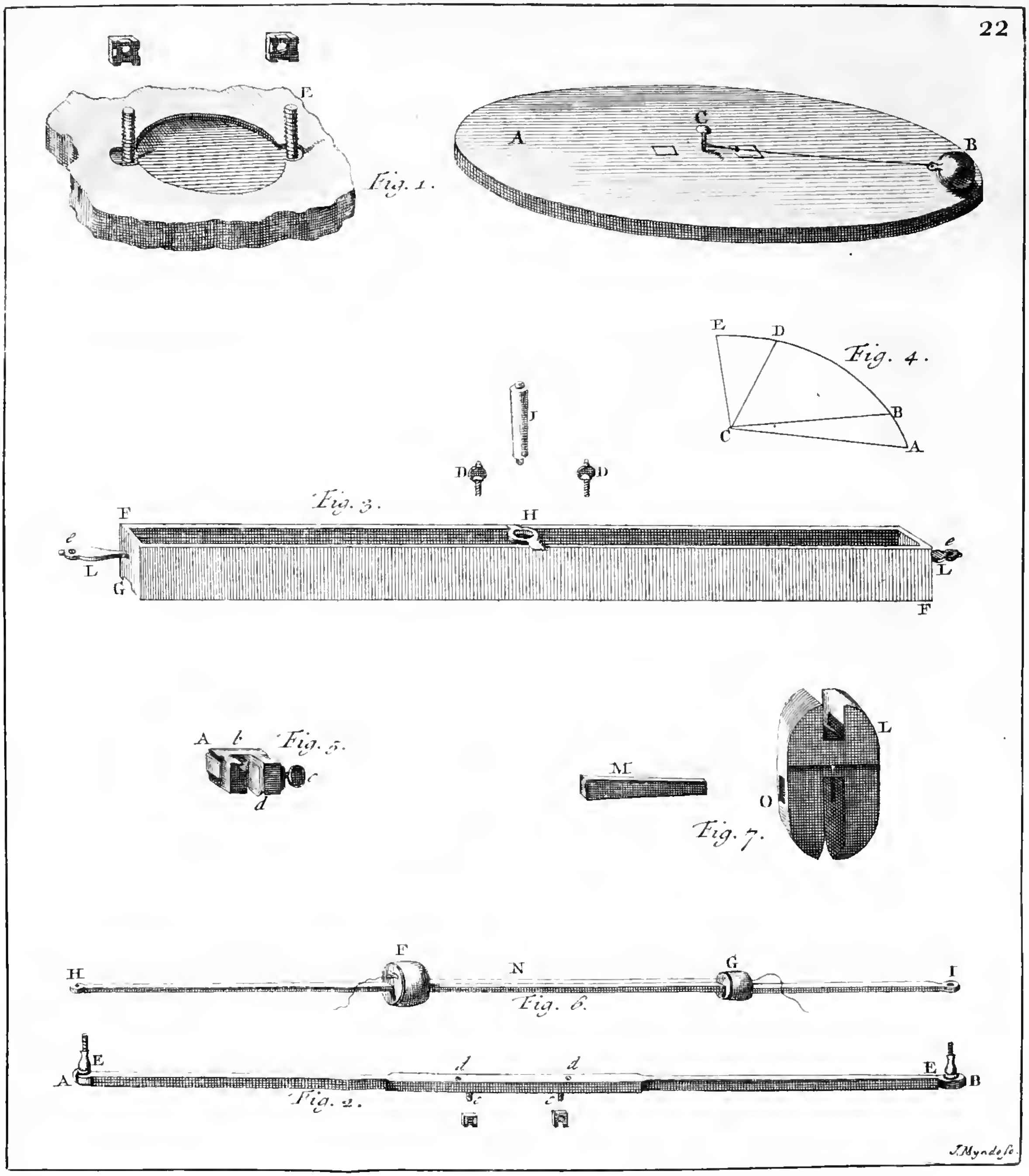
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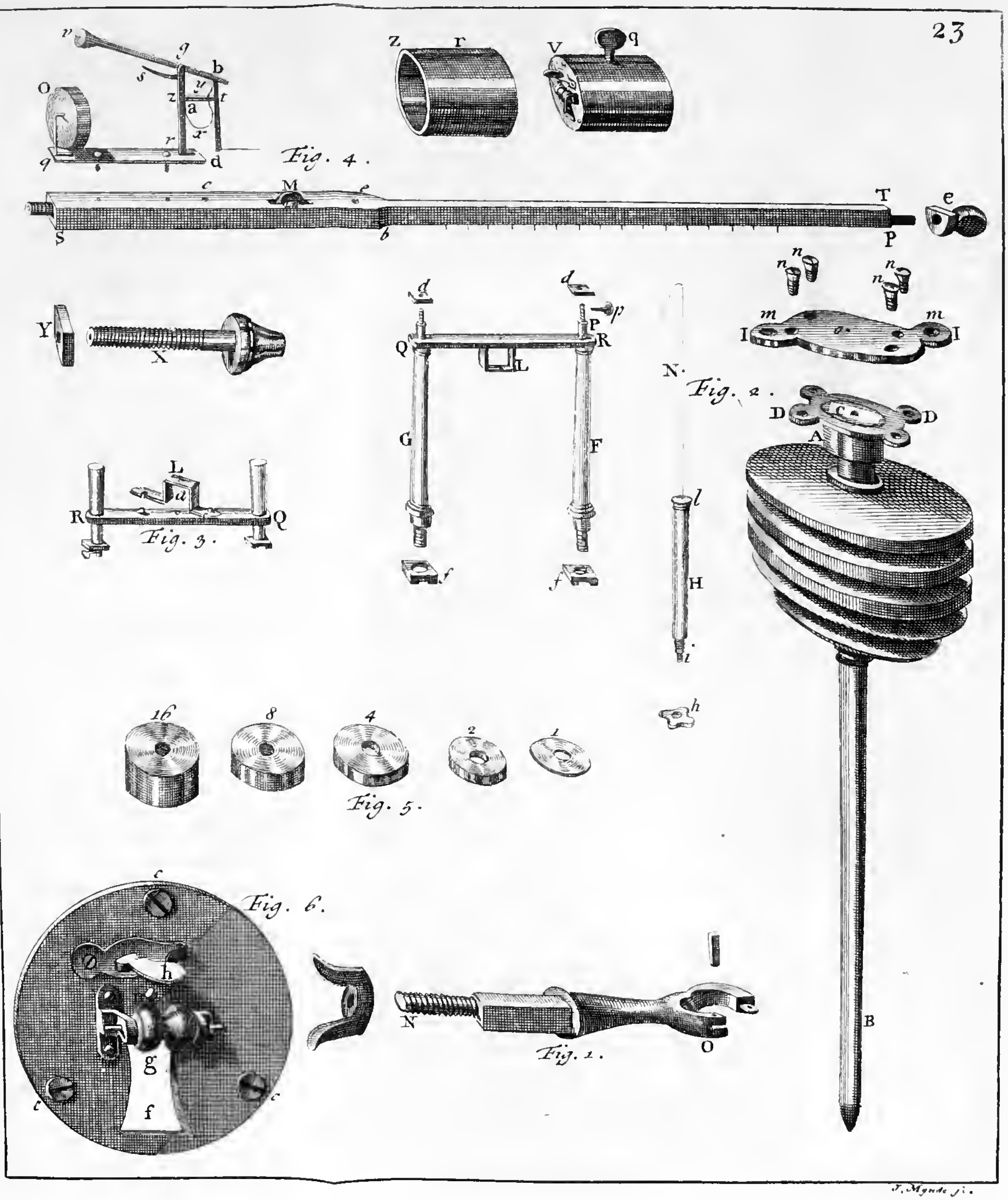
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J. Mynde sc.

It appears alſo, that a Body projected, and acted upon by a Force tending towards the Center, is mov'd in a Plain, which goes thro' a Line along which the Body is projected, and thro' the Center of the Forces, as we ſhew'd in N° 581. 640.

Now let us conceive the Moments between two Strokes to be diminiſh'd, as alſo the Strokes themſelves, thoſe Moments ſtill remaining equal among themſelves, while the Strokes are unequal in any Proportion, the Demonſtration will hold good. If the Diminution be *in infinitum*, the Strokes are chang'd into a continual Preſſure, and the Body in every Point is turn'd out of the ſtreight way; yet it is ſubjected to the Law determin'd in the preceding Demonſtration. If therefore the Body be mov'd in the Curve A B D E, and the Time be conceiv'd divided into very ſmall Moments, equal to one another, the Area of the mix'd Triangle A C B will contain ſo many equal little Triangles as there are Moments in the Time, in which A B is run thro'; and the Area of the mix'd Triangle D C E will in the ſame manner contain as many Triangles, equal to one another, and to the former, as there are Moments in the Time in which D E is run thro'; therefore the Times in which a Body runs thro' A B, and D E, are to one another as the Number of equal Triangles, contain'd in the Areas A C B, D C E; that is, they are as the Areas themſelves. Whence we deduce the general Propoſition mention'd N° 585. 641. Plate XXII. Fig. 4.

The Inverſe of which Propoſition, which is contain'd in N° 586, is alſo demonſtrated. If a Body, mov'd along A B, in a following and equal Moment, goes thro' B D, becauſe by the firſt Motion in that Moment, it would have continued its Motion along B C, equal to A B, it was neceſſarily remov'd from its way along the Direction L D *; but if the Triangles A B C, B C D, are equal, B D C, B L C, will alſo be equal; therefore the Line L D will be parallel to B C †; that is, the Direction of the Force, which turns the Body from the right Line, is directed towards the Center C. 642. Plate XXIV. Fig. 2. * 360. † 39 El. I.

If now we conceive any Curve to be divided, by Lines drawn to the Center of the Forces, into very ſmall equal Triangles, their Baſes are run thro' in equal Times by a Body, *which is retain'd in the Curve by a central Force* *; therefore *the Velocities of the Body, in various Points of the Curve,* are as thoſe Baſes †, which *are inverſely as the Perpendiculars, drawn from the Center of the Forces* to the Baſes continu'd ‖; that is, *to the Tangents to the Curve in the Points conſider'd.* 643. * 639. † 119. ‖ 15 El. VI. 38 El. I.

The Things hitherto demonſtrated in this Scholium are very general; what I ſhall add now only obtains, if in this the Force agrees with Gravity, ſo as to act on Bodies in motion, as it does upon Bodies at reſt; but we ſuppoſe the Bodies equal: but if the Force alſo agrees in that with Gravity, ſo as to act in the ſame manner on all the Particles of Matter, it is of no Conſequence whether the Bodies be equal or not. 644.

Infinitely ſmall Lines, gone thro' by equal Forces, acceding to the Center, are as the Squares of the Times in which they are run thro'. For a Force 645.

may be consider'd as uniform, in an infinitely small Space, and what has
* 374. been demonstrated of falling Bodies *, may be referr'd hither.

646. *If the Forces differ, but the Times are equal, the Spaces run thro' will be*
* 587. 133. *as the Forces.*

647. Therefore *Spaces infinitely small, run thro' by central Forces, are as the Forces themselves, and as the Squares of the Times*; that is, in a Ratio compounded of those two Ratios.

648. Hence we deduce, that *a Body, which is retain'd in a Curve by a central Force, in every infinitely small Moment, is mov'd according to the Laws ex-*
* 541. 542. 545. *plain'd concerning Projectiles* *. For tho' a Body tends to the Center, if the Space gone through be infinitely small in respect to the Distance from the Center, the Lines drawn to the Center may be consider'd as parallel.

Plate XXIV. Fig. 3. Let A F G E be the Curve in which the Body is mov'd; C the Center of the Forces; A D the Tangent to the Curve in the Point A; let us suppose A D infinitely little, and the Lines B F and D G to be parallel to A C,
* 542. 545. these will be as the Squares of the Lines A B, A D *, which are as the Times, in which A F, A G, are run thro'.

SCHOLIUM II.

Concerning Motion in a Circle.

649. A*NY Force whereby a Body is retain'd in a Center*, if it be directed towards the Center of the Circle, acts always perpendicular to the Di-
* 18 El. III. rection of the Motion; for the Tangent is perpendicular to the Radius *. Therefore the Action of that Force never conspires with the Motion of the Body, or acts contrary, and it always *acts in the same manner as it would do on a quiescent Body*; for that Reason, it is of no Conseqence, whether such a Force, which retains the Body in the Circle, be of the same Nature as Gravity or not, and acts in every Case in the same manner on a Body in motion as it does on a Body at rest, it retains the Body in the same manner.

650. Let a Body be mov'd in a Circle, whose Diameter is G L; C the
Plate XXIV. Fig. 4. Center of the Circle and of the Forces. Let an equal Body be projected along A D, with the Velocity with which the Body is mov'd in the Circle. These Bodies in equal Times, run thro' the equal Lines A B, G H, infinitely small; they also in equal Times run thro' the little Lines BE, H I; the first, by its Weight, the second, by its central Force; supposing B E to be vertical and H I parallel to G C; which little Lines are to one another, as the Weight of the Body to the central Force, which retains the Body in
* 587. 133. the Circle *.

Let D F be the Height from which a Body falling acquires a Velocity, with which the Projection is made, the Body goes through that Space in falling, whilst with an uniform projectile Motion it goes through twice
* 376. that Length *; if therefore D F is vertical, and A D the double of D F,
† 541. the projected Body will go thro' F †; therefore AB^q, or GH^q is to AD^q,
* 120. 374. or $4 \times DF^q$, as B E to D F *.

In

In the Circle drawing I i parallel to G H, that is, perpendicular to the Diameter *, G i or H I, G I, or G H, and G L will be in continu'd Proportion *, wherefore H I : B E : : D F : $\frac{1}{4}$ G L. 18 El. III. 8 4. El. VI. * 120. 374.

The Proportion above-mention'd is therefore chang'd into this, 651.
H I × G L : 4 × D F ^{q} : : B E : D F : : B E × G L : D F × G L. By Alternation H I × G L : B E × G L : : 4 × D F ^{q} : D F × G L. Whence we deduce H I : B E : : D F : $\frac{1}{4}$ G L.

That is, *The Force whereby a Body is retain'd in a Circle, is to the Weight of the Body, as the Height from which the Body falling, acquires a Velocity, with which the Projection is made to the fourth Part of the Diameter.*

If the same Body, in the same Circle, be carried with another Velocity, the Consequents of the Proportion remain; therefore the Antecedents are chang'd in the same Ratio: that is, *the central Force* varies, as the Height from which a Body falling acquires the Velocity with which it moves, which Height *follows the Proportion of the Square of the Velocity* *. 652. * 374.

But as long as we speak of the same Circle, the periodical Time is so much the less, as the Velocity is greater, and so on the contrary, and that Time is inversely as the Velocity, whence the Demonstration of N° 616 is made plain, cæteris paribus, to be inversely as the Squares of the periodical Times. 653.

We have said in N° 607, that the central Forces, supposing the Bodies as well as the periodical Times to be equal, are as the Distances from the Center; which, to demonstrate, we suppose two equal Bodies to describe in equal Times the concentric Circles B I L, A F M; in very small, and equal Moments they run thro' the similar Arcs B I, A F. But the Bodies, in the same Moments, would move along the Tangents B H, A D, equal to the Arcs, if there was no central Force. Therefore the Bodies in equal Moments, are by the central Forces, transferr'd from H to I, and from D to F; and indeed, because of the infinitely small Arcs, along the right Lines H I, D F, in whose Ratio the central Forces are, but it is very plain that those Lines are as the Distances from the Center B C, A C. 654. Plate XXIV, Fig. 5.

There remains concerning the Motion in a Circle, that we demonstrate the Proposition of N° 621. 655.

Let the Distances from the Center be D and d; the periodical Times T, t; the central Forces V, v: let us suppose $T^{q} : t^{q} :: D^{c} : d^{c}$; therefore $\frac{D}{T^{q}} : \frac{d}{t^{q}} :: \frac{D}{D^{c}} : \frac{d}{d^{c}} :: \frac{1}{D^{q}} : \frac{1}{d^{q}}$. But $V : v :: \frac{D}{T^{q}} : \frac{d}{t^{q}}$ *; therefore $V : v :: \frac{1}{D^{q}} : \frac{1}{d^{q}}$. Q. E. D. * 620.

SCHOLIUM III.

Of the Motion in an Ellipse.

IN this and the following Scholiums we intend to treat of that Force, which acts on Bodies in motion as if they were at rest.

656. Plate XXIV. Fig. 6. Let D A E be an Ellipse; C the Center; let a Body be mov'd in the Ellipse, in which it is retain'd by a Force, which is directed to its Center; we are to determine what this Force is.

Let the Body be at A, and A I be the Tangent to the Ellipse; A B the Diameter; E D its conjugate Diameter parallel to the Tangent *; A L an Arc describ'd in a small constant Moment of Time; I L, parallel to A C, the Space run through in the same Moment by the central Force, which Space follows the Ratio of the central Force *.

* *La Hire Sect. con.* Book II. Prop. 10.
* 646.

Draw L G parallel to I A, and L H perpendicular to A C; as also A F perpendicular to E D; and let also C L be drawn.

The rectangular Triangles L H G, A F C, are similar, because of the equal Angles L G H, A C F *. Therefore L H : L G :: A F : C; and $LH \times AC = LG \times AF$.

* 29 El. I.

But the Quantity $LH \times AC$ is constant; for it is the double of the Area of the Triangle A L C *, which is proportionable to the constant Moment, in which A L is described †.

* 41 El. I.
† 585, 641.

In the Ellipse also $ED \times AF$ *, is a constant Quantity; therefore $ED \times AF$, is to $LH \times AC$, or to $LG \times AF$, that is, E D to L G, always in the same Ratio wherever a Point as A is taken in the Ellipse; therefore there is also a constant Ratio between ED^q and LG^q. But in the Ellipse $ED^q : LG^q :: AB^q : AG \times GB$ *, or $LI \times AB$; because of A G and L I being equal, and the Difference infinitely small between G B and A B; therefore the Ratio between AB^q and $LI \times AB$, that is, between A B and L I, is also a constant Ratio: therefore L I is increas'd and diminish'd, that is, the central Force, in the same Ratio that A B is increas'd and diminish'd, or its half A C, which is equal to the Distance of the Body from the Center, as we have observ'd in N° 633.

* *La Hire Sect. con.* Book V. Prop. 21.
* Ibid. Book III Prop. 3.

657. But if whilst the Body is mov'd in the Ellipse, the Force be directed to the Focus; this Force in receding from the Center of the Forces, decreases in an inverse Ratio of the Square of the Distance, as we have it in N° 626, the Demonstration of which Proposition we shall give here.

Plate XXIV. Fig. 7. Let D A B be a Semi-Ellipse; B D the Axis; C the Center; F the Focus, to which the Force is directed; A I a Tangent to the Ellipse in any Point as A; A L an infinitely small Arc.

Having drawn A C, A F; let L G and C E be parallel to the Tangent A I; L I parallel to A C; and L *i* æquidistant from A F, L I and A G will be equal, as well as L *i* and A *g* *. But A E will be equal to C D the greater Semi-Axis; for having drawn A *f* to another Focus, and *f* M also parallel to A I, the Angles A M *f*, A *f* M will be equal †; and the Sides

* 34 El. I.
† *La Hire Sect. con.* Book VIII. Prop. 8.

A

A M, A f, equal *; E M, E F are alſo equal becauſe C F, C f are equal †: therefore E M + M A, that is, E A is equal to F E + A f, and E A is half the Sum of the Lines F A, A f, which taken together are equal to B D *. * 5 El. I. † 2 El. VI. * 625.

Beſides, draw L H perpendicular to A C, and L b making right Angles with A F; and join the Points H, b. By reaſon of the right Angles A b L, L H A, the Points H, b, are in the Circumference of a Semi-circle whoſe Diameter is A L *; therefore the Angles b L H, b A H, are in the ſame Segment, and therefore equal †: the Angles L H b and L A b are alſo in the ſame Segment and equal; now here becauſe A L is infinitely ſmall, it coincides with the Angle I A b, and is equal to the Angle A E C; wherefore the Triangles L b H, A E C are ſimilar; and L b : L H : : A C : A E or C D *. * 31 El. III. † 21 El. III. * 29 El. I.

Likewiſe, becauſe of the ſimilar Triangles A g G, A E C, A G is to A g, or L I to L i, as A C to A E, or C D. Theſe Things being laid down, let us conceive two Bodies running along this Ellipſe, in the ſame time, one of which is retain'd by a Force, which is directed to C the Center of the Ellipſe, and the other by a Force tending to F one of the Foci.

When both Bodies run along the little Arc A L, the firſt is moved by the central Force, along I L, and the ſecond by the central Force runs thro' i L, but the Times in which the Bodies go through theſe little Lines, are to one another as the Areas L A C, L A F *, for we ſuppoſe the whole Ellipſe to be run thro' by each of the Bodies in the ſame time; therefore that in each Caſe, the ſame periodical Time is repreſented by the whole Area. But thoſe Areas are to one another as their double A C × L H, A F × L b; now theſe Products, becauſe of L H : L b : : C D : A C, are as A C × C D, and A F × A C, that is, as C D to A F. * 585. 641.

The Spaces I L, i L gone through by the central Forces, which we have ſhewn to be, as A C to C D, are alſo in a Ratio compounded of the Forces, and the Squares of the Times *, or of the Lines C D, A F. The Force along A C is proportional to that Line, as we have demonſtrated †, and may be denoted by that very Line; we call the Force along A F, V: therefore A C : C D : : A C × C D^q : V × A F^q. Whence we deduce $V = \frac{CD^c}{AF^q}$; it appears therefore by reaſon of the conſtant Quantity C D^c, that changing the Point A, the Force V is changed in an inverſe Ratio of the Square of the Diſtance A F. *Q. E. D.* * 647. † 656.

In regard to the Motion in an Ellipſe, we have obſerved two Things more in N° 627, which we ſhall now demonſtrate.

658. Plate XXIV. Fig. 8.

Let B A D be a Semi-Ellipſe, B D the greater Axis; C A the leſſer; F the Focus, the Center of the Focus. With the Center F, and Radius F A let the Circle A P be drawn; we muſt demonſtrate that the periodical Time in the Circle is equal to the periodical Time in the Ellipſe; for the Radius F A is equal to the greater Semi-Axis of the Ellipſe, as follows from its Deſcription. * 625.

Let there be two Bodies at A, of which one moves in the Circle, the other in the Ellipse, and let A L, A M be very small Arcs describ'd in the same time; the Spaces gone through by the central Force will be equal; because both Bodies are at A F the same Distance from the Center: But those Spaces are i L, N M, supposing A i Tangent to the Ellipse, and A N to the Circle; as also N M, and i L, parallel to A F. Let also I L be parallel to A C, as O M to N A, G L to A I, and draw L C, L F, M F.

* 31 El. III. 8. 4 El. VI. In the Circle OM^q is equal to $2\,MN \times AF$*; for A F and O F are taken as equal, and A O, M N are actually equal.

La Hire Sect. Con. Book 3. Prop. 3. In the Ellipse $AC^q : BC^q$ or $AF^q :: 2\,IL \times AC : GL^q \frac{2\,IL \times AF^q}{AC}$. for A G, I L are equal; and A C, G C differ but by an infinitely small Quantity.

The Triangles I i L, A C F are similar, because their Sides are respectively parallel, therefore $FA : AC :: i\,L$, or $MN : IL = \frac{MN \times AC}{FA}$.

Substituting instead of I L its Value in this Equation $GL^q = \frac{2\,IL \times AF^q}{AC}$, we have $GL^q = 2\,MN \times AF$; to which Quantity also OM^q is equal: therefore G L and O M are equal: whence it appears, that in an Ellipse a Body at the End of the lesser Axis moves with the same Velocity, that another is carried in a Circle, whose Diameter is equal to the greater Axis of the Ellipse, if both are retain'd in their Curves with the same central Force, which is directed to the Focus of the Ellipse, and this is the first Part of N° 627.

659. * 37 El. I. Because the Curve at A is parallel to the Axis B D, the Triangles C A L, F A L are equal*; the right-angled Triangles C A L, F A M, whose Bases are equal, are to one another, as the Heights A C, A F, or C D; the Areas of the Circle, and the Ellipse, are in the same Ratio to one another. Therefore by Alternation, the Area of the Triangle C A L, or F A L, is to the Area of the Ellipse, as the Area of the Triangle F A M is to the Area of the Circle: therefore the Time in which a Body is mov'd along A L is to the periodical Time in the Ellipse, as the Time in which * 585. 641. A M is run thro' to the periodical Time in the Circle*; the Antecedents are equal, therefore the Consequents also. Which was the last Thing to be demonstrated.

SCHOLIUM IV.

Of Motion in an agitated Orbit.

LET a Curve A F be given, describ'd by a Body acted on by a central Force; and C be the Center of the Forces. Let the Curve be divided by Rays drawn from the Center C, as C A, C B, C D, *&c.* containing between themselves Angles exceeding small. 660. Plate XXIV. Fig. 9.

Let us conceive every one of the Angles to be continually increasing or diminishing, whilst the Radii keep their Length, and that a new Curve *a f* is produc'd passing thro' the Ends of the Radii.

The Triangles A C B, *a c b*, because of their equal Bases C A, *c a*, are to one another as their Heights *; which are as the Angles A C B, *a c b*; but all the Angles in one Curve are to the corresponding ones in the other, in the same Ratio; for in all the Curves they are all equal to one another; therefore any corresponding Triangles, as A C B, *a c b*; B C D, *b c d*, are in the same Ratio *; so that those mix'd Triangles are proportional, A C E : *a c e* :: E C F : *e c f*; and by Alternation A C E : E C F :: *a c e* : *e c f*.

* 1 El. VI.

* 12 El. V.

Now let us suppose a Body to be mov'd in the Curve *a f*, whilst a Body acted on by a central Force tending to C goes thro' the Curve A F; and let us conceive besides, that whilst one Body runs thro' A B, the other is carried along *a b*; whilst the first is come to D, that the other is at *d*, and so on; therefore in the same Time A E, *a e*, and in the same Time also E F and *e f* are gone thro'; therefore the Times in which A E, E F are gone thro', are as those in which the Body is mov'd thro' *a e*, *e f*. But those Times are as the Areas A C E, E C F *, which are as the Areas *a c e*, *e c f*; in which Ratio therefore are the Times in which the Body is carried through *a e* and *e f*: which Demonstration also, since it obtains, taking any Arcs whatever, it follows, that a Body carried in the Curve *a f*, describes Areas by Lines drawn to the Center *c*, proportionable to the Times, and is retain'd in its Curve, by a Force tending to the Center *c* *.

* 585. 641.

* 585. 642.

Now let us conceive the Curve A C so to move about the Center C, that the angular Motion of the Curve follows the Proportion of the angular Motion of the Body agitated in that Curve, when the Body moves in the Curve from A to F, its angular Motion is A C F; let us suppose the Curve in the mean time to be transferr'd by an angular Motion; and that the Line *a* C is come to the Situation A C; and that the Angles A C F, A C *a*, as they increase, keep the same Ratio to one another; the Angles *a* C F, A C F *, will also be in a constant Ratio. Plate XXIV. Fig. 10.

* 12 El. V.

Now if this be the Ratio, which in the former Figure (9) is given between the Angles *a c f*, A C F, and the Body be mov'd, and be retain'd by the central Force in the quiescent Curve A E F, and another Body in the same manner runs thro' a like Curve, and equally agitated, as has been said,

ſaid, that laſt, as plainly appears, will really move in the quieſcent Curve *a c f*.

661. Hence we deduce, *that every Body that deſcribes any Curve by a central Force, may by another central Force, deſcribe the ſame Curve, moveable about the Center of the Forces.*

Now we muſt treat of the Difference between thoſe central Forces.

662. Plate XXIV. Fig. 11. Let A, B, D, very little diſtant from each other, be three Points of any Curve run thro' by a Body tending to C by a central Force; and let G B H be a Tangent to the Curve in the Point B; and let G D, H A be parallel to B C: we ſuppoſe G B, B H to be equal to each other, and therefore that A B, B D are run thro' in equal Times.

Becauſe of the Diſtance between the Points A, B, D, being infinitely ſmall, the central Force in its Motion thro' thoſe Points is not chang'd; therefore in the equal Times in which A B, B D are run thro', the Body is equally deflected by the central Force from the right Line; that is, its Way is equally curv'd from which equal Deflection it follows, that H A, G D, are equal to one another.

The Angle which any Curve makes with the Tangent is infinitely ſmall, therefore H A and D G are infinitely ſmall in reſpect of H B, B G; wherefore as theſe are equal, and infinitely ſmall, the Angles B C A, B C D are equal.

Beſides, let the Angles A C *a*, D C *d* be equal; and with the Center C let the circular Arcs A *a*, D *d*, be deſcrib'd. It appears evidently that the Points *a*, B, *d*, are the Points of the Curve in which the Body is mov'd, if it be carried in the moveable Curve A B D, ſuppoſing the angular Motion of the Curve to be to the angular Motion of the Body, as the Angle *a* C A to the Angle A C B*; and in that Motion the Body is carried from *a* to B, in the ſame Time that it goes from A to B in the quieſcent Curve.

* 660.

Let us ſuppoſe F B I to touch the Curve *a* B *d*, in the Point B, and that I *a*, F *d* are parallel to B C; becauſe *a* B, B *d*, are run thro' in equal Times, I B, B F are equal, which might be gone thro' in the ſame Time, the central Force being taken away; F *d*, I *a* are alſo equal; which is ſhewn by the ſame Demonſtration, by which we prov'd H A, G D to be equal.

Join F, G, and H, I; and draw D E parallel to F G, and A L parallel to H I; produce E D to O cutting B C at N.

Becauſe of the equal B H, B G, and B I, B F, and alſo the equal Angles H B I, F B G, the Triangles F B G, B H I are equiangular and congruous *, wherefore F G, H I are equal, which are alſo parallel †: wherefore A L, E D, and F E, G D, H A, I L are alſo equal and parallel ‖; and L *a*, E *d* are alſo equal, as they are the Differences of Quantities reſpectively equal: A *a* and D *d*, the Meaſures of equal Angles, in Circles whoſe Radii differ but infinitely little, are alſo equal; therefore the Triangles A L *a*, D E *d* *, are equiangular, and the Angles A L *a*, D E *d* equal; but this laſt is equal to the Angle O N C, and the other to the

* 4 El. I.
† 27 El. I.
‖ 30. 34 El. I.
* 8 El. I.

2 Angle

Angle D N C *; becauſe the Legs are parallel; wherefore the Angles O N C, D N C are equal and right Angles. * 29 El. I.

In the Time that F d is run thro' in the moveable Curve by the central Force, G D, which is equal to F E, is run thro' in the quieſcent Curve by the central Force; therefore the Space run thro' in the ſame Time by the Difference of the Forces is E d. But the Point E in this Figure is determin'd by drawing a Perpendicular to B C thro' D. 663.

These Things being ſuppos'd, let C be the Center of the Forces, and let the Body move in the Curve A E G, ſo agitated about the Center C, that the angular Motion of the Curve may be to the angular Motion of the Body in the Curve, about the ſame Center C, as the Angle a C A to the Angle A C E: Let F G be a Continuation of the Curve A E; with the Center C and Radius C G deſcribe the Arc F G g; and drawing E C, G C, let the Angle G C F be made to the Angle E C G, as the Angle a C A to A C E. Whilſt the Body runs thro' E G in the Curve A E, by the Motion of the Curve, the Point G is transferr'd to F, and the Body runs thro' E F in the Time that it could have gone thro' E G in the quieſcent Curve. Let G H be drawn thro' G perpendicular to E C, which continued any how cuts E C at H, and C F continued at f; and f F will be the Space run thro' by the Difference of the Forces, ſuppoſing the Angles F C G and G C E infinitely ſmall *. 664. Plate XXIV. Fig. 12. * 633.

If taking any other Point E, E G and E F are ſo determin'd, as to be deſcrib'd in an equal Time, wherever the Point E is; that is, if the Areas E G C, E F C, have a determin'd Bigneſs *, the little Line f F will be proportional to the Difference of the Forces †. * 585. 641. † 646.

Let the Area E G C be call'd N, and the Area E F C, M; ſuppoſing N and M to be determin'd Quantities. We have $EC \times GH = 2N$, and $EC \times fH = 2M$; whence we deduce $GH = \frac{2N}{EC}$, and $fH = \frac{2M}{EC}$; as alſo $fH + GH$; that is, $fg = \frac{2M + 2N}{EC}$, and $fH - GH$; that is, $fG = \frac{2M - 2N}{EC}$. By the Property of the Circle $fG \times fg = fF \times fI$, taking F C and C I for equal *. * 36 El. III.

This Equation ſubſtituting for fG and fg their Values, is chang'd into this $\frac{4M^q - 4N^q}{EC^q} = fF \times fI$; but by reaſon of f F infinitely ſmall, f I is equal to 2 F C, and becauſe C F and E C differ infinitely little, one may be taken for the other: therefore the Equation is again chang'd into this $\frac{4M^q - 4N^q}{CF^q} = 2fF \times CF$: therefore $fF = \frac{2M^q - 2N^q}{CF^c}$. The Numerator of this Fraction is a conſtant Quantity; therefore fF, which expreſſes the Difference of the Forces, is in the inverſe Ratio of the Denominator; namely, of the Cube of the Diſtance from the Center.

This

This Force is the Excefs whereby the central Force in a moveable Curve exceeds the Force in the quiefcent Curve, and the Motion of the Curve confpires with the Motion of the Body.

When the Point f falls between G and H, the fame Demonftration ferves, but the central Force in the quiefcent Curve exceeds the other, and the Motion of the Curve is directed contrary-wife. But if the Point f falls between H and g, or beyond g, then we confider the Motion of the Body towards contrary Parts from E to A.

From all this we deduce, That *if a Body, by any central Force, defcribes*
665. *a Curve, it will defcribe the fame Curve, moveable about the Center of Forces, if you add, or take away the Force which follows the inverfe Ratio of the Cube of the Diftance.*

666. *If the Force be added, the Motion of the Curve, and the Motion of the Body, tend the fame way.*

667. *If the Forces be taken away, they are directed towards contrary Parts.*

SCHOLIUM V.

Of Motion in an agitated Ellipfe.

668. A BODY is retain'd in an Ellipfe by a central Force, tending towards
* 626. 657. the Center, and decreafing in an inverfe Ratio of the Square of the
* 665. 666. Diftance *; if a Force be added, which decreafes in an inverfe Ratio of the Cube of the Diftance, the Body will defcribe the fame Ellipfe; but fo carried, that its Motion will be directed the fame way with the Motion of the Body. The laft Force decreafes more by a Diftance being encreas'd, than the firft; therefore the Sum of the Forces, decreafes fafter than in the inverfe Ratio of the Square of the Diftances, whence the Propofition of N° 631 is prov'd.

669. By a like Demonftration appears the Truth of N° 632; for if from the Force, which follows the inverfe Ratio of the Square of the Diftance, be taken, the Force which follows the inverfe Ratio of the Cube of the Diftance; that is, which decreafes fafter than the firft, that which remains diminifhes more flowly, than according to the inverfe Ratio of the Square of the Diftance, that being encreas'd.

670. In N° 630, 631, 632, we have treated of Forces, decreafing according to a Ratio differing but little from the duplicate inverfe Ratio of the Diftance, or of Curves nearly circular; becaufe in thefe Cafes there are no fenfible Errors in the Propofitions, tho' the Forces follow the Ratio of fome other Power of the Diftance; in which Cafe, mathematically fpeaking, the Curve is not an Ellipfe, mov'd according to the Laws explain'd; for which is requir'd a Force which is the Sum or Difference of the Forces, of
* 626. 657. which one follows the inverfe duplicate Ratio *, and the other the triplicate
† 665. Ratio of the Diftance †.

Now

Now to determine from the given angular Motion of the Ellipse, the Force to be added, or taken away, and *vice versâ*, from that being given to find the Motion of the Curve, let A be the Extremity of the greater Axis; F the Focus, the Center of the Forces; aA a Portion of the Circle describ'd by the Center F, and Radius F A; A L a Portion of the Ellipse. 671. Plate XXIV. Fig. 13.

Let us suppose while the Body is carried in the Ellipse along A L, that the Curve is carried by the angular Motion aFA; and that the Angles aFL, AFL are to one another, as M to N. We also suppose those Angles to be infinitely small. At a and A, to the Circle aA draw the Tangents ai, E A I, running together at E, and the last of which also touches the Ellipse at A; draw also A B, L I, parallel to aF, the last because of the infinitely small Arcs aA, A L, may be look'd upon as parallel to A F; lastly, let A C be parallel to aB, and L G to A I.

E a and E A * are equal, therefore also aE and E B, which is equal to E A, by reason of the similar Triangles E B A, E i I, E B or $\frac{1}{2}aB$: E i or $ai - \frac{1}{2}AB :: BA : iI$; but aB is to ai, as the Angle aFA to aFi; that is, as M — N to M: therefore $BA : iI :: \frac{1}{2}M - \frac{1}{2}N : \frac{1}{2}M + \frac{1}{2}N :: M - N : M + N$. From the Property of the Circle aC or B A, aA or aB, and the Diameter, are in a continued Proportion *; therefore $BA = \frac{AB^q}{2AF}$. The Ellipse in the Extremity of the greater Axis, falls in with a Circle, whose Diameter is the Parameter of the Axis *; therefore if this Diameter be call'd 2 R, I L will be $= \frac{AI^q}{2R} = \frac{Bi^q}{2R}$: but $\frac{aB^q}{2AF}$ is to $\frac{Bi^q}{2R}$, as $\frac{\overline{M-N}^q}{AF}$ to $\frac{N^q}{R}$: therefore $IL : AB :: \frac{N^q}{R} : \frac{\overline{M-N}^q}{AF}$.

* 36 El. III.

* 31 El. 3.

*8.4. El. VI. *La Hire Sect. con.* B. 7. P. 6.

But as we have seen $AB : Ii :: M - N : M + N$; therefore by the Composition of the Ratio $IL : Ii :: \frac{N^q \times \overline{M-N}}{R} : \frac{\overline{M-N}^q \times \overline{M+N}}{AF}$ $= \frac{\overline{M^q - N^q} \times \overline{M-N}}{AF} :: \frac{N^q}{R} : \frac{M^q - M^q}{AF}$, I L and iL are gone thro' in the same Time, by the first Force, which is that with which the Body is retain'd in the quiescent Ellipse; and the second, the Difference of that Force with the Force by which the Body is retain'd in the moveable Ellipse: therefore the Force in the Ellipse is to that Difference, as $\frac{N^q}{R}$ to $\frac{M^q - N^q}{AF}$ *.

* 646.

Let $\frac{N^q}{AF^q}$ be call'd the Force by which a Body in an Ellipse is retain'd in the Point A, and let $\frac{N^q}{R} : \frac{M^q - N^q}{AF} :: \frac{N^q}{AF^q}$, to the Difference of the Forces $\frac{RMM - RNN}{AF^c}$, at the End of the greater Axis.

 If

If we ſpeak of any other Diſtance, and call it D, the Force by which the

* 626. 657. Body is retain'd in the Ellipſe, is diſcover'd by this Analogy *, $\frac{1}{AF^q} : \frac{1}{D} :: \frac{N^q}{AF^q}$ to the Force required $\frac{NN}{DD}$.

* 665. The Difference of the Forces is diſcover'd by this Rule *, $\frac{1}{AF^c} : \frac{1}{D} :: \frac{RMM - RNN}{AF^c}$ to the Difference requir'd $\frac{RMM - RNN}{D^c}$.

672. Therefore the whole Force by which the Body is retain'd in a moveable Ellipſe follows the Proportion $\frac{NN}{D^q} + \frac{RMM - RNN}{D^c}$, when the Body and the Ellipſe go the ſame way.

673. If theſe Motions are contrary, the Force is proportional to $\frac{NN}{D^q} - \frac{RMM + RNN}{D^c}$.

SCHOLIUM VI.

Of the Computation of the Motion of the Apſides in Curves differing very little from a Circle.

674. THE Ends of the greater Axis of an Ellipſe, in which a Body is mov'd, which is retain'd by a Force tending to the Forces, are call'd the Apſides. We are now about to determine the Motion of the Apſides; that is, the angular Motion of the Ellipſe, ſuppoſing any Force, different from that which we have treated of in the foregoing Scholium; in which Caſe, the Motion can't be referr'd to a moveable Ellipſe, unleſs

* 670. we ſpeak of a Curve very little differing from a Circle *.

675. We muſt premiſe a Lemma. The Square of this Quantity $a - b$ is $aa - 2ab + bb$; that the Cube may be form'd, all the Quantities of this Square muſt be multiplied by $a - b$; the Product of the two firſt by it is $a^3 - 3aab + 2abb$, and in the remaining Part of the Product b riſes to a greater Power than the firſt.

That a fourth Power may be form'd from the Cube, every one of the Quantities of the Cube muſt be multiplied: the firſt two being multiplied, we have $a^4 - 4a^3b + 3aabb$, and in all the remaining Quantities, b is rais'd beyond the firſt Power.

676. Continuing thus, it appears clearly, that in *relation to the Power of the Quantity* $a - b$, *whoſe Index is* n, *the firſt Terms are* $A^n - na^{n-1}b$, *and in all the remaining Terms* b *is rais'd to an higher Power.*

677. Now ſuppoſing what has been demonſtrated in the foregoing Scholium, let AF the greateſt Diſtance of all be call'd H; and the undetermin'd Dif-

3 ference

ference between H and D be call'd X. By reducing the two Fractions $\frac{NN}{D^q} + \frac{RMM - RNN}{D^c}$ to one, we have $\frac{DNN + RMM - RNN}{D^c}$; ſubſtituting in the Numerator inſtead of D, the Value H — X the Force in the moveable Ellipſe is proportional to $\frac{RMM - RNN + HNN - NNX.}{D^c}$

Now let any Force be given, which, that the Solution may be more univerſal, we will conceive to be form'd of two Forces join'd, (if there were more, one might proceed the ſame way) which have any Ratio to one another, which is given between *a* and *b*; and which ſeparately follow the Ratio of any Power of the Diſtance.

Let the firſt be as the Power m — 3, the ſecond as the Power n — 3: therefore the Force propos'd is as $\frac{aD^m + bD^n}{D^c} = \frac{a \times \overline{H - X}^m + b \times \overline{H - X}^n}{D^c}$.

Inſtead of $\overline{H - X}^m$ we put $H^m - mH^{m-1}X +$, &c.* In the remaining Terms X riſes beyond the firſt Power; therefore all theſe are ſmall in reſpect of thoſe that are put here, becauſe X is very little in reſpect of H: for we have ſuppos'd the Curve to differ very little from the Circle. * 676.

In the ſame manner $\overline{H - X}^n = H^n - nH^{n-1}X$, and the whole Force is as $\frac{aH^m - amH^{m-1}X + bH^n - bnH^{n-1}X.}{D^c}$

If now the Motion of the Body, which is retain'd in the Curve by that Force, muſt be referr'd to a Motion in a moveable Ellipſe, that Force muſt be ſuppos'd analogous to the Force whereby a Body is really retain'd in ſuch an Ellipſe; for by analogous Forces, that is, which conſiſt of correſpondent and proportional Parts, ſimilar Curves are deſcrib'd. Theſe Quantities therefore are analogous, $\frac{RMM - RNN + HNN - NNX}{D^c}$, and $\frac{aH^m - amH^{n-1}X + bH^n - bnH^{n-1}X}{D^c}$; that is, becauſe of the common Denominator the Numerators are analogous. 678.

In an Ellipſe differing but little from a Circle, H ſcarce differs from the Semi-Parameter, as follows from the Generation of the Ellipſe *, and the Definition of the Parameter †; therefore — R N N + H N N deſtroy one another, and R M M becomes H M M; and theſe are analogous Quantities HMM—NNX and $aH^m - amH^{m-1}X + bH^n - bnH^{n-1}X$; that is, the conſtant Parts are to one another as undetermin'd, which are multiplied by X; therefore H M M : N N X : : $aH^m + bH^n$: $amH^{m-1}X + bnH^{n-1}X$. When we muſt compute arithmetically, we put the greateſt Diſtance to be expreſs'd by Unity, that is H = 1; we alſo divide the Conſequents by X, by which the Proportion is not diſturb'd: and we have M M : N N : : $a + b$: $am + bn$. 679.

* 625.

† *La Hire Sect. Con.* def. poſt. P. 3. B. 3.

Therefore N : M : : 1 : $\sqrt{\frac{a+b}{am+bn}}$. 680.

 This

This is the universal Rule, which is easily applied to particular Cases.

681. Let there be a Force, which follows the Ratio of any Power of the Distance, and let its Index be n — 3; the Force therefore is, as $\frac{D^n}{D^c}$, we put this Expression equal to the general Expression $\frac{aD^m + bD^n}{D^c}$; and it appears that a and m, are not given, and they are therefore = 0; b is one, and N D

* 680. expresses any Number: therefore this Proportion $N : M :: 1 : \sqrt{\frac{a+b}{am+bn}}$*, is chang'd into this $N : M :: 1, \sqrt{\frac{1}{n}}$, or $M : N :: 1 : \sqrt{n}$.

682. *That is, the angular Motion of a Body, in an Ellipse carried along, is to its angular Motion in the same Ellipse at rest, as one to the square Root of a Number, which exceeds by three the Index of the Power, whose Ratio the Force follows.*

Therefore from the given angular Motion of the Curve, the Power that the Force follows is discover'd; and *vice versâ*, from the given Power is discover'd the angular Motion of the Curve.

I shall give but one Example, which has its Use in Astronomy.

683. Let a Body be given, which moves in an Ellipse, which goes forward three Degrees in each Revolution; that is, the Motion of the Body in the Curve is transferr'd 363 Degrees, whilst it would go 360 Degrees in the quiescent Orb; M therefore is to N, as 363 to 360; or as 121 to 120; and M M to N N, as 14641 to 14400: therefore $N = \frac{14400}{14641}$, and the Power of the Distance whose Proportion the Force follows is $\frac{14400}{14641} - 3 = \frac{29523}{14641}$; wherefore the Force is reciprocally as $D^{\frac{29523}{14641}} = D^{2\frac{241}{14641}} = D^{2\frac{4}{243}}$ nearly.

684. If the Progress of the Apsides at every Revolution, was 3 Deg. 2′, 38″, the Force would be reciprocally as $D^{2\frac{7}{416}}$, nearly.

685. I will propose another Case, which will also be of use in what follows. Let there be a Force given, whereby a Body is retain'd in a quiescent Ellipse, and which tends to the Focus; that is, which follows the inverse

* 626. 657. Ratio of the Square of the Distance*, and let the Force be substracted which follows the direct Ratio of the Distance; from the given Forces it is requir'd to find the Motion of the Apsides, and *vice versâ.*

* 677. The Force is as $\frac{1}{DD} - bD = \frac{D - bD^q}{D^c} = \frac{aD^m + bD^n}{D^c}$*: therefore

† 680. $a = 1$; $b = -$b, m 1; n = 4. And $N : M :: \sqrt{\frac{1-b}{1-4b}}$†; whence b

$$= \frac{MM - NN}{4MM - NN} = \frac{\overline{M+N} \times \overline{M-N}}{\overline{2M+N} \times \overline{2M-N}}.$$

The Motion of the Apsides is directed the same way as the Motion of

* 631. 668. the Body *; because as the Distance increases, the Force increases whereby it is taken off, whereby the Diminution in the Recess from the Center is greater than in the inverse Ratio of the Square of the Distance.

In

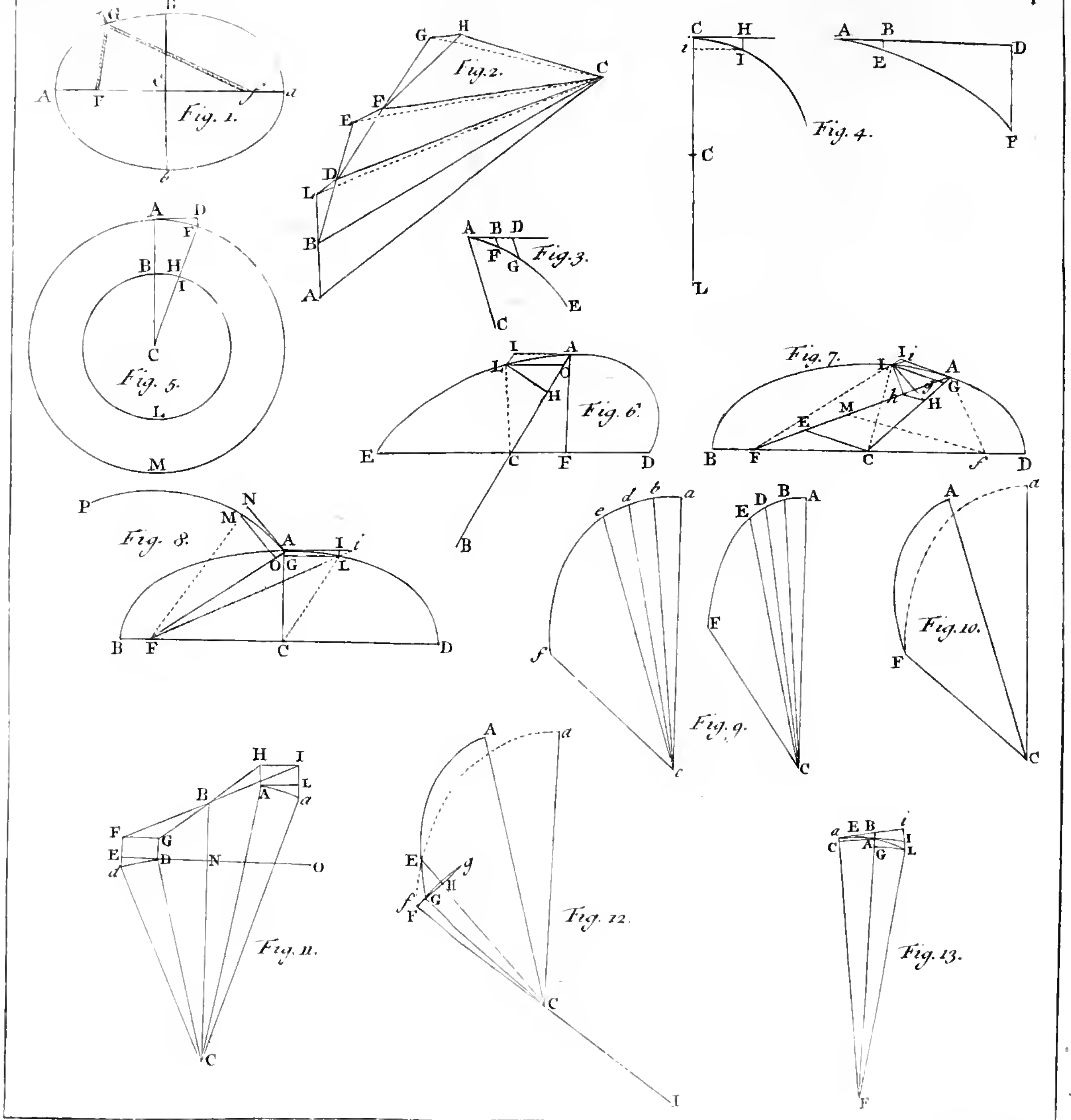
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.

In the foregoing Example, in which M : N : : 121 :: 120, $= \frac{241 \times 1}{362 \times 122}$ 686.
$= \frac{1}{183,24}$.

If the angular Progrefs of the Curve, at every Revolution of the Body 687.
in the Curve, was 3 Deg. 2′, 38″; *b* would be equal to $\frac{1}{180,66}$; that is, the Force fubftracted would be equal to fuch a Part of the other Force, as would retain the Body in the quiefcent Ellipfe.

The End of the Firft Book.

BOOK II.

PART I. Of Innate Forces.

CHAP. I.

Of the Nature, Generation, and Deſtruction of Forces in general, and their Differences from Preſſures.

A Quieſcent Body reſiſts Motion, not whilſt it is at reſt, but
* 19. whilſt it is acquiring Motion *: A Body in motion, reſiſts Acceleration and Retardation; not whilſt it keeps its Velocity, but whilſt its Velocity is changing, whether it is increas'd, or di-
* 364. miniſh'd *.

688. Therefore in general, *A Body, that acquires, or loſes Velocity, reſiſts*; which Reſiſtance, in the laſt Caſe, is call'd an Action.

For a Body which acquires Motion is ſaid to reſiſt by its Inertia; if it loſes Motion, it is ſaid to act by its innate Force: but theſe
689. differ only relatively. *The acquiring, and loſing Velocity, often expreſs the ſame Change in Motion.*

690. A Body which has ten Degrees of Velocity, and loſes four, acquires the ſame Velocity in a Ship, mov'd towards the ſame Part as the Body, with the Velocity ten, or a greater; and that, which in the Ship is taken for the Action, whereby Motion is communicated, if we do not attend to the Ship is call'd the Effect, by which that ſame Motion is conſum'd: that is, the Reſiſtance of the Body,
* 689. of which we are ſpeaking, whilſt the Motion is chang'd *, is looked upon as the Effect of the Inertia by him, who is in the Ship, and refers the Motion to it; but as to him who, without attending to the Ship, conſiders the Motion, the Body acts by its innate Force.

691. Therefore *Inertia and Force differ relatively only*; *and the ſame*
* 689. *Reſiſtance, ariſing from the Change of Motion *, is referr'd to the Inertia, or Force, according as this Change is look'd upon as an Increaſe, or Diminution of Motion.*

This relative Diſtinction between Inertia and Force being laid
692. down, we ſee that *a quieſcent Body has no Force*; for it cannot loſe Motion,

Motion, nor act by its own Force; but *a Body can reſiſt by its Inertia, whether it is at reſt or in motion.* 693.

Therefore *Force is that,* by which a Body in motion is diſtinguiſh'd from one at reſt, and *by which a Body acquires a Faculty of acting upon an Obſtacle.* But theſe Words expreſs Relations, *to be mov'd and to be at reſt, to act and reſiſt, differ relatively only.* 694. 695.

Whence it follows, that thoſe things, which belong to this matter, may be conſider'd two ways, by attending to the Generation, or Deſtruction of Forces.

We eaſily deduce from what is ſaid before, that *Preſſure generates Force*; for we ſaw, that a Body is mov'd out of its Place by it, *if it* (viz. *the Body*) *be not retain'd by a contrary Action* *. 696. 697.
With whatſoever Celerity a Body yields, it will keep this for ever, * 363.
whilſt it is not deſtroy'd by any outward Cauſe †. If the Preſſure † 355.
upon the Body be continued, *the Celerity* already acquir'd *is increas'd*, and that *as long as the Body is preſs'd.*

There is never any Preſſure without a Reaction equal to that Preſſure *; when a Preſſure is not deſtroy'd by a contrary Preſſure, * 361.
but moves an Obſtacle, and generates Force, the Reſiſtance, or Reaction of the Obſtacle, is to be attributed to its Inertia, as we have ſeen *. * 364.

A Preſſure is often deſtroy'd in part by a contrary Preſſure, what remains in this Caſe moves the Obſtacle, and generates Force; thus a Ship drawn by a Rope, ſuffers Reſiſtance from the Water: as long as this is leſs than that Preſſure, by which the Rope is drawn, the Celerity of the Ship is increas'd, and the Reaction, which is equal to the Action, as the Rope is equally ſtretch'd both Ways, is to be attributed in part to the Inertia of the Ship. When, the Celerity being increas'd, the Reſiſtance of the Water has increas'd ſo much, as alone to deſtroy the Action, whereby the Ship is drawn, it proceeds by its innate Force, with an equable Motion; two Preſſures acting upon it, which mutually deſtroy one another; as I obſerv'd before of the Carriage *. 698. * 364.

In every Caſe in which an Obſtacle is mov'd by a Preſſure, or its Motion increas'd, the Preſſure is not entirely deſtroy'd by the contrary Preſſure, wherefore *Force is generated.* 699.

A Preſſure, in an Inſtant infinitely ſmall, can only communicate a Velocity infinitely ſmall, and therefore a Force infinitely ſmall to a Body; therefore *the Force* is the Effect of a Preſſure, which has acted upon the Body in a finite Time, and *is equal to the Action of the Preſſure, which communicated it*; for the Effect anſwers to the whole

whole Cauſe: therefore the Force is equal to the whole Action, which the Preſſure exerts, whilſt it acts in a finite Time. But the Preſſure itſelf is deſtroy'd in all the Moments, infinitely ſmall; and when we ſpeak of its Greatneſs, that Action only ſhould be conſider'd, which it performs in ſuch an Inſtant; for this is diſtinct from the Action of the Preſſure, which acted in the foregoing Moment, or ſhall act in the following. Whence it follows, That
701. *the Force exceeds Preſſure, as much as a finite Time exceeds an infinitely ſmall Moment.*

702. Therefore *all Preſſure in reſpect of innate Force is infinitely ſmall.*

703. Therefore *the ſmalleſt Force can exceed the greateſt Preſſure.*

704. Thoſe who have endeavour'd to compare Preſſure with the innate Force, conſider'd that Effect of Preſſure, in which a Body was broken, or the Parts preſs'd inwards, which could be done, without a local Motion, and therefore a Generation of innate Force *; (* 699.) the Effect of which innate Force was compar'd with the Effect of the other Force.

And it agrees with daily Obſervations, that the ſmalleſt Stroke is greater than any Preſſure whatſoever. For let there be given a Preſſure, how great ſoever, if an Obſtacle be oppos'd to it, which can't be overcome by it in a ſmall Time, it will not be overcome at all. But a Stroke how ſmall ſoever, often repeated, can deſtroy every Obſtacle.

In all theſe I don't ſpeak of a Preſſure infinitely great, which generates a Force infinitely great in a finite Time.

705. When a Preſſure generates Force, equal Degrees of Velocity are not communicated by an equal Action in the Acceleration; for *that equal Degrees of Velocity may by equal Actions be communicated to equal Bodies,* one of which is at reſt, the other in motion; *it is requiſite that that, which acts upon the Bodies, ſhould have the ſame Relation in reſpect of each*; that is, it is requiſite that the moving Cauſe be carried with the ſame Velocity as the Body in motion, which it
706. can then act upon as upon a Body at reſt: but *the Action, by which the moving Cauſe is transferr'd, is to be ſuperadded to the Action of this, that we may have the whole Action by which the Body is mov'd.* For
707. both Actions are employ'd towards moving the Body. Hence I deduce that a Body is accelerated with more difficulty than it is mov'd, and with ſo much the greater difficulty, as it has the greater Velocity already.

708. As the Preſſure increaſes the Velocity, and therefore *the Force,* ſo alſo it is plain enough, that this *may be diminiſh'd by the Preſſure*; and

and that a contrary Action destroys the Force, whilst it is consum'd: which Destruction of the Resistance is call'd the Effect of the Force, as we have said already *. * 691.

709. But whilst a Body itself acts, it suffers no Action, except the Reaction from the Resistance of the Obstacle; which Reaction, as it is equal to Action *, it follows, that *a Body is acted upon as much as it acts*; *and* that *the Effect of the Action upon an Obstacle follows the Ratio of the Force lost*; for the Diminution of the Force is the Effect of Reaction: whence I deduce, that the whole Forces are proportional to the Effects by which they are consum'd, which is also evident from another Consideration. * 361.

710. A Body never acquires, or loses Motion, in an indivisible Instant; this always is done successively: and as the whole Action by which the Motion is communicated, is equal to the Sum of all the small Actions, by which acting successively the Motion was communicated *; so the whole Action of the Body is equal to the Sum of the small successive Actions, by which the Body consum'd its Motion. * 697. 700.

711. The greater the Resistance is, which the Body suffers in a certain Moment, the greater its Action is in this very Instant. Therefore, if the Resistance be continued, the greater this shall be, the sooner the Body will lose its whole Force; yet the Effect will not be different: for the Force which is destroy'd by the Resistance, follows the Proportion of the Resistance, and of the Time in which it acted; that is, the Force lost follows the Ratio compounded of the Resistance and Time; which same Ratio the Action of the Body follows, and the Effect which it produces.

712. So that it again appears that *the Force lost is proportional to the Effect, which it produces whilst it is destroy'd, whether it be destroy'd in a shorter or longer Time.* Which we shall afterwards find to agree with Experiments. I have demonstrated above, that the Force is equal to the Action by which it is communicated *; but from this it appears that the Force is equal to the Action by which it is consum'd; whence I deduce, that *any Degree of Velocity may be taken away by the Action, by which it can be communicated.* By an Action, equal to that, by which a tenth Degree of Velocity is superadded to a Body, which has nine Degrees, if it had ten, it would be reduc'd to nine. * 700. 713.

714. Whence it follows, that *a Body is with more difficulty accelerated than retarded.* If a Body has ten Degrees of Velocity, a tenth is more easily taken away than an eleventh communicated *. * 707. 713.

 From

From theſe more general Things concerning innate Forces their Meaſure muſt be deduc'd; but we muſt not immediately refer to Forces what is demonſtrated of the Meaſure of Preſſures; for Preſſure and Force differ entirely.

715. 1. *A Preſſure can act in a Place*, but *the Action of an innate Force is from one Place to another:* for unleſs the Body is in motion, it cannot act by its innate Force; that it may reſiſt by its Inertia alſo, it may be mov'd out of its Place.

716. 2. *The Intenſity of a Preſſure* upon a Point, to which it is immediately apply'd, is determin'd in ſingle Moments, and *depends upon the Preſſure itſelf. The Intenſity of the Action of* any *innate Force, does* not *depend* upon its Greatneſs, but *upon the Obſtacle*, which may be varied *in infinitum*, the Force remaining the
* 711. ſame *.

717. 3. *The Action of a Preſſure* is undetermin'd, *is chang'd according*
* 141. *to Circumſtances* *, *and*, cæteris paribus, *follows the Ratio of the Time in which it acts. But the innate Force of a Body, its Maſs and Velocity being given, is determin'd, and can only produce a determin'd Effect*, which is perform'd in a ſhorter, or longer Time, according to the greater, or leſs Reſiſtance, which it ſuffers *.
* 711.

718. 4. There is no *Preſſure* without Reſiſtance *; *if it does not act*,
* 362. *it is not a Preſſure.* But *innate Force is inherent in a Body, whilſt* the Body continues in motion with the ſame Velocity, keeping its Direction, *it does not act* by its own Force, but keeps it entire.

719. 5. *Preſſure and Force are incommenſurable to one another*; this is
* 702. infinitely great in reſpect of that *.

720. 6. Laſtly, *A Preſſure immediately deſtroys an oppoſite Preſſure*; but I ſhall demonſtrate in what follows, that *a Force can never immediately deſtroy a contrary Force.*

CHAP. II.

Of the Meaſure of Forces from their Generation.

721. WE have ſeen that Force is generated by Preſſure, and that this is equal to the whole Action, by which it was com-
* 700. municated *; wherefore I muſt now ſpeak about determining this whole Action.

722. It is manifeſt that the Action of a Preſſure, *cæteris paribus*, fol-
* 142. lows the Ratio of its Intenſity *.

We

We have also seen that *an Action, the Intensity of the Power being given, follows the Ratio* of the Space pass'd thro', in a certain Time, by the Point to which it is apply'd *. All that has been demonstrated of the Æquilibrium depends upon this Foundation; a Weight of one Ounce sustains the Endeavour of a whole Pound, when, the Agitation of the Points to which they are applied, being given, the Space pass'd thro' by the former exceeds the Space which the whole Pound passes through in the same time sixteen times. But the Space, pass'd thro' in a certain Time, follows the Ratio *of the Velocity of the Point mov'd* *. 723 * 141. 361. * 119.

Lastly, it is also plain, that the Action of a Pressure, which acts in Time, follows the Ratio of this Time. 724.

Therefore the whole Action of a Pressure is, as its Intensity *; *as the Velocity of the Point to which it is applied* †, *and as the Time in which it has acted* *, that is, it follows the Ratio compounded of these three Ratios; and is as the Product which is had by multiplying these three; and it appears plainly enough, that nothing besides is to be consider'd in this Determination. 725. * 722. † 723. * 724.

If, during the Action, the Intensity of the Power, or the Velocity of the Point be varied, *the Action is to be determin'd in each of the Moments, and the Sum of all the small Actions will be equal to the whole Action*; to which the Force communicated will be equal *; if it produces no other Effect, besides the Generation of the Force itself. 726. * 700.

The Space pass'd thro' follows the Ratio of the Time and Velocity *; whence I deduce, that the Consideration of the Time and Velocity, in the Computation shewn, may be neglected, if we attend only to *the Space pass'd thro' by the Point which presses*; and this Space *multiplied by the Intensity of the Pressure, will express the Force communicated.* * 121. 727.

If *a Point, whilst it passes thro' a determinate Space* A B, presses with a certain Force, that is, *if the Intensity of the Pressure is determin'd, it will produce the same Action, whether it be mov'd faster, or slower*; for the Time is diminish'd, as much as the Velocity is increas'd, *and* vice versâ; that is, as much as the Action is diminish'd in respect of one, so much is it increas'd in respect of the other *; therefore *in this Case we don't attend to the Time.* 728. Plate XXV. Fig. 1. * 723. 724.

If the Intensity of the Pressure is varied; but determin'd in each of the Points of the Line; the Action will also be determin'd in each of the small Spaces; and the Sum of the small Actions will be the 729.

ſame alſo, in whatſoever Time the Line ſhall be paſs'd over by the Point preſſing.

730. This is the Caſe, when an elaſtic Plate, bent, (which we ſhall call a *Spring* in what follows) is let looſe. Then the Point preſſing paſſes thro' a Space as A B; and in different Relaxations, the ſame bending of the Spring being given every Time, the Action is the ſame; becauſe every time the ſame Space is paſs'd through, and in any determinate Point as *c*, of the Way paſs'd thro', there is the ſame Intenſity of the Preſſure, in each of the Relaxations.

731. Therefore *the ſame Spring, bent in the ſame manner, whilſt it is relax'd always communicates equal Force to a Body, whether it be relax'd ſlower or faſter*, if the Inertia of the Spring be not ſeparated from the Inertia of the Body; which obtains, if the Spring makes part of the Body. We ſhall always obſerve this in Experiments, and it takes place in all elaſtick Bodies.

A Machine,

Whereby Experiments are made on a Pendulum, mov'd by the Action of a Spring.

732. Plate XXV. Fig. 2. The vertical Board A B C, made of thick Wood, ſtands upon the Foot D F E, the Supports H, H, being put between. That this Board may be the firmer, there is join'd to the hinder Part of it; the vertical Piece I, that reaches almoſt to A: this is faſten'd to the Board L, which is triangular, rectangular, whoſe Baſe is applied to the Piece F of the Foot.

733. * 567. The Foot is ſupported by three Rollers, like thoſe deſcrib'd above *. The Machine is fix'd by three Iron Screws *c*, *c*, (the third could not be repreſented) by raiſing it a little; and by theſe it is put into a vertical Situation, and the Line B C is horizontal; which Situation the Plumb Line T ſhews.

The Machine is rais'd only to a ſmall Height above the Baſe D F E, that it may be the more firm.

734. A ſquare Iron Bar M goes through the upper Part of the Board perpendicular to it, which is ſo faſten'd behind the Board as to be immoveable. Upon this is mov'd a ſquare braſs Tube, which is faſten'd by four Screws

735. To this Tube two Plates are join'd, which ſupport an Iron Ruler O Q, ſufficiently thick, that it may not have a tremulous Motion when mov'd; the Motion is made about an Axis, join'd to the upper

upper part of the Ruler. This Axis is of Steel, well polish'd, and fasten'd tight; its Extremities are conical, and go into Cavities or Boxes very smooth, of the same Figure, and turn in them. These conick Cavities are in the Ends of Screws, which go thro' the said Plates, and in which they stick so that they can't be mov'd without difficulty; by a small Motion of one Screw the tremulous Motion of the Axis in the Boxes is hinder'd.

The Ruler O Q, which, together with those Things which are join'd to it, (and of which afterwards,) makes a Pendulum, is at rest in a vertical Situation, parallel to the Surface of the Machine; and when it is mov'd, it moves in a Plane parallel to this Surface.

From the angular Motion of this Pendulum we determine the Force with which it is mov'd; and we determine this Motion by two Indices, moveable in the Slits *m m*, *n n*, the last of which only is here represented; they are fasten'd by Screws applied to the hinder Part of the Board. Its Surface is made a little hollow near the Slits, by which the Motion of the Indices is directed. These are made of a thin Plate *e h*, to which a Screw is join'd at the hinder Part, at the End *h*, and upon which the Index itself *e f* stands at right Angles; upon this is moveable a small Cursor, or sliding Piece *i*, which is at pleasure fasten'd at any Part of the Index. Such an Index is represented by itself at G. 736.

This Index is fasten'd, to determine the Height, to which the Pendulum ascends in a certain Motion, or that from which it is let go in other Experiments. The Velocity depends upon this Height, which the Pendulum acquires by descending; and in the Ascent it determines the Velocity, with which the Pendulum is driven upwards.

We measure these Velocities by the Brass, divided Rulers, V X, Y Z, which are applied to the Board in an horizontal Situation; the Surface of which is hollow'd in such manner, as to agree with the Surfaces of the Rulers. There are Screws join'd to the hinder Part of the Rulers, which go through Slits in the Board, that the Rulers may be fasten'd; Slits are used instead of Holes, that the Rulers may be manag'd more accurately, whose Ends X, Y, ought to answer to the Surfaces of the Pendulum; namely, each to the Surface, which is given at the same Part with it; which is exactly determin'd by help of the Square *l o g p*; for the Lines *o g*, *g p*, being applied to the Table, we shall have *g l* rais'd at right Angles, which if it be applied to the Surface of the Pendulum, whilst at rest, the Point *g* should agree to the End of the Ruler. The Pendulum 737.

dulum being rais'd any how, we meaſure its Inclination, if the Square be ſo applied, that *g l* may be perpendicular upon the Surface of the Board, and touch the lateral Surface of the Pendulum rais'd higher; then, if the Point *g* falls upon the divided Line of the Ruler, it will agree to the Diviſion, which ſhews the Inclination ſought.

The Diviſions of the Rulers begin at the Ends X, Y, and ſhew the Angles of Inclination of the Pendulum, whoſe Subtenſes,
* 441. which follow the Ratio of the Velocities *, are as the Numbers mark'd upon the Diviſions; the whole Ruler contains 24 Diviſions, each of which is divided again into ten ſmaller.

738. Plate XXVI. Fig. 1. Three Curſors are applied to the Pendulum; which are moveable along it, and may be faſten'd at pleaſure in any Place; two are repreſented at A and B, and the third is like A; the Holes *e*, *c*, *d*, of which *c* only can be ſeen, and which are equal, and diſpos'd in the ſame manner, have a Screw, that the ſolid Pieces F, G, H, I, L, (Pl. XXVIII. Fig. 7.) may be applied to the Curſors, which ſolid Pieces have a Tail with a Screw on it: there is a ſixth like and equal to H; of theſe I ſhall ſpeak ſeparately afterwards, when I explain each of their Uſes. Four of theſe Solids are join'd to the two firſt Curſors as A; to the third, as B, there is added a fifth at *e*. All theſe Solids weigh equally, and are of equal Heights.

739. Plate XXVIII. Fig. 1. To the third Curſor B there is join'd at *f*, by help of two Screws *g*, *g*, going into the Holes *i*, *i*, a Steel Spring O O. Its Plate M M is applied to the Surface of the Curſor, and the Screws, by which it is faſten'd, go thro' the Holes *v*, *v*. The Elaſticity is in the Rings O, O; that is, theſe are elaſtic; and therefore the Plates P, P, when they are mov'd towards the Plate M M, recede from it of their own accord, when they are relax'd.

In the middle of the Plate M M a Piece like a little Tongue *r* S of Steel ſticks, having Teeth on its ſides, and whoſe Head S ſtands out a little beyond P, P, and is perforated.

Plate XXV. Fig. 2. The three Curſors (the two firſt with their Solids, and the third with its Solid and Spring) weigh equally; and the Parts of each are in Æquilibrio in ſuch manner, as, when applied to the Pendulum O Q, not to alter its Situation: the third with its Spring, and Solid join'd to it, is repreſented join'd to the Pendulum at R; and the Pendulum may be mov'd by the Spring; which that it may be done, the iron Plate S is applied to the Board A B C.

I repreſent this Plate by itſelf at A B; to this another B C is join'd at right Angles, of Iron alſo, which is perforated in the middle. 740. Plate XXVI. Fig. 2.

Upon this two ſmaller Copper Plates, *d e*, *n i*, ſtand at right Angles alſo; which ſupport a ſmaller ſteel Plate *f g*; this is repreſented by itſelf at F G, drawn according to its true Bigneſs; it has four Ears *b*, *b*, *b*, *b*, which Screws go thro', when the Plate is faſten'd; in its middle there is an oblong Hole L. The anterior Surface, which is repreſented at *Fig.* 4. is poliſh'd; the hinder Surface is repreſented *Fig.* 3. and is alſo made ſmooth. To this are applied two Catches *p q*, *p q*, moveable about Screws at *p*, *p*, which retain them; the weak Springs faſten'd by the Screw Pins *r*, *r*, bring together the foreſaid Catches, and then the Heads *q*, *q*, come together; and when theſe are ſeparated, if they are left to themſelves they return to the ſame Situation. The hinder Faces of the Catches which we ſee in *Fig.* 3. are plane, but thoſe Parts of the anterior Faces, (*Fig.* 4.) which we diſcover in the Hole, in the middle of the Plate, are convex towards L. Fig. 3. 4.

The Heads *q*, *q*, are ſeparated when the Hammer *m* is depreſs'd; which is mov'd by the Motion of the Tail *v t*; by which Motion the Axis *t s* is turn'd, upon which the Tail of the Hammer *m o* ſtands at right Angles. 741. Fig. 2.

The Plate A B is perforated in *z z* and *y y*, that it may be faſtened by two Screws, made of Braſs, as *h*, going through Holes beyond the Wood, by Help of the outward Screw *q*, the Copper Plate *l* being put between, that the Wood may not be damag'd. 742.

This Plate, as I ſaid, is repreſented at S, and the Heads of the Screws at *b*, *b*; and it may be faſten'd in four other Places by Screws going through the Holes *a*, *a*; *a*, *a*; *a*, *a*; *a*, *a*; the Curſor R is faſten'd at ſuch an Height, that the Tongue of the Spring may anſwer to the Hole of the Plate having the Catches *; the Tongue is thruſt into this Hole, whereby the Catches, by reaſon of the Obliquity of the Teeth in the anterior Part, are a little ſeparated, but immediately return when the firſt Teeth come beyond the Catches; then the Spring bent is join'd to the Plate. If the Tongue be thruſt farther in, the following Teeth come to be uſed, and the Spring is bent more; and by this Method the bending may be varied. 743. Plate XXV. Fig. 2. * 740.

The Plate S muſt now be faſten'd in ſuch manner, that the Spring remaining bent, the Pendulum may be put into a vertical Situation. This is perform'd if to the Pendulum, when being left to itſelf, it has of its own accord acquir'd this Situation, the Index *e f* be firſt applied, and faſten'd, that after the Spring is bent it may determine the Situation of the Pendulum; which may be determin'd alſo by help of the Square *g l o p*. The Place of the Plate S, which is diſcover'd by this Method, is chang'd, if the Spring be bent in another manner.

744. Beſides the three Curſors mention'd *, two Weights alſo P, T,
* 738. are join'd to the Pendulum in the ſame manner, the firſt of which is repreſented at P, (in Plate XXV. Fig. 2.) This firſt Weight is two Pounds, and its Height an Inch and an half; the ſecond T is twice as high, *viz.* three Inches; but it weighs only half a Pound.

These Weights are moveable along the Pendulum, and are faſten-ed at pleaſure by Screws.

Experiment I.

745. The Pendulum above-mention'd * is ſuſpended, the three Cur-
Plate XXIX. ſors are applied to it †, which are faſten'd, at different Heights,
Fig. 1, 2, 3. ſo as to anſwer to the Holes in the great Board, thro' which the
* 735. † 738. Screws *b*, *b*, are put; the Curſor R has the Spring join'd to it.

Fig. 1. R is twenty ſix Inches diſtant from the Point of Suſpenſion, and the Spring is bent ſo, that the firſt Teeth of the Tongue go behind
* 743. the Catches *: the Spring is relax'd by the Motion of the Ham-
† 741. mer †; and, Trials being repeated, it is requir'd where the Index
* 736. muſt be plac'd *, that the Pendulum may come to it, without running againſt it; which may be determin'd to the greateſt Exactneſs.

Fig. 2. The Situation of the lower Curſors is now chang'd; R is plac'd where B was, at the Diſtance of twenty Inches from the Center of Motion; and, *vice verſâ*, B where R was; the Situation of the Plate S being alſo chang'd. The Spring is now bent, as in the foregoing Caſe; and, being relax'd, the Pendulum aſcends to the ſame Height.

Fig. 3. Let R be plac'd at the diſtance of eight Inches from the Center of Motion, in the Place of A, and *vice verſâ*; if other Things be perform'd in the ſame manner, the Height will alſo be the ſame. In theſe three Trials the Spring is bent in the ſame manner, but is relax'd in unequal Times, and performs the ſame Effect each Time, and generates the ſame Force.

746. In theſe we muſt attend well to every thing, the leaſt Neglect diſturbs the Experiment; therefore we have ſeldom a perfect Equa-

lity

lity of the Angles. In my Experiments, here explain'd, the greateſt Angle a little exceeded 44 ſmaller Diviſions of the Braſs Ruler *; which Diviſions are ſcarce equal to the fifteenth Part of an * 737.
Inch; and the Difference of this Angle, with the ſmalleſt Angle of the three, was leſs than one ſuch Diviſion. It once happened to be a ſmaller Difference: but I obtain'd this with difficulty: but the greateſt Difficulty of all is, when the Spring is join'd to the lower End of the Pendulum; in which Caſe a very ſmall thing diſturbs the Effect ſenſibly. But it is never neceſſary to chuſe this Situation of the Spring; the Propoſition is abundantly confirmed by this Experiment of mine *. * 745.

Let us now ſee what belongs immediately to the comparing of 747.
Forces.

It is plain, that all Particles of Matter, mov'd in the ſame manner, are mov'd by equal Forces, therefore *if two Bodies are carried* 748.
with equal Velocities, the Forces are, as the Number of Particles in each, that is, as the Quantities of Matter, or *as the Maſſes*; for by this Name we expreſs the Quantity of Matter in a Body.

If the Maſſes agree, the Forces are, as the Actions, by which different Velocities are communicated to them *. * 700.

But a determin'd Velocity is not ſuddenly communicated to a 749.
Body which acquires Motion *; it ſucceſſively paſſes thro' all the * 710.
leſs Degrees of it, whilſt there is a continued Action on it. Let us ſuppoſe this to be a Preſſure, whoſe Intenſity remains, whilſt its Action continues in ſuch manner, that the immediate Action upon the Body is continually the ſame; which can't be obtain'd, unleſs the preſſing Point be continually carry'd with the ſame Velocity with the Body *; but in this Caſe, equal Degrees of Velocity are * 705
communicated to the Body, in equal Times †; and the Velocity is † 705.
as the Time in which the Preſſure acted upon the Body: we ſuppoſe the Body not to be retain'd, and that the Preſſure produces no other Effect.

Let us ſuppoſe the Line A B to repreſent the Time, in which 750.
the Preſſure acted; B C the Velocity communicated in the Time Plate XXXII. Fig. 1.
A B; D E, parallel to B C, will repreſent the Velocity impreſs'd on the Body in the Time A D.

If we conceive A B divided into innumerable equal Parts, infinitely ſmall; in each of theſe, by reaſon of the equal Times, and the equal Intenſities, the Actions will be as the Velocities *. * 723.

Therefore the Reſiſtances of the Body are in the ſame Ratio †. † 361.
Whence we deduce this general Concluſion, That *a Body, which*

acquires a determinate Degree of Velocity, infinitely ſmall, reſiſts Acceleration in the Ratio of the Velocity, which it has.

752. Whence it follows that *the Action, whereby the Velocity of a Body, which is already mov'd with a finite Velocity, is increas'd by a Degree infinitely ſmall, infinitely exceeds that Action, whereby an equal Degree infinitely ſmall can be communicated to a Body at reſt.*

The Action in a Moment, which anſwers to the Inſtant of Time D, is repreſented by the Line D E; and all Lines repreſent ſimilar Actions in the Moments, anſwering to them; and all together repreſent the whole Action. The Ratio of theſe Lines is not
* 1 El. VI. chang'd, if we allow the ſame Breadth to each *, and indeed that, which is equal to a ſmall Line, whereby one of the infinitely ſmall Moments mention'd is repreſented; but in this Caſe all the Lines together make the Surface A B C; which therefore follows the Ratio
* 700. of the whole Action, and therefore of the Force communicated *.
753. Therefore the Velocities being put as D E, B C, *in the ſame Body, the Forces are* as the Surfaces ADE, A B C; that is, *in a duplicate*
* 19 El. VI. *Ratio, or as the Squares of the Velocities* *.

The Caſe, which we have examin'd, is like that of falling Bodies, and of which we have demonſtrated, by a manner of reaſoning like this, that the Spaces paſs'd thro' in the Fall, meaſur'd from the Beginning of the Fall, are to one another, as the Squares
* 374. of the Velocities acquir'd by falling *; whence we deduce, that
754. *the Force, acquir'd by falling, is as the Height, from which the*
755. *Body fell* *; and hence it follows, that *the Gravity, which, in equal*
* 393. *Times, communicates equal Degrees of Celerity to the Body* †, *does*
† 372. *not communicate equal Degrees of Force to it* *; but that That,
* 751. whereby the Body tends towards the Earth, is mov'd with the
† 749. Body †; whilſt it acts upon the Body in motion, as it would if it
* 371. were at reſt *.

756. It will appear from other Demonſtrations alſo, deduc'd from Principles, which have nothing common to one another, nor to theſe Things from which we have now reaſon'd, that the Forces are to one another in the ſaid duplicate Ratio of the Velocities, when I ſhall treat of compound Motion, and the Reſiſtance of Fluids.

757. *The innate Forces of Bodies in motion*, cannot differ except in reſpect of the Quantity of Matter in the Body, or the Velocity with which it is carry'd; whence I deduce an univerſal Rule for comparing Forces; for they *are in a Ratio compounded of the*
* 748. † 753 *Maſſes* *, *and the Squares of the Velocities* †.

Wherefore

Wherefore *the Forces are equal; if the Squares of the Velocities are inverſly as the Maſſes.* 758.

Such alſo are the Velocities, which by equal Actions, (ſuch are the Relaxations of equal, ſimilar, and equally bent Springs, when the Inertia of the Springs does not differ from the Inertia of the Bodies *), are communicated to unequal Bodies. 759. * 731.

A MACHINE,

Whereby many Experiments, of innate Forces, and the Colliſion of Bodies, are made.

This Machine, which is made of Wood, conſiſts of a vertical Board C B, about three Foot long, and about nine Inches broad, or high. This is ſupported by two Pillars D, D, which are faſten'd by a Croſs-piece put between. To this Board there is join'd a ſmaller horizontal Board A, whoſe anterior Part is ſuſtain'd by the Board itſelf C B, and its hinder Part by the Pillar E, whoſe Diameter is almoſt three Inches and an half, and whoſe Situation is eaſily known, by comparing the Figures. Upon this there is put another Pillar M, and indeed in ſuch manner, that both exactly anſwer to each other, and one is as it were the Continuation of the other. The lower Part N of the Pillar M is divided into two Parts, which go thro' the Holes *x*, *x*, of the horizontal Board A, that they may join themſelves to the upper Part b of the Pillar E, in which Situation it is faſten'd by the Wedge d, going thro' the Holes y. When the Pillar M is thus diſpoſed, the lower Side of the Board P P, made of thinner Wood, join'd to this Pillar, is applied to the Board A; that the Situation of the Pillar may be determin'd more accurately. 760. Plate XXVII. Fig. 1. 2. Plate XXV. Fig. 3.

The three Pillars E, D, D, ſtand upon the horizontal Foot G G H. The whole Machine is ſupported by the three Rollers I, I, I *, that it may be eaſily mov'd; but when we are to uſe it, it is rais'd a little by the Screws *t*, *t*, *t*, and with great Exactneſs is put into a vertical Situation; the Plumb Line Q, applied to the anterior Part of the Pillar M, ſhewing this Situation. 761. * 567.

We repreſent by itſelf the upper Part of the Pillar M, leſs diminiſh'd than in the other Figures. Plate XXV. Fig. 4.

This Part is ſquare, and the Bracket O is join'd to it, that the Iron Ruler S T T may be ſuſtain'd, whoſe End T T reſembles a 762.

Crofs. The Ruler is faften'd by the Iron Screws f, f, which go into the Wood, in which their outward Parts being alfo Iron are faften'd. The Copper Plates *e*, *e*, are perforated; but the Circumference of each Hole, in the upper Part, has an Incifion, that a Thread may conveniently be put into it.

763. Upon the upper fquare Part of the Pillar M, there is put a Piece of Wood m, to whofe anterior Surface the Copper Ruler A A is applied (Plate XXVIII. Fig. 2.) to which are join'd the Screws L, L; thefe go through Holes in the Piece of Wood M, one of which is feen at *x*: the Ruler is faften'd by help of the outward Screws *m*, *m*, the Copper Plates *n*, *n*, being put between, that the Wood mayn't be damag'd. This Ruler is mark'd with the Letters *a a* in *Fig.* 1. *Plate* XXVII.

Plate XXVIII. Fig. 2. To the Extremities of this Ruler are applied the two Cylinders Y, Y; to which another Copper Ruler B B is join'd, which is faften'd by the Screws S S, going through the Holes *d*, *d*, into the Holes *c*, *c*.

764. Along this Ruler are mov'd the fmall fquare Tubes G, G, G, F, F, F, one of which is reprefented by itfelf at O; to the lower Part of each of thefe there is join'd an Hook; and the Tubes may be faften'd at pleafure by a Screw. The upper Plates of the Tubes are greateft; when they are brought together, the Diftance of the Hooks is an Inch and an half.

But that they may on either fide be equally remov'd from the middle of the Ruler B B, in the middle, there is applied to the lower Surface of the Ruler, at *v*, the Copper Plate P, refembling
765. a Crofs, which is faften'd by the Screw *q*. When the fhorter Arms of it are applied to the Ruler, the upper Plates of the middle Tubes, in the middle of the Ruler, meet exactly in the upper
766. Part. But when the longer Arms of the Crofs P, are applied to the Ruler, the middle Hooks are feparated, as much as is requir'd in many Experiments, as we fhall afterwards fee.

767. Plate XXVIII. Fig. 3. There is another Copper Ruler C C, like the Ruler A A, to which another alfo D D is join'd, with its Tubes and Hooks; but it will eafily appear what Difference there is between thefe Rulers and the foregoing, by comparing *Fig.* 2. with this, in which a Tube is reprefented by itfelf at R.

768. The Ruler C C is join'd to the Iron defcrib'd above *, and indeed to the lower Surface of the Part T T, as we fee it in *Fig.* 1. *Plate* 27.

* 762.

The

The Ruler itself is *c c*, fasten'd by the Screws *e e*. This Ruler, and that join'd to it *d d*, are parallel to *a a*, and *b b*, describ'd above *, and all are parallel to the Plane C B; the Hooks of the Ruler *b b* answer to the Hooks of *d d*; that is, in both Rulers they are disposed in the same manner. Plate XXVII. Fig. 1. * 763.

All the Hooks are in the same horizontal Plane, and a Line, which would pass thro' two corresponding Hooks, would be perpendicular to the Surface B C, if this be conceiv'd to be continued.

769 The Bodies *s* and *r*, with which the Experiments are made, consist of Copper Rectangles, one of which is represented at A B. This is suspended by Threads, put into the Incisions *c*, *c*, *d*, *d*. The Distance between *c*, *c*, or *d*, *d*, is three Inches, that it may answer to the Distance between the first and third Hook, when three are join'd *; but the manner of its Suspension is manifest enough, by comparing *Fig.* 1. *Plate* XXVII. and *Fig.* 3. *Plate* XXV. with one another. The external Threads are sustain'd by the Hooks *i*, *i*, *h*, *h*, and pass through the Holes *e*, *e*, (Plate XXV. Fig. 4.) that they may be brought to the little Pegs, or Wedges *n*, *n*; other Threads descend directly from their Hooks to the little Pegs *m*, *m*; by turning of the little Wedges the Rectangles are brought to the Situation desir'd; which that it may be done more conveniently, as there are many Threads, they are all of different Colours. The Threads are requir'd to be small and strong enough, that they may be able to bear the Weights to be applied to them, therefore I make use of silken ones; and prefer those which are wove to those which are twisted. Plate XXIV. Fig. 4. * 764. Plate XXV. Fig. 1.

770. In the middle of the anterior Surface of the Rectangle A B, there is a Cavity *e*, having a Screw, and to which the truncated Cone *f* answers, that it may be the deeper; of which Cone we shall presently see another Use also. Plate XXVIII. Fig. 4.

771. To the same anterior Surface of the Rectangle various Bodies are applied, of which I shall speak separately, when they shall come to be used in the Experiments: each of these Bodies is equally prominent; they weigh equally also; that the Weight of the Rectangle may be always determinate, and the same. Fig. 7. 9 10.

772. There are two such Rectangles, which differ in this only; that which is here represented, besides the Cavity *e*, which we have mention'd, has two small Holes *i*, *i*, in its anterior Surface, into which Screws are put, as we shall afterwards see.

The

773. The ſecond Rectangle, like the firſt, equal to it, and of the ſame Weight, has alſo two ſmall Holes in its anterior Surface, not above and below *e*, as *i*, *i*, but at the Sides of this Cavity *e*.

774. Fig 5. The Weight of the Rectangle, with the Body join'd to it, is doubled, tripled, or quadrupled, by the Copper Cylinder T, T, or T being join'd to it; in this Caſe the Cone f is put into the Cavity *y*, which it fits; and the Cylinder is faſten'd by the Screw *g*, going into a ſmall Hole, as *x*, into its oppoſite End: if the Weight is to be increas'd more; for Example, ſix, eight, nine, or ſixteen times, ſolid Pieces of Lead V, V, V, or X, are applied, which are faſten'd in the ſame manner.

775. Plate XXVII. Fig. 1. * 737. In the Motion of the Rectangle its Velocity is determin'd by the Diviſions of the Ruler X V, or Y Z, as has been explain'd in another Machine *; but theſe Things are to be obſerv'd; that the Slits, thro' which the Screws join'd to the Rulers go, muſt be longer, at leaſt, nine or ten Inches. We do not want the Square there made uſe of; becauſe the correſponding Threads, the anterior or poſterior, are in the ſame Plane perpendicular to the Surface C B, (we call thoſe anterior, which are leſs diſtant from the middle of this Surface) and by directing the Sight along both Threads, we diſcover the Point to which theſe anſwer in the Ruler itſelf.

776. The Indices, by which we determine the Heights, from which Bodies are let down in the Experiments; or ſhew thoſe, to which they aſcend, are applied to the copper Ruler R R, plac'd on the Table A longwiſe, and which is but a little diſtant from the anterior Extremity of this Table.

777. Plate XXVIII. Fig 6. The ſeparate Figure of the Indices ſufficiently ſhews, how they are mov'd along the Ruler; there are four Inciſions in *e*, *e*, &c. that the Box *a b*, which receives the Ruler, may be ſtraiten'd at its Extremities, whereby the Index, by reaſon of the Elaſticity of the Copper, is faſten'd; yet ſo, that its Motion along the Ruler is not hinder'd.

Two larger Indices are requir'd, which are repreſented at O and Q; theſe differ only in their Conjunction with the Boxes *a b*; beſides theſe two, three ſmaller, as P, are requir'd. Upon the greater are mov'd the Curſors *c*, *c*, which are faſten'd at pleaſure by the Screws *d*, *d* *.

* 736.

778. Plate XXVIII. Fig. 1. 4. * 769. † 739.

Experiment 2.

To the copper Rectangle A B *, the Spring O O is join'd †, by Screws like thoſe, which are repreſented at *g g*, (*Plate* XXVI. *Fig.*

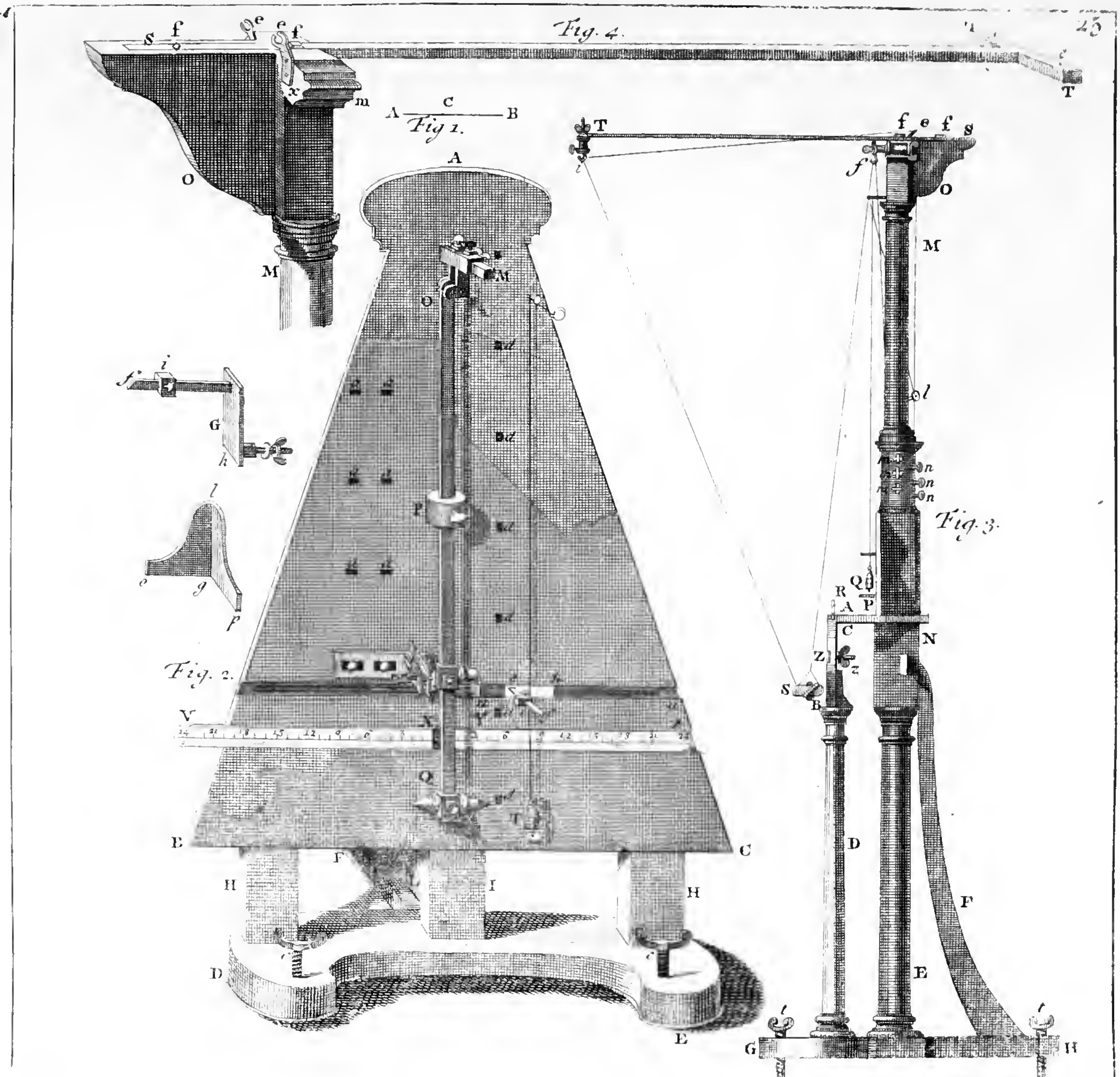
Fig. 4.
Fig. 1.
Fig. 2.
Fig. 3.

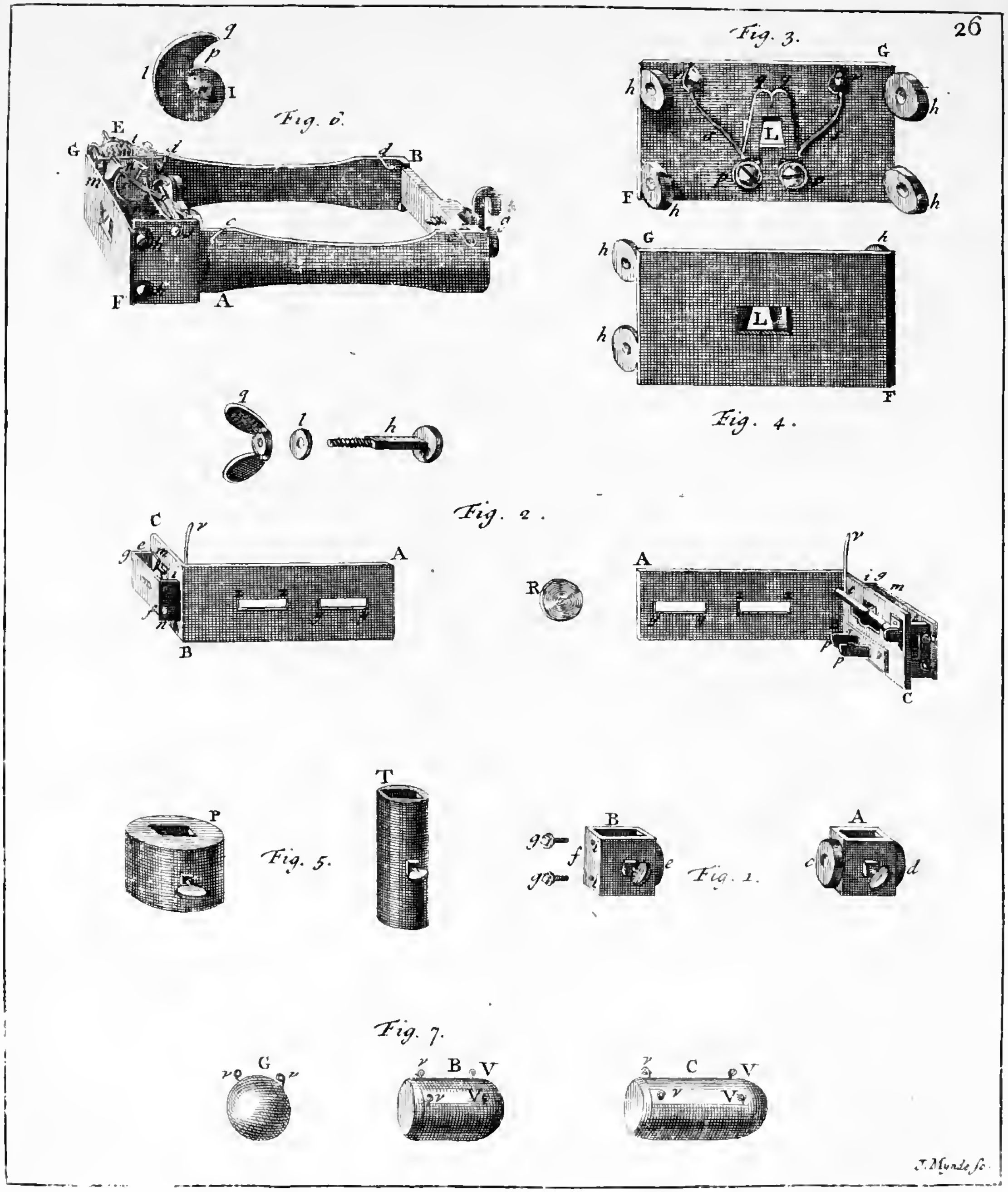
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Fig. 6.
Fig. 3.
Fig. 4.
Fig. 2.
Fig. 5.
Fig. 1.
Fig. 7.
J. Mynde sc.

Fig. 1.) thefe go thro' the Holes *v*, *v*, into the Holes *i*, *i*, which contain a Spiral, that they may receive the Screw.

It is neceffary, that the Rectangle fhould have its determinate Weight *; if any thing is wanting, this is made up, by putting a thin copper Plate between, which is alfo perforated, that the faid Screws may go thro' it. * 771.

The Rectangle is now fufpended, inftead of that which is reprefented at *s*; but as in this Cafe fomething peculiar is to be obferv'd in the Difpofition of the anterior Threads, we reprefent this feparately at g. Plate XXVII. Fig. 1.

To the Board B C we apply the iron Plate, of which above *; it is faften'd by Screws †, which go thro' the Holes n, n: the Plate, which has Catches, is turn'd towards the Spring; and the Rectangle, to which this is join'd, is to be difpos'd in fuch manner, by turning the little Wedges, *m*, *m*, *n*, *n*, as to be horizontal, and to have its longer Sides parallel to the Surface B C; and to be at that Diftance from this Surface, and at fuch an Height, that the little Tongue of the Spring may anfwer to the Hole in the middle of the Plate which has the Catches. * 740. † 742.

The Plate is remov'd a little, that the Body may be fufpended freely; and, when it is at reft, the Sight is directed along the hinder Threads, and the Ruler Y Z is mov'd, till its End Y anfwers to the Threads. Then the Plate being brought to a, the Tongue of the Spring is put into its Hole, by which the Spring is bent, and join'd to the Plate *, which is fo difpos'd, and faften'd, that the Threads may again anfwer to the End of the Ruler. * 743.

779. In this Situation I now reprefent it; *f g* is the Plate with Catches, which being join'd to the greater Plate S, is faften'd to it by the Screws *b*, *b* *. By preffing the Tail *v* of the Hammer *m*, it is deprefs'd, and the Spring is relax'd †, which is driven forwards together with the Rectangle join'd to it. The Velocity communicated is difcover'd by making Trials; the greater Index * is difpos'd at that Diftance which we have judg'd the Threads will be rais'd to at firft; by a fecond Trial the Situation is corrected, till we at laft come to this, that the Thread may come to the Index, without running againft it; we had the Velocity 16,8. the laft Number expreffes the fmaller Divifions *. Plate XXIX. Fig. 4. * 743. † 741. * 777. * 737. 775.

780. All Things remaining, the Cylinder T is put into the Rectangle, that the Weight of the Body mov'd may be quadruple *: other Things are perform'd as in the foregoing Trials, and the Velocity is difcover'd, which is half of the former, namely 8,4. Plate XXIX. Fig. 5. * 774.

781. The Cylinder T being taken away, the leaden Weight V muſt
Plate XXIX. be applied, whereby the Maſs becomes nine times the former *,
Fig. 6. and the Velocity is diſcover'd to be 5,6. Which is a third Part of
* 774. the firſt.

If the Maſs applied be ſixteen times the firſt, the Velocity is equal to 4,2.

782. In all theſe Caſes the Action, which communicates Motion, is the Relaxation of the ſame Spring, bent in the ſame manner, and therefore the Action is the ſame; and the Squares of the Velocities are inverſly as the Maſſes; that is, the Product of the Maſs by the Square of the Velocity is always the ſame.

783. Some Philoſophers are of Opinion, that the Action of a Spring is not the ſame, if the Times, in which it is relax'd, are not
* 731. equal; I have demonſtrated that the Thing is not ſo *; and con-
† 745. firm'd this Demonſtration by an Experiment †. But I ſhall now conſider the Thing in another manner.

I ſhall move different Bodies by the ſame Spring, bent after the ſame manner, and relax'd in equal Times, and we ſhall ſee that the Effect will agree with the Propoſition N. 758. and that the Product of the Maſs by the Square of the Velocity is every Time the ſame.

784. It will appear, that a Spring, bent in the ſame manner, is relax'd in equal Times, if it communicates the ſame Velocity, every time, to the Point, to which it is applied. For the Part of the Spring which is relax'd, is mov'd with the ſame Velocity with the Point to which it is applied; as we now ſuppoſe the Velocity of this Point, and therefore the Velocity of the Spring itſelf, at the End of the Relaxation, to be the ſame every time, it will be the ſame alſo in the ſame Degrees of Expanſion: for the Relaxation is made in all Caſes according to the ſame Laws; ſo that, in equal Moments, the ſame ſmall Spaces are paſs'd thro', and the whole Relaxation, every time, is made in the ſame Time.

Experiment 3.

785. This is demonſtrated by the ſame Machine, as the firſt Experi-
Plate XXXI. ment in this Chapter *. The Weight T is applied to the Pendu-
Fig. 1. lum †, which is equal to half a Pound *; it may be applied at
* 745. any Diſtance from the Center of Suſpenſion; let this be 30 Inches,
† 735. meaſur'd from the middle Point of the Weight: the Curſor with
* 744. the Spring is faſten'd at R; another Curſor A, with two Solids *,
* 733. is join'd to the lower End of the Pendulum. The Spring is bent and

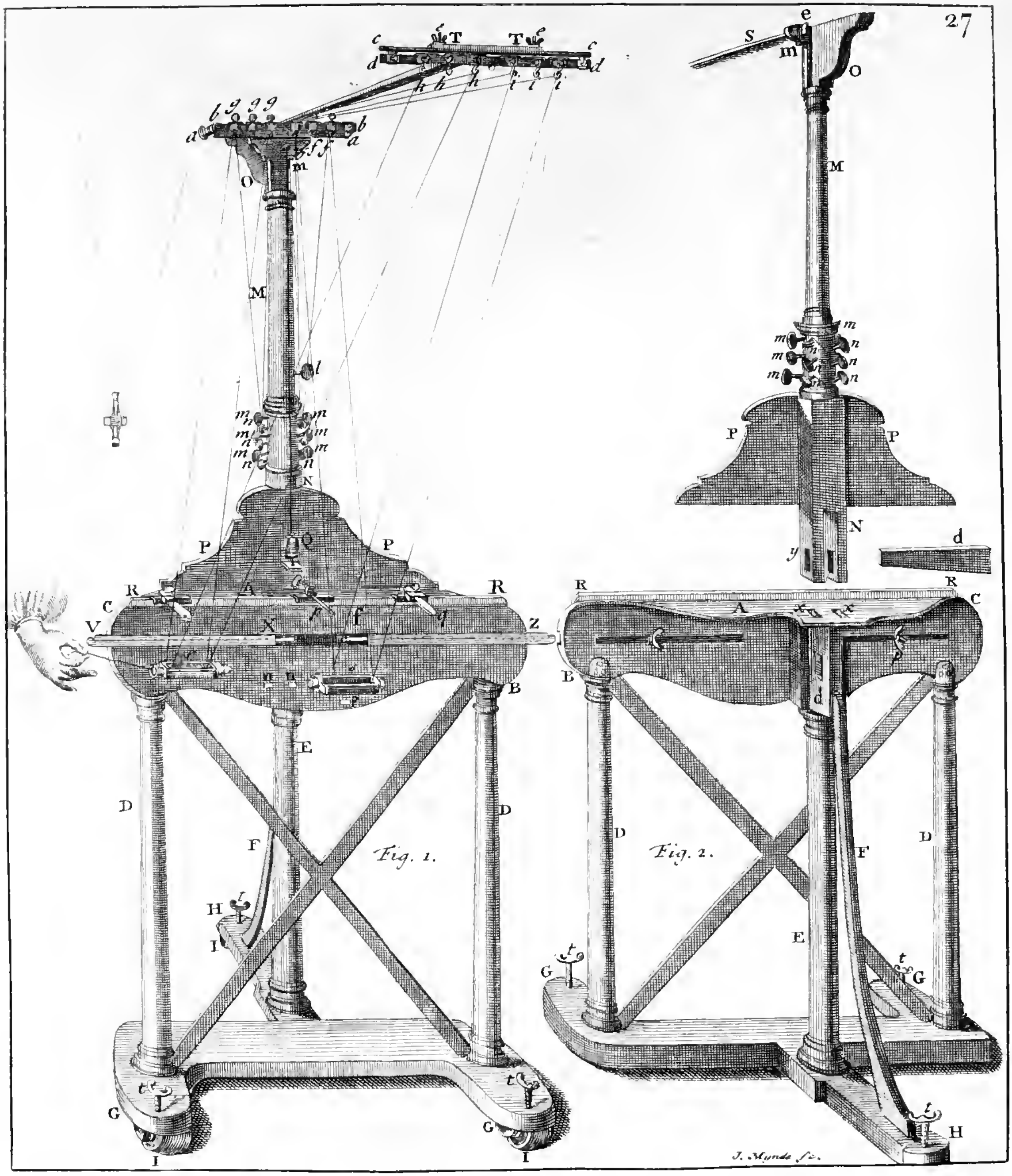
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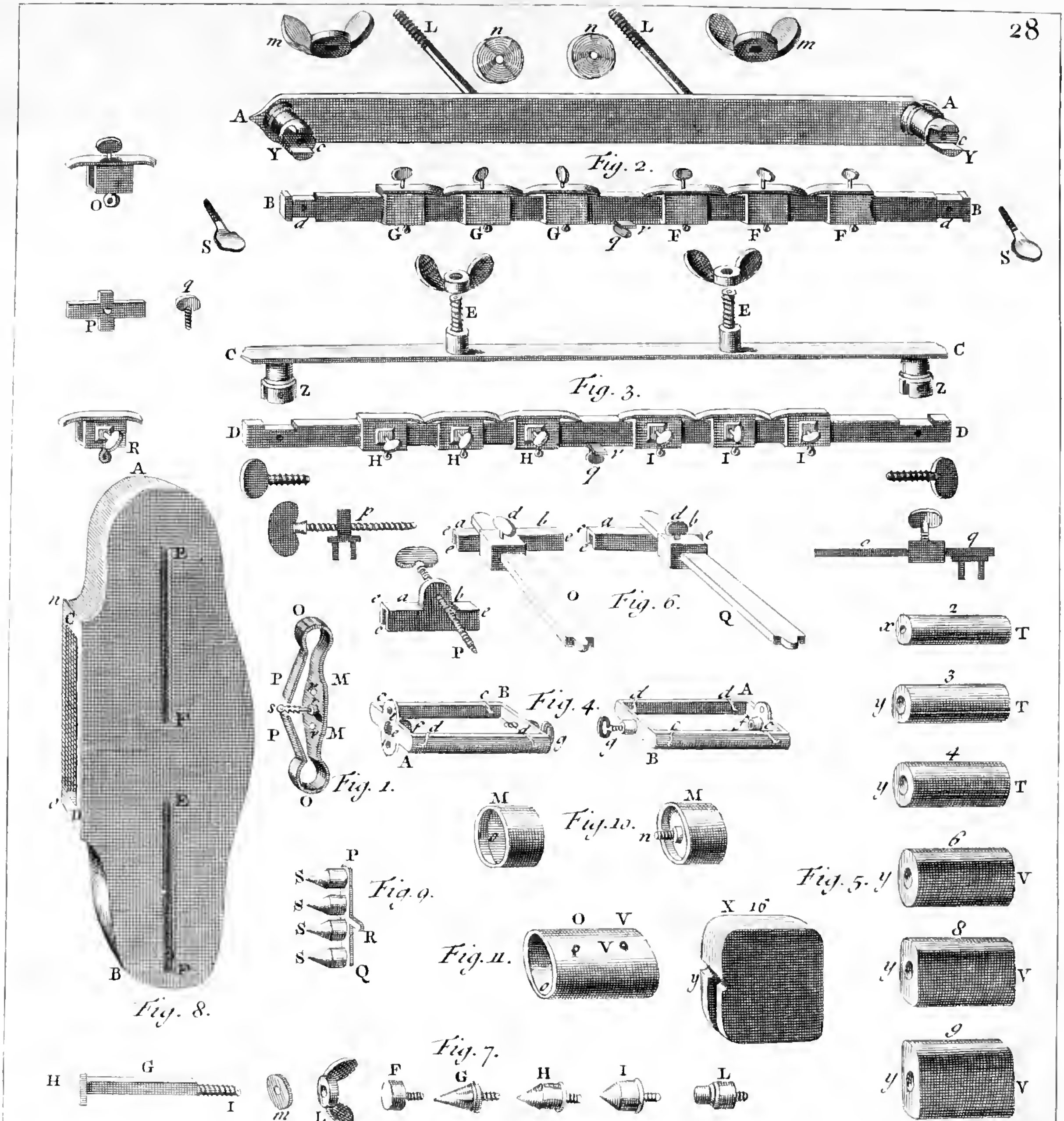
Fig. 1.
Fig. 2.
J. Mynde sc.



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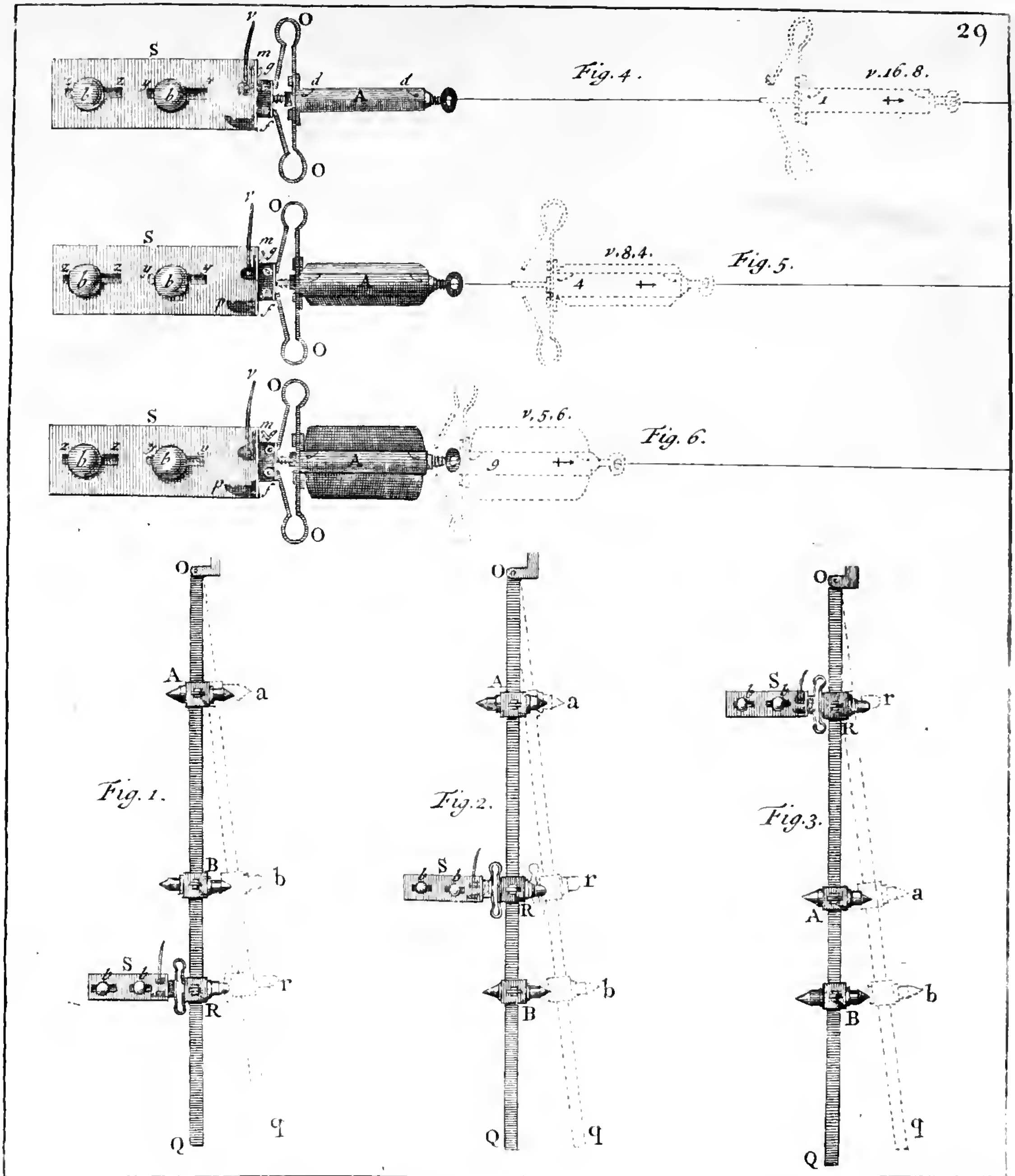
Fig. 4.
v. 16. 8.
Fig. 5.
v. 8. 4.
Fig. 6.
v. 5. 6.
Fig. 1.
Fig. 2.
Fig. 3.

and then relax'd, and the Angle is measur'd as was said in the first Experiment *; I had an Angle of 40,5 Parts. * 745.

The Weight T being taken away, we applied the Weight P †, 786.
at the Distance of fifteen Inches from the Center of Suspension; Plate XXXI.
and the rest being manag'd as in the foregoing Case, we had the Fig. 2.
Angle 37,8. † 744

When the Distances of the Weights T and P are different, but in the same Ratio of 2 to 1, the Angles are different, and in a different Ratio from that now discover'd; but the same Conclusion must be deduc'd from the Experiment *, because the angular Velocities are always equal to one another. We shall demonstrate by Computation, and Experiment, that they were such in the present Experiment; we shall give the Computation in the Scholia, the Experiment here. * 788. 789.

Experiment 4.

To the Pendulum we join the Weight T, the Cursors R, and A, 787.
as in the first Case of the foregoing Experiment. Instead of the Plate XXXI.
Solid join'd to R, we make use of another F (Plate XXVIII. Fig. Fig. 3.
7.); and both the Solids join'd to A are remov'd.

Between the Plates, between which the Ruler O Q is suspended *, another small Plate is horizontally dispos'd, in which there is a narrow Hole; thro' which the Thread is put, which sustains the Ball G, by which the simple Pendulum is made. The Weight of this Ball is equal to the Weight of the two Solids remov'd from the Cursor, that the Ball, join'd to the Cursor A, may weigh as much as this Cursor with its Solids, which was used in the foregoing Experiment. Moreover, the Length of the simple Pendulum *o* G is such, that the Center of the Ball may answer to the middle Point of the Cursor A; and the Ball is suspended in such manner, that being join'd to the Cursor A, the Thread may be parallel to the Ruler O Q. * 735.

This Pendulum O Q is rais'd, the Ball G is join'd to it, and it is let go from the Height of 40,5 Divisions: the Center of Oscillation of the Pendulum A O is between A and O; therefore it would descend in a shorter time than the simple Pendulum *o* G *, and it drives forward G *o* in its Motion; so that the same Quantity of Matter, descends in the same manner, as would be mov'd in a like Descent of the Pendulum O Q (Fig. 1.): for this reason, when *a* and *g* have got to the lowest Point, they have the Velocity, which, in the first Case of Experiment third *, was communicated to the * 418. * 785.

Curſor A. But *g* has now a greater Velocity, than if it had only deſcended from the Height 40,5; for it has been accelerated, therefore it will aſcend to a greater Height; upon this account it is ſeparated from *a*, and aſcends to *g* to the Height of 46 Diviſions. But that we may meaſure this Angle, the Ruler Y Z (Plate XXV. Fig. 2.) is diſpos'd in ſuch manner, that its End Y anſwers to the Thread of the ſimple Pendulum when it is at reſt.

Therefore we ſee that the Velocity communicated to the Curſor A, in the firſt Caſe of Experiment, third is that, by which the Body G, of the Pendulum *o* G, can aſcend to the Height 46.

Plate XXXI. Fig. 4. After the ſame manner, we examine the ſecond Caſe of the ſame third Experiment *; we let go the Pendulum O Q with the Ball G, from the Height 37,8, and it aſcends to the Height 46, as in the foregoing Caſe.

* 786.

788. Therefore the Spring was relax'd in the ſame Time in both Caſes, whilſt it communicated the ſame Velocity to the Pendulum *: the Angles in Experiment third were unequal, becauſe the Pendulum was not retarded in the ſame manner in both Caſes; but the Retardation does not relate to the Action of the Spring, that happens after the Spring is ſeparated from the Plate fix'd.

* 784.

Therefore the Velocity, impreſs'd upon the Body T in the firſt Caſe, is to the Velocity, which was communicated to the Body P in the ſecond Caſe, as 2 to 1; for ſo the Diſtances from the Center of Motion were; which alſo, if every Body be ſuppos'd divided along-wiſe into an equal Number of Parts, may be referr'd to the correſponding Parts; becauſe the Heights of the Bodies are in the ſame Ratio of 2 to 1; but the Maſſes are as 1 to 4; that is, inverſly as the Squares of the Velocities.

789. But it immediately appears, to one who conſiders the Thing, that theſe Velocities were communicated by equal Actions. The Spring, in each Caſe, produc'd two Effects. 1. It communicated Motion to the Ruler and Curſors. 2. Motion to the Body. The Ruler, with the Curſors was, every time, projected with the ſame Velocity; therefore the Parts of the Actions of the Spring, by which this was effected, were equal, and ſince the whole Actions of the Spring in both Caſes were entirely ſimilar, and equal; the Bodies themſelves T, and P, were alſo mov'd by equal Parts of the Actions.

From this Propoſition that the Forces are equal, when the Squares of the Velocities are in an inverſe Ratio of the Maſſes *, which we have confirm'd by theſe Experiments, we eaſily deduce, That

* 758.

That *the Forces acquir'd by falling are* alſo *equal, if the Heights are inverſly as the Maſſes* *. 790. * 754.

If two Bodies are mov'd with Velocities, which are inverſly as the Maſſes, the Forces will be in the ſame inverſe Ratio of the Maſſes, that is, *as the Velocities*. For, in this Caſe, the Product of the Velocity by the Maſs is the ſame for each Body *. Let the Bodies be A and B; if this Product be multiplied by the Velocity of the Body A, the Force of this Body will be given *; the Force of the Body B is had, by multiplying the ſame Product by the Velocity of B; therefore the Forces are as theſe very Velocities *. 791. * 12 El. VI. * 757. * 2 El. VI.

SCHOLIUM I.

Of the Forces of Pendulums.

WHAT I have ſaid in this Chapter of the Actions of Springs, I ſhall not illuſtrate any further; becauſe all, that might be added, concerning the unequal Bendings of the ſame Spring, and of comparing, and determining the Times of the Relaxations in different Circumſtances, belongs to the laſt Chapter of this Book. In this Scholium I ſhall illuſtrate what relates to the Forces of Pendulums, when they are mov'd, whether they are ſimple, or compound; but we conſider the Force only in the loweſt Place, that is, where the Velocity is greater, than in the other Points of the ſame Vibration. We alſo ſuppoſe that we have to do with ſmall Vibrations. 792.

The Force of a Body is as its Maſs, which is as the Weight *, and as the Square of the Velocity †. Thence it follows, that *the Force of a ſimple Pendulum follows the Ratio of the Weight, of the Length, of the Square of the Angle*, and the Gravity which acts upon the Body *. But as we treat of the Motion of Pendulums in the ſame Place, we neglect the laſt Ratio. * 156. 793. † 157. * 445.

If we conſider the compound Pendulum, the Difficulty is greater, and that this ſame Rule may be applied to ſuch a Pendulum; inſtead of the Weight the Sum of the Weights is to be uſed, and inſtead of the Length the Diſtance between the Point of Suſpenſion and Center of Gravity muſt be taken; but we don't attend to the Center of Oſcillation, which determines the Length of the Pendulum in other Caſes *; for the Center of Gravity and the Center of Oſcillation coincide in the ſimple Pendulum: wherefore we muſt otherwiſe determine, what is to be uſed in the compound Pendulum. But ſome Things muſt be premis'd, before the Demonſtration of this Propoſition can be made appear. 794. * 451.

Let us ſuppoſe, in the compound Pendulum, every Particle of Matter to be multiplied by the Square of its Diſtance from the Center of Suſpenſion; we call the Sum of all the Products P *d d*. 795.

We ſuppoſe alſo every Particle of Matter to be multiplied by its Diſtance from the ſame Center: the Sum of the Products is equal to the Product of the Sum of all the Weights by the Diſtance of their Center of Gravity from the Center of Suſpenſion *. We call this Product Cc. We call the Sum of the Weights C, and the Diſtance of their Center of Gravity c.

* 223.

We call the Diſtance of the Center of Oſcillation from the Center of Suſpenſion o.

* 415. The Angle of the Pendulum * is call'd a.

† 437. The Velocity of the Pendulum †, which in the compound Pendulum is the Velocity of the Center of Oſcillation, is call'd v.

The angular Velocity is b.

The whole Force of the whole Pendulum; that is, the Sum of the Forces of all the Parts of the Pendulum, when it has the greateſt Velocity in a Vibration, ſhall be call'd e.

We have now the following Equations.

796. * 474. $\frac{Pdd}{Cc} = o$ *.

797. † 447. $\frac{a}{\sqrt{o}} = b$; or $aa = obb$ †.

798. * 445. $aao = vv$ *; therefore $ob = v$ †.

† 797. * 757. * 448. That we may determine the Force of the Pendulum, we muſt multiply every Particle of Matter by the Square of its Velocity, and the Sum of the Products will expreſs the Force *. The Velocity of every Point follows the Ratio of its Diſtance from the Center of Suſpenſion, and the Ratio of the angular Velocity *; therefore each of the Points muſt be multiplied by the Square of its Diſtance, and the Sum muſt be multiplied by the Square of the angular Velocity; and this Product will expreſs the Force itſelf.

* 795. † 797. * 796. Therefore $Pdd \times bb = e$ *. Whence, inſtead of bb putting $\frac{aa}{o}$ †, and then the Value inſtead of o *, we deduce theſe other Equations $Cc \times aa$

779. $= e$; and $aa = \frac{e}{Cc}$, the firſt of which agrees with thoſe mentioned

* 793. above *.

800. From this Equation we alſo deduce, That *the Force of a Pendulum follows the Proportion of the Product of the Sum of the Weights by the Height, from which their common Center of Gravity deſcends, or to which it aſcends*; for this Height is as the Diſtance of this Center from the Point of Suſpenſion, this ſame Height is alſo as the Square of the Angle; for, *cæteris paribus*, the Velocity of the Point is as the Angle *, and the Square of the

* 442. * 374. 393. Velocity is as the Height †.

SCHOLIUM

SCHOLIUM II.

Computations of the Motions of the compound Pendulum, used in the 1. 3. *and* 4. *Experiments of this Chapter.*

WHEN we are to make Computations of these Motions; the Weights and Measures of the Parts ought to be explor'd in the first Place; afterwards some Things in general are to be determin'd by Computation. We express the Weights by Ounces; Inches give the Measure of the Lengths; and the smaller Divisions of the Brass Rulers shew the Magnitudes of the Angles *. 801.

* 775.

The Weight of the Iron Ruler O Q (Plate XXV. Fig. 2.) † is 55,5; its Length 36,14; Length below the Axis 35,92 and the Weight of this Part 55,16. 802.

† 735.

The Weight of the Cursor (Plate XXVI. Fig. 1.) *, without the Solids is 5; with them 7,5. The Height of the Cursor is 1,5.

* 738.

The Weight of the Spring, O O (Plate XXVIII. Fig. 1.) † is 1,25. Its Height 4.

† 739.

The Weight of T (Plate XXVI. Fig. 5.) is equal to 8; and P weighs 32 Ounces †. The Height of the first is 3; and of the second 1,5.

† 744.

In many Computations, *all the heavy Points must be multiplied by the Squares of their Distances from the Center of Motion.* I shall determine *the Sum of all the Products, for the Iron Ruler* O Q (Plate XXV. Fig. 2.); because this Sum will afterwards come to be of use. We do not attend to the Part above the Axis, but only consider the Length 35,92. *; the Error arising thence is entirely insensible. 803.

* 802.

If this Length be call'd l, the Sum which we seek will be $\frac{1}{3} l^3$ †; but l is once used for the Weight of the Ruler; therefore the Number sought is equal to a third Part of the Weight, multiplied by the Square of the Length; that is, it *is equal* to 23740.

† 480.

If the Bodies, which are applied to this Pendulum, should not take up a sensible Space along its Length, the Square of the Distance from the Point of Suspension, would be to be multiply'd by the whole Weight of the Body applied; but as we make use of such Bodies, whose Height does not seem fit to be neglected, we must now examine, what follows from this Height; for all the Parts are not equally distant from the Center of Motion. If we make a Computation, we discover that *to the Product, of the Square of the Distance of the Center of Gravity of the Body by its Weight, a Supplement must be added*; which is the same, whatsoever that Distance is; but is small for all the Bodies, which we have made use of in the Experiments. 804. 805.

Let l be the Distance of the Center of Gravity of the Body from the Point of Suspension; the Height of the Body $2a$; we suppose the Body

to

to be continued uniformly, that it may extend itself to the Center of Suspension; its whole Length will then be $l+a$; and the Length of the Body added $l-a$. I require the Sum of the Products for each of these Bodies, and the less being substracted from the greater, there remains the Sum, that relates to the Body itself;

† 480. The Sums are † $\begin{cases} \frac{1}{3}l^3 + all + aal + \frac{1}{3}a^3. \\ \frac{1}{3}l^3 - all + aal - \frac{1}{3}a^3. \end{cases}$

The Difference is $+2all \quad + \frac{2}{3}a^3.$

In this Computation $2a$ expresses the Weight of the Body applied; therefore $2all$ is the Product of the Weight by the Square of the Distance, to which, whatsoever the Distance l be, we should always add the Supplement $\frac{2}{3}a^3$, *which is equal to the Product of the third Part of the Weight, of the Body applied, by the Square of half its Height.*

806. But these Supplements, if determin'd for the Bodies, which we use,
* 803. are found to be so small, in respect of the Number now discover'd *, that they may be neglected without any sensible Error; for the greatest does not exceed 6.

807. In many Computations also there is requir'd the Product *of the Weight of the Iron Ruler*, often mention'd, *by the Distance between the Centers of Suspension and Gravity*; therefore I will take notice of this Product also.

The Center of Gravity of the Ruler is in its middle, and is distant from its End 18,07. The Distance of the upper Extreme from the Center of Suspension is 0,22; therefore the Distance between these two Centers is
* 802. 17,85, which should be multiplied by the Weight 55,5 *, *the Product is* 991.

We proceed now to peculiar Problems.

808. We discover by one Experiment the Force, which a Spring, bent in a certain manner, whilst it is relax'd, communicates to a Pendulum. This
* 799. is equal to $Cc \times aa$ *.
† 785. Let us put the first Case of the third Experiment of this Chapter †.

Two Cursors are applied at the Distances 35 and 26 from the Point of
802. Suspension; the Weight of each is 7,5 *; the Products of the Weights by the Distances are equal to 262, and 195. The Distance of the leaden
† 744. Weight applied is 30; and this weighs 8 †; the Product is 240. I collect
* 807. these Products into one Sum, and add 991 *; and I have Cc †; whose
† 223. 795. Value therefore is 1688. The Angle a in the Experiment is discover'd to be 40,5 Parts. The Square of the Angle is 1640, whose Product by 1688 gives the Force $e = Cc \times aa = 2763820$.

809. In this Measure we express by Unity the Force, which the simple Pendulum would acquire, if each of these, the Weight, Length, and Angle, were denoted by Unity. Then the Weight would be equal to one Ounce; the Length would be one Inch; and the Angle, in my Machine, would
* 737. answer to one smaller Division of the divided Ruler *, and would be 0 Degrees 7′ 24″. But the Force which such a Pendulum wou'd acquire wou'd be equal to that, which one Ounce acquires, by falling from the Height
810. of 0,000002354 Inches: and *the whole Force, which the Spring communicates*

cates to the Pendulum, coincides with that, which Gravity impreſſes upon one
Ounce, when it deſcends ſix Inches and an half *. * 754.

I will now give the Computations of the other Angles, mention'd in the
Experiments of this Chapter.

I will conſider the ſecond Caſe of the third Experiment * firſt, and de- 811.
termine the Angle. * 786.

The Numbers 262, 195, and 991, mention'd above †, come to be of † 808.
uſe here alſo; but we make uſe of another, inſtead of that, which the
Leaden Weight gave; becauſe this Weight was chang'd, and weighs in
this Caſe 32; the Diſtance, by which it muſt be multiplied, is 15; the
Product 480, I add to the other three, and have $Cc = 1928$. By this
Number I divide the Force, diſcover'd by the foregoing Computation,
$Cc \times aa = 2768320$ *: and $aa = 1436$, whoſe ſquare Root 37,9 agrees * 808.
as to Senſe with the Meaſure of the Angle, which we had in the Ex-
periment.

After the ſame manner we proceed in the Computation of the firſt Ex- 812.
periment *; we make uſe of three Curſors applied at the Diſtances from the * 745.
Point of Suſpenſion 8, 20, and 26. Each of theſe is multiplied by the
Weight of the Curſor, and the Sum is 405. I add 991 *, and $Cc =$ * 807.
1396; by this Number I divide the Force 2768320, and have the Square
of the Angle 1983; whoſe Root 44,5 ſomewhat exceeds the Angle diſ-
cover'd in the Experiment; but the Difference is ſmall.

In Experiment fourth * we demonſtrated, that the angular Velocity was 813.
the ſame in each Agitation of the Pendulum in the third Experiment; this * 787.
ſame thing will now appear by Computation alſo.

From the Æquations $\frac{Pdd}{Cc} = o$ * and $\frac{a}{\sqrt{o}} = b$ † we deduce the angular * 796.
Velocity b, from the given Angle a: by putting the Value, for e, in the † 797.
ſecond Æquation, we have, $\frac{Cc \times aa}{Pdd} = bb$. But in the Caſes, which we
examine, the Product Pdd was the ſame; for the Parts of it, which relate
to the Iron Ruler, and Curſors, are not varied; the other Parts alſo,
which relate to the Leaden Weights, do not differ, $30 \times 30 \times 8 = 15 \times 15 \times 32$.
Therefore bb is as $Cc \times aa$. In the firſt Caſe $Cc = 1688$, and $aa = 40,$
$5 \times 40, 5 = 1640$ *; the Product of which Numbers coincides with the * 808.
Product of thoſe correſponding in the ſecond Caſe, $Cc = 1928$, and aa
$= 1436$ *; as we ſaw in the foregoing Computations †. Therefore the * 811.
angular Velocities, which are in a ſubduplicate Ratio of theſe Products, † 807. 810.
are equal.

By Computation alſo we eaſily diſcover the Angle of the ſimple Pendu- 814.
lum in Experiment fourth *, from the one, or other Angle, of Experi- * 787.
ment third, being given; that is, from the given Height, from which the
compound Pendulum in Experiment fourth is let go; but the Center of
Oſcillation of this Pendulum muſt firſt be determin'd.

* 474. 795. The Diſtance of this Center from the Point of Suſpenſion is $\frac{Pdd}{Cc}$ *.

The Numerator of this Fraction conſiſts of four Parts. The firſt has relation to the Iron Ruler, and is 23740 *. The ſecond is referr'd to the lower Curſor, and is $35 \times 35 \times 7{,}5 = 9187$. The third relates to the Curſor with the Spring, and is $26 \times 26 \times 7{,}5 = 5070$. Laſtly, the fourth is in the firſt Caſe $30 \times 30 \times 8, = 7200$; in the ſecond Caſe $15 \times 15 \times 32, = 7200$; which Products are equal.

* 803.

I collect into one Sum 23740; 9187; 5070; and 7200; and I have $Pdd = 45197$.

* 808. In the firſt Caſe $Cc = 1688$ *; in the ſecond Caſe $Cc = 1928$ †.

† 811. Therefore, in the firſt Caſe, the Diſtance of the Center of Oſcillation from the Center of Suſpenſion is 26,78.

In the ſecond Caſe 23,44.

Now 26,78 is to the Length of the ſimple Pendulum 35, as 40,5x40,5
* 450. 451. is to the Square of the Angle ſought 2134 *; whoſe ſquare Root ſcarce exceeds 46.

815. We have the ſame Angle 46, if we make a Computation for the ſecond Caſe; which is again a Confirmation that every Point of the Pendulum, in each Caſe, had the ſame Velocity.

816. In what follows we ſhall have two Experiments, in which three Curſors will be to be applied to the Pendulum, and nothing beſides, as in the firſt
* 745. Experiment of this Chapter *. But in the laſt of thoſe Experiments the middle Curſor will be to be diſpos'd in ſuch manner, that its middle Point may coincide with the Center of Oſcillation of the whole Pendulum. We require the Diſpoſition of the Curſors.

This Problem is indeterminate; but, among poſſible Caſes, we ſhould chuſe ſuch, as anſwer the Deſign of the Experiment; for this reaſon we ſuppoſe, three Curſors being applied, that the Center of Oſcillation coincides with this Center, when all the Curſors are remov'd; that is, that the Diſtance of the Center of Oſcillation from the Point of Suſpenſion, (a
* 427. 802. ſmall Fraction being neglected), is 24 Inches *.

In this very Center we apply the middle Curſor, by which this Center is not alter'd; and the two other Curſors being join'd, we conſider the Pendulum as form'd of two Pendulums join'd together, which have the ſame Point of Suſpenſion; the firſt of which would conſiſt of the Iron Ruler, and middle Curſor; the ſecond of the two other Curſors, join'd by a right Line, that is inflexible, and without Weight. In the firſt Pendulum the Diſtance of the Center of Oſcillation is 24; therefore in the ſecond alſo the Diſtance of this Center from the Point of Suſpenſion will be the ſame. I ſhall now examine the ſecond Pendulum only, and ſhew the Situation of the Curſors, that is, of the Weights.

Let their Diſtances from the Point of Suſpenſion be x and y; the firſt is the greateſt; let the Weight of the Curſor be $p = 7{,}5$. $\frac{pxx+pyy}{px+py} = \frac{xx+yy}{x+y}$
* 474. $= 24$ *.

Therefore

Therefore $xx - 24x = 24y - yy$. We determine y at pleasure, and discover x.

Let y be $= 8$, and x will be 28,5. If y be $= 10$, x will be $= 28,7$. And thus further, $y = 12$, $x = 29$; $y = 14$, $x = 28,7$. &c.; but we may chuse any one of these Dispositions of the Cursors. x never exceeds 29; and we chuse this Situation.

CHAP. III.

Of the Actions of Forces, and their Destruction.

WE have seen that the innate Force of a Body is consum'd by acting; and that the Action follows the Proportion of the Force lost*; whence it follows, that the Force may be measur'd by its Effect †; for this is equal to the whole Resistance, or contrary Action by which it is destroy'd *. Now by considering the Pressure, whose Intensity remains, and by which the Force is destroy'd, it will appear also by a Demonstration like that, which we propos'd about the Generation of Forces *, that the Forces of the same Body are as the Squares of the Velocities, as we have seen it †. But it is not necessary to determine the Measure of the Forces again; what we had in the first Chapter of this Book, we deduce from those things, which relate to the Measure of the Effects *. 817. * 709. † 712. * 361. * 750. † 753. * 713.

If Bodies by acting lose their whole Forces, the Effects follow the Ratio compounded of the Masses, and the Squares of the Velocities *. 818. * 757.

We must now illustrate this with Experiments; but we should chuse such Effects, which may be brought to an accurate Measure. Such are the Bendings of the Parts of elastick Bodies; but we have not yet examin'd the Laws of such Bendings, they are consider'd in the last Chapter of this Book. One Case only can be of use here; namely, when the Bendings are equal, and similar. That we may have these, equal Forces are requir'd; that is, the Bodies should be mov'd with Velocities, whose Squares are inversely as the Masses *; or, if the Bodies acquire their Velocities by falling, they are to let down from Heights, which are in that inverse Ratio of their Masses †. 819. * 758. † 790.

EXPERIMENT I.

Two Cylinders A B, D C, are made of Ivory, whose Diameters are an Inch and an half; their Extremities A, D, are hemispherical; the others B, C, are conical. The smaller is almost two 820. Plate XXX Fig. 1.

Inches and an half long; the other is two Inches longer, and its Weight is exactly double that of the other. To these are fasten'd Threads at their conical Extremities.

It is requir'd, that in the Extremities A and D of the Axis, that the Ivory may have the same Elasticity; which is easily obtain'd, if the Cylinders are made of the same Ivory, and we must attend to that; *viz.* that the Points A and D coincide with the Axis of the Vertex.f

All Scruple about this Equality of Elasticity may be remov'd, it the two Cylinders be made equal, and like the Cylinder D C; le; these be let down from different, yet always equal Heights for both which, that it may be done, they are retain'd by Threads, as C *c*, which being relax'd, the Parts of the Cylinders, as D, strike upon an horizontal Surface of a heavy Piece of blue Marble, well fasten'd; the Surface must be made wet a little, that the Colour may be more intense. In the Strikings the elastick Parts are press'd inwards, and the Cylinders impress upon the Marble, or rather the Moisture with which it is cover'd, very plain Spots and circular. If the Spots of both Cylinders, when they descend from equal Heights, are equal in every Case, there will be no doubt left of the Cylinders having the same Elasticity, in the Places D. These things being try'd, one of the Cylinders must be diminish'd towards C, that it may have the Magnitude A B; that is, lose half of its Weight.

Now if the Cylinder C D be let down from the Height of nine Inches, and A B from the Height of eighteen, the Spots on the Marble will be exactly equal.

If A B be let down from an Height of three Feet; *viz.* an Height quadruple of the former, that the Velocity may be double, the Spot will be greater, and the Diameters will be as 5 to 6 nearly.

821. We have the Effects of the Forces also, which are reduc'd to a Measure, if the Forces are consum'd by pressing the Parts of soft Bodies inwards. Clay is the most convenient of all to be used; but we choose that, of which the most common, and meaner earthen Vessels are made. This is requir'd to be pure, and is to be so temper'd with Water mixt with it, as to daub the Hand indeed, but not stick to it. Moreover, it is requir'd to be similar to itself in every Part; which, that it may be obtain'd, the Parts are well work'd together.

When a Mass of such Clay is bent, it gapes, and in some Places separates; when it has this Property, the Parts, which are preſs'd inwards, whilſt they yield, penetrate into thoſe that are next to them.

If we make uſe of other Clay, which is whiter, and almoſt of the nature of Chalk, it does not eaſily gape, and the Parts alſo, whilſt they yield, penetrate into thoſe next to them with difficulty; but they rather remove them; which happens differently, according to the different Nature of the Clay. For this reaſon, I only make uſe of the Clay firſt mention'd *; becauſe we can diſcover by reaſoning what ſhould happen to it; all the Effects are ſubject to fix'd Laws, and may be foreſeen, and Experiments confirm the Reaſonings about them. If we make uſe of other Clay, we have different Effects, according as it more or leſs agrees with the Clay mention'd. I happen'd upon this Obſervation by chance only; for when I had many Years made uſe of the Clay, that I could moſt eaſily get, I always obſerv'd, that all the Experiments exactly anſwer'd to one another, and agreed with the Rule, to which the Experiments themſelves had brought me. But a few Years ſince, when I made uſe of another kind of Clay, and the Experiments did not anſwer one another as before, I examin'd the thing with Care; I eaſily perceiv'd, that in this laſt Caſe, a Cavity was form'd in part, not by the Introceſſion, but rather by the Receſs of the Parts, and that the Effect ought to be meaſured by ſome other Rule unknown to me. 822.

* 821.

For this reaſon I perceived that I muſt return to my firſt Clay, and that ſoft Bodies only were to be uſed, which have the Property ſhewn above *, for I ſpeak of theſe only in the following Reaſonings. 823.

* 821.

If the Breadth of a Cavity be great, in reſpect of its Depth, the Reaſonings don't take place in this Clay; becauſe in this Caſe, whatſoever the Nature of the Clay be, the Parts eaſily yield ſidewiſe, and Part of the Cavity only, is to be attributed to their Introceſſion. 824.

When a Body, by making a Cavity in a ſoft Body, whoſe Parts are ſimilar, and cohere equally, and being compreſs'd yield in ſuch manner, as to pierce into thoſe next to them, as I have ſhewn above *, *loſes Motion*, it overcomes the Preſſure, whereby the Parts cohere together; and by the Reſiſtance, which the Body in motion ſuffers by overcoming this Preſſure, its Force is diminiſhed, and at length totally deſtroy'd: therefore *the Effect of the Force* in this Caſe, whilſt the Body loſes its Motion, is the Separation of the Parts of a ſoft Body, which are mutually moved amongſt one an- 825.

* 821.

 other;

other; which Effect *follows the Proportion of the Number of Particles moved, and the Space run thro' by them, in their Motion near one another*; and whether this be done ſlower, or faſter, the ſame
826. Coheſion is to be overcome: whence we deduce, that *the Forces are equal, which are conſumed in forming equal and ſimilar Cavities, in the ſame ſoft Body; whether theſe be made in a longer, or ſhorter Time.*

Experiment 2.

827. In this Experiment we make uſe of the Machine, explained in
* 760. the foregoing Chapter *: to this we join the Box, or rather wooden
Pl. XXVIII. Fig. 8. Solid A B, almoſt two Inches and an half thick; it is made hollow in C D: this Cavity is above four Inches long, two Inches broad, and one Inch deep; two Slits E F, E F, go thro' the Wood. This Solid is faſten'd by two Screws, as G, going thro' the Board, to which it is applied, and the Slits. The Head H at the hinder Part of the Table retains the Screw, and the Extremity goes beyond the Slit, that it may be faſten'd by help of the outward Screw L, which, the Braſs Plate *m* being put between, compreſſes the anterior Surface of the Solid, to make it faſt.

828. The Cavity of the Solid is fill'd with Clay, of which we ſpoke
* 821. above *; we pare off the Clay which ſtands too high with a wooden Knife anointed with Oil, in order to make the Surface exactly flat.

PlateXXVII. Fig. 1. The Solid is applied to the Board B C, Screws going thro' the Holes f, f, and thro' the Slits of the Solid, as we ſaid. In this Situation, the Line *o n*, (*Plate* XXVIII. *Fig.* 8.) which touches the Board, is in a vertical Situation, and agrees with the middle of the Board. The Solid, by means of the Slits, may be rais'd and depreſs'd, and faſten'd at any Height, within certain Limits, its vertical Situation being kept.

829. The Rectangle *s* is now remov'd, we make uſe of *r* only, which
* 769. is ſuſpended, as we ſaw before *.

† 771. To this we join † one of the Solids, of which before ‖, and in-
‖ 738. deed that which is repreſented at H, (*Plate* XXVIII. *Fig.* 7.) This is cylindrick, but terminated by a Cone, whoſe Section thro' the Axis gives an Angle of 85 Degrees. When *r* is at reſt, in the Situation which it acquires of its own accord, the Vertex of this Cone exactly touches the Surface of the Clay, if in the Diſpoſition of the Hooks, by which the Threads join'd to *r*, are ſuſtain'd, we attend to what was deliver'd in N° 766.

The

The Rectangle *r* is drawn by the Thread, that it may be rais'd; and when it is relax'd, it ſtrikes the Clay, and the Cone makes a Cavity. The Velocity, with which the Body ſtrikes the Clay, is determin'd by the Diviſions of the Ruler V X *; this Ruler muſt be faſten'd in ſuch manner, that its End X, when the Body is at reſt, may agree with its exterior Threads *: The Rectangle only with the Cone, which Maſs we call *one*, ſtrikes the Clay with the Velocity twelve, and a Cavity is made, whoſe true Magnitude is repreſented at A. 830.

* 737. 775.

* 775.

831. Plate XXXI. Fig. 5.

The Situation of the Box is chang'd, which contains the Clay, that the Cavity may be impreſs'd in it, at the Diſtance of an Inch at leaſt from the firſt.

The Maſs of the Body mov'd is chang'd in ſuch manner, as to be equal to nine *; the Threads which ſuſtain the Body, now become longer; wherefore this muſt be rais'd †, that it may be exactly at the ſame Height at which it was in the firſt Trial. Then if this Body ſtrikes the Clay with the Velocity four, it will make a Cavity exactly equal to the former one A. Plate XXXI. Fig. 6.

* 774.

† 769.

The Velocities are in theſe two Caſes as 12 and 4; that is, they are as 3 to 1; the Maſſes are as 1 to 9; that is, they are inverſely as the Squares of the Velocities; therefore the Forces, which were deſtroy'd by making equal, and ſimilar Cavities, were equal *. 832.

* 758.

We demonſtrate the ſame Thing, by making uſe of Bodies falling directly.

A MACHINE, *whereby the Forces of Bodies, falling directly, are compar'd.*

The Board AB is a Foot long; ten Inches broad; and two Inches thick. It is made hollow in *a b c d* an Inch and an half deep, and is join'd faſt to the Feet E E, E E, by which it is ſupported. 833. Plate XXXII. Fig. 2.

Upon theſe Feet, at the Angles of the Board, four wooden Pillars C D, C D, C D, C D, ſtand. The Pillars are ſomewhat above three Feet high. Two which are join'd to the ſame Foot, which is plac'd broadwiſe to the Board, are join'd by the ſmall Rulers *e e*, *e e*; *f f*; *g*, *g*; *h*, *h*; in ſuch manner, that the Ruler R R, being plac'd between the ſmall correſponding ones, may be parallel to the Surface of the Board.

Three Balls *(Fig. 3.)* that are equal, made of Braſs, of an Inch and an half Diameter, are made uſe of: one C is ſolid, the other two hollow; theſe conſiſt of two Hemiſpheres A, *a*, and B, *b*, which

which are join'd by a Screw. The Weights of the Balls are to one another as one, two, three.

* 821. When Experiments are to be made, the Cavity *a b c d* is fill'd with Clay *, and what Clay ſtands above the reſt, is ſcrap'd off by a piece of Board, or wooden Knife, that its Surface may not only be exactly plane, but may alſo make the ſame Plane with that Part of the Board which ſtands above, and ſurrounds the Cavity.

The Ruler mention'd R R, is made hollow a little underneath longwiſe, that it may receive any of the Balls, whilſt it is held by the Hand M, as is repreſented at G. In this Situation the loweſt Point of the Ball is diſtant nine Inches from the Surface of the Clay. This Diſtance is double, if the Ruler R R paſſes between the Rulers *f*, *f*, *f*, *f*; if between the Rulers *g*, *g*, triple; and quadruple if between *h*, *h*.

But this Diſtance for the moſt part muſt be diminiſh'd a little, but unequally in different Circumſtances; then the Ball is applied to the End of the Screw I, which goes thro' the Ruler R R, and may be put thro' it more or leſs.

EXPERIMENT 3.

We call the lighteſt Ball the firſt; we call that the ſecond, whoſe Weight is double; laſtly, we call'd the ſolid Ball the third, whoſe Weight is triple of the firſt.

834. The Ruler R R being put between the Rulers *e*, *e*, the ſecond
Plate XXXII. and third Balls are let down ſucceſſively, being firſt oil'd; theſe
Fig. 2. 4. in part ſink into the Clay and make Cavities, ſo much the greater as the Balls are heavier. The Cavities are B, C, which are repreſented in *Fig.* 4. the Dimenſions being reduc'd to half. The Lines mark'd with Points ſhew the Depths of the Cavities.

If the Ruler R R be put between the Rulers *f*, *f*, and the firſt Ball be let down, the Cavity will again be B, (*Fig.* 4.)

If R R be between *g*, *g*, and the firſt Ball be let down, the Cavity will be C, *Fig.* 4.

And in general the Cavities don't differ when the Heights are in-
* 790. verſely as the Maſſes, in which Caſe the Forces are equal *.

835. That all Scruple, which may ariſe from the Depth of the Cavity may be remov'd, the Ball is applied to the hollow Surface of the Ruler, and let down; the Diameter of the Cavity is meaſur'd, and by having recourſe to the Table contain'd in the firſt Scholium following, the Depth of the Cavity is diſcover'd, which is expreſs'd in hundredth Parts of the Diameter of the Ball. The Screw I is advanc'd

advanc'd beyond the hollow Surface of the Ruler, as much as is equal to the Depth diſcover'd; the Experiment is repeated, the Ball being applied to the End of the Screw, a new Cavity is form'd in another Place of the Clay; and, the firſt being neglected, we conſider this.

We ſaid further, that we muſt not regard the Time in which 836.
the Cavity is made; becauſe the Effect is determin'd. The Preſſure deſtroys the Force, if it acts in a leſs Time, it acts faſter; and when the Space paſs'd thro' is the ſame, the Action is the ſame *; * 728.
which ſhould be referr'd to all the ſmall Parts of the Effect. But the Force, which is deſtroy'd, is equal to the Action, which deſtroys it *; all theſe things ſpontaneouſly flow from thoſe things, * 709.
which we treated of before; yet I will illuſtrate the thing itſelf by Experiments.

Experiment 4.

We make uſe of the Machine with the compound Pendulum de- 837.
ſcrib'd above *. We apply three Curſors † to the Pendulum O Q ‖, * 732. Plate XXV.
at Diſtances from the Center of Motion taken at pleaſure; but ſo, Fig. 2.
that the two extreme ones may be at leaſt ſix Inches diſtant from † 738.
the Ends of the Ruler O Q. ‖ 735.

We join two Solids * to each of the Curſors. * 738.

We make uſe of the Box that contains the Clay †, as in the † 827.
ſecond Experiment foregoing. This Box is join'd to the Table A B C, and may be fix'd at any Height, the Screws being put thro' Holes, as *d*, *d*.

The Box is vertical, as in the ſecond Experiment, and its Side *o n* (*Plate* 28. *Fig.* 8.) agrees to a vertical Line, drawn thro' the middle of the Board; and when the Pendulum is at reſt, if the Curſor anſwers to the Box, the Vertex of the Cone, by which the Solid is terminated, join'd to the Curſor, reaches to the Surface of the Clay.

The three Curſors mention'd, applied to the Pendulum O Q, Plate XXX.
are A B C; the Cones *g*, *h*, are ſimilar; the Box is faſten'd ſo as Fig. 2.
to anſwer to the Curſor A, the Pendulum is rais'd to a Height which we determine by the Index; for Example, to the Height of 40 or 45 Diviſions; it is left to itſelf, and loſes its Force, whilſt it makes a Cavity in the Clay.

The Situation of the Box is chang'd, that it may anſwer to the Plate XXX.
Curſor B; but it is faſten'd in ſuch manner, that the Curſor may Fig. 3.
anſwer to another Part of the Surface of the Clay. The Pendulum

is

is rais'd to the ſame Height in the foregoing Caſe, and the ſame Force is deſtroy'd, the Curſor B acting upon the Clay.

Plate XXX. Fig. 4. Laſtly, the Curſor C, whoſe Cone is chang'd by joining *g* or *h* to it, in whoſe Place *i* is ſubſtituted, makes a third Cavity, whilſt the Pendulum, being mov'd in the ſame manner as in the two foregoing Trials, loſes its Force.

Theſe three Cavities are ſimilar, and equal; tho' the Times, in which they are made, differ.

Experiment 5.

838. Plate XXX. Fig. 5. Thoſe things being laid down, which were explained in the foregoing Experiment; to the Pendulum O Q we join the two Curſors A, B, with their Solids; to the firſt at *h* is joined one of the Cones, by which the Cavities, in the foregoing Experiment, were made.

* 744. The Weight P *, of two Pounds, is applied at the diſtance of fifteen Inches from the Point of Suſpenſion, and the Pendulum being raiſed to an Height little ſhort of thirty-eight Diviſions, let it loſe its Force, the Cone *h* running againſt the Clay.

Plate XXX. Fig. 6. * 744 P is taken away, and the Weight T, which is equal to half a Pound, is faſtened at the diſtance of thirty Inches from the Center of Motion; other things remain. The Pendulum is raiſed to the Height of 40 Diviſions and an half, and the Cone *h* alſo makes a Cavity; the Cavities will be equal. The Diſtances of 15 and 30 Inches are meaſured from the middle Points of the Weights.

839. By Experiment 4th, of the foregoing Chapter, it is plain, that the Velocity of the Cone *h* was the ſame, in both Strikings; therefore, when the Cavities are equal, and ſimilar, it is manifeſt that they were made in equal Times. Now if we refer to this, what * 788, 789. was ſaid * upon occaſion of the 3d Experiment of the foregoing Chapter, it will appear, that equal Parts of theſe Cavities are to be attributed to the Actions of the Bodies P, and T, which loſt * 785. their whole, and equal * Forces; for they were moved with Velocities in the Ratio of 1 to 2, whilſt the Maſſes were as 4 to 1, inverſely as the Squares of the Velocities.

840. When a Cavity is made, each of the ſmaller Increaſes are to one another as the Number of Particles which yield, and as the Spaces, through which they are moved between others; that is, theſe Increaſes are as the Forces, which the Body loſes by making theſe * 825, 712. Increaſes *: therefore the Sum of the Increaſes, that is, *the* whole *Cavity,*

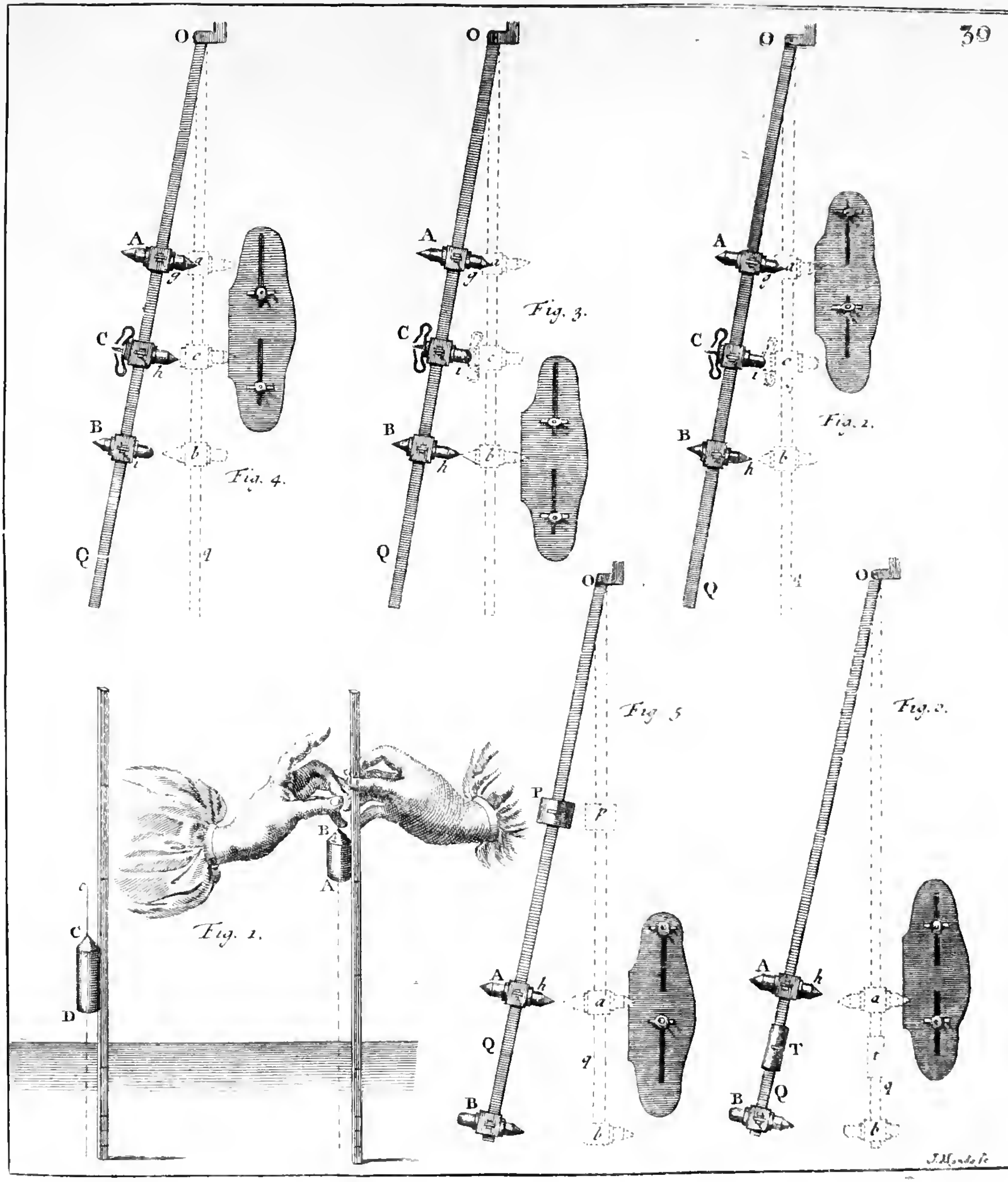
Fig. 4.
Fig. 3.
Fig. 2.
Fig. 1.
Fig. 5
Fig. 6.
J. Mynde sc.

Cavity follows the Proportion of the Sum of the Forces lost, that is, *of the Force lost in the Formation of the whole Cavity.* 841.

Therefore *the same Body moved with a determinate Velocity, if it consumes its Force by pressing inwards the Parts of a soft Body, will make a Cavity of a determinate Magnitude, whatsoever Figure this has.*

EXPERIMENT 6.

We make use of the Machine, by which the 2d Experiment of this Chapter is demonstrated; it is managed in the same manner as that; but there is a different Cone, which, joined to the Body, runs against the Clay. 843. Plate XXXI. Fig. 7.

We make use of two different Cones successively, which are represented at G and I (Plate XXVIII. Fig. 7.) If we cut the first through the Axis, we have an Angle of 55 Degrees; the Section of the second gives an Angle of 102 Degrees.

The first Cone being joined to the Rectangle, and the Cylinder being added, that the Mass may be three *, let this run against the Clay, with the Velocity ten, the Body is at rest, and the Cavity is represented at B. * 774.

The Cone being taken away, and the second being used, let the Experiment be repeated, with the same Velocity, the Situation of the Box being altered, we have the Cavity C, which being compared with B, the Diameters are as 3 to 4. Plate XXXI. Fig. 8.

The same Body, moved in the same manner, made both these Cavities; the Forces destroyed were equal; the Figures of the Cavities differ; yet they are equal. For from the Angles of 102 and 55 Degrees mentioned, it follows, that the Depths of the Cavities, the Diameters being as 4 to 3, are as 9 to 16, as every one will discover, who draws the Figure, or makes his Computation from the Tables of Sines; therefore the Depths are inversly as the Squares of the Diameters of the Bases; that is, inversly as the Bases themselves *: and therefore the Cavities are equal †. 844. * 2 El. 12. † 15 El. 12.

But the Demonstration of the Formation of the Cavity is universal *; whence it follows, that *the Cavities which in a soft uniform Body*, whose Parts are similar to one another, and equally cohere, and being compressed, yield among other Parts, (for we speak here of such a one,) *are made by Bodies, which consume their whole Forces by these Actions, are to one another in a Ratio compounded of the Masses of the Bodies, and Squares of the Velocities, whatsoever Figure the Cavities have* *. 845. * 480. * 841. 757.

Experiment 7.

846. This Experiment alſo is performed as the ſecond of this Chapter,
Plate XXXI. the Cone being kept, whoſe Angle is 85 Degrees, which was
Fig. 9. uſed in that Experiment. The Body, whoſe Maſs is four, ſtrikes the Clay with the Velocity ſix, and makes a Cavity.

Plate XXXI. The Maſs is doubled, and the Velocity, and the Body again
Fig. 10. loſes its Motion, ſtriking againſt another Part of the Clay.

847. The Force in this laſt Caſe is octuple the former *; the Diameter of the Cavity is double, and the Cavity itſelf alſo octuple †; for the Cavities of the Cone are ſimilar.

* 757.
† 12 El. 12.

Experiment 8.

848. If the Body, by which the firſt Cavity was made, in the 7th
Plate XXXI. Experiment, ſtrikes upon the Clay eight times ſucceſſively, in
Fig. 9. the ſame manner, and always acts upon the ſame Place in ſuch manner, that it continually increaſes the Cavity; after eight Strokes the Cavity will be equal to that, which was formed the ſecond time in Experiment ſeventh; that is, it will be octuple that, which is made by one Stroke. Which is another Confirmation, that the Force deſtroyed follows the Ratio of the Cavity itſelf.

849. Twenty-ſeven equal Strokes make a Cavity, whoſe Diameter is triple, and which exceeds the firſt twenty-ſeven times *.

* 12 El. 1.

850. We muſt obſerve concerning this Experiment, that ſometimes, the Strokes being repeated, which are made upon the ſame part of the Surface of the Clay, this acquires an Elaſticity; then the Cone does not ſtick in the Cavity after the Stroke, and the Experiment does not ſucceed; but it always anſwers well, every time that the Cone ſticks in the Cavity, even to the laſt.

851. In the two laſt Experiments the Cavities were ſimilar, therefore I will add the following ones in which the Figures differ.

Experiment 9.

852. This is managed as before, only we change the Weight put into the Rectangle, and let the Maſs be now ſix; the Weight might have been kept. Let the Body run againſt the Clay, with the Velocity eight; let the Diameter of the Cavity be meaſured: this was in the Experiment which we mention ninety-eight parts, an hundred of which are contained in half an Inch.

Pl. XXVIII. We take away the Cone; this was that which is marked H,
Fig. 7. whoſe Section through the Axis in the Vertex gives an Angle of

85

85 Degrees *, we substituted the Solid L, which is terminated * 738. 829.
by an Hemisphere, whose Diameter is equal to half an Inch.

The Body strikes the Clay again, with the same Velocity eight; Pl. XXXI. Fig. 11.
it consumes its Force, by making a Cavity, which has the Figure of a Segment of a Sphere; the Diameter of the Cavity is also measured, by using hundredth Parts of an half Inch, and is equal to ninety-four Parts.

If these Diameters being given, of 98, and 94 Parts, we have recourse to the Table, which is had in the first Scholium following *, we discover the Magnitudes of the Cavities to be 514 and * 867.
508; that is, that they are as to Sense equal.

The Experiment being repeated with the Velocity six, we had 853.
the Diameter of the Cone 81 and the Diameter of the Segment 85. The Magnitudes of the Cavities now were 290, and 283; again equal as to Sense; and the Tables demonstrate, that nothing more accurate can be given in them. But these Forces are to the first, as 36 is to 64 *; in which same Ratio these last Cavities are to the * 753.
first 36 : 64 : : 288 : 512.

As in this Experiment we make use of the smaller Sphere, (we 854.
can't use the greater with the Machine, which we made use of) it may be suspected, that the smaller Differences can't be sufficiently discovered by this Method; I will now consider greater Cavities, whose Figures are different; for it is well known, that unequal Segments of the same Sphere are not similar.

Experiment 10.

We must return to the third Experiment of this Chapter *. 855.
We have seen that the Cavities are equal, which were made by * 834.
equal Forces; I speak now of comparing those with one another, which were impressed by unequal Forces.

We had the Cavities B, and C, the second and third Balls being Pl. XXXII. Fig. 4.
let down, from the Height of nine Inches; I let down the first Ball from the same Height, and there is given the Cavity A, the Forces, by which these three were impressed, are as one, two, and three *. * 748.

The second and third Balls being let down, from the Height of eighteen Inches, double the former, the Forces are as four and six *; and the Cavitiesare D and E. * 754.

The Diameter of the Ball, which is equal to an Inch and an half, being divided into an hundred equal Parts, the Diameters of the

Cavities, will be denoted by these Parts, A, 65; B, 76; C, 82 $\frac{1}{2}$; D, 87; E, 93 $\frac{1}{2}$.

* 867. Therefore the Segments are 80, 162, 243, 320, 489 *; nearly as 1. 2. 3. 4. 6. that is, as the Forces by which the Cavities were impressed.

856. From the same Proposition, that the Cavities are proportional
* 841. to the Forces *, which we now have illustrated by various Experiments, we also deduce, that what has been said of one Cavity only, may be referr'd to many; and from the Force given by which the Cavity is formed, we shall determine the Number of Cavities equal to this, which may be made by any other given Force, whatsoever.

Expreiment II.

857. Pl. XXXI. Fig. 12. This Experiment scarce differs at all from the last but one, and some others.

A Rectangle is suspended; the Mass is determined at pleasure; let this be for Example two. We use the Cone G (Plate XXVIII. Fig. 7.) whose Section through the Axis gives in the Vertex an
* 843. Angle of 55 Degrees *. It strikes against the Clay with the Velocity five.

Pl. XXVIII. Fig. 9. The Cone is taken away, and the Copper-plate P Q R is substituted; it resembles a Cross, whose horizontal Arms, one of which only appears R, are shorter than the others. These smaller Arms also are perforated, as is seen in *l.* Two Screws go through the Holes, by which the Plate P Q R is joined to the Rectangle suspended. But that Rectangle is used, in whose anterior Surface the
* 773. Holes, which receive the Screws, are in an horizontal Line *; so that P Q may be vertical.

To this Part P Q are applied four Cones S, S, S, S, whose Bases are Cylindrick, that they may not take up too much room. The Cones are like that, which we have already used in this Experiment.

Pl. XXXI. Fig. 13. The Mass of the Rectangle is kept, which is equal to two; the Vertices of the four Cones, when the Rectangle is at rest, touch the Clay, but are not supported by it.

Now when the Body strikes against the Clay, the four Cones penetrate equally into it, and make four equal Cavities. If the Velocity be ten, double the former, that is, if the Force be qua-
* 753. druple *, these Cavities will be equal to the former: which only the Body impressed with the Velocity five.

We

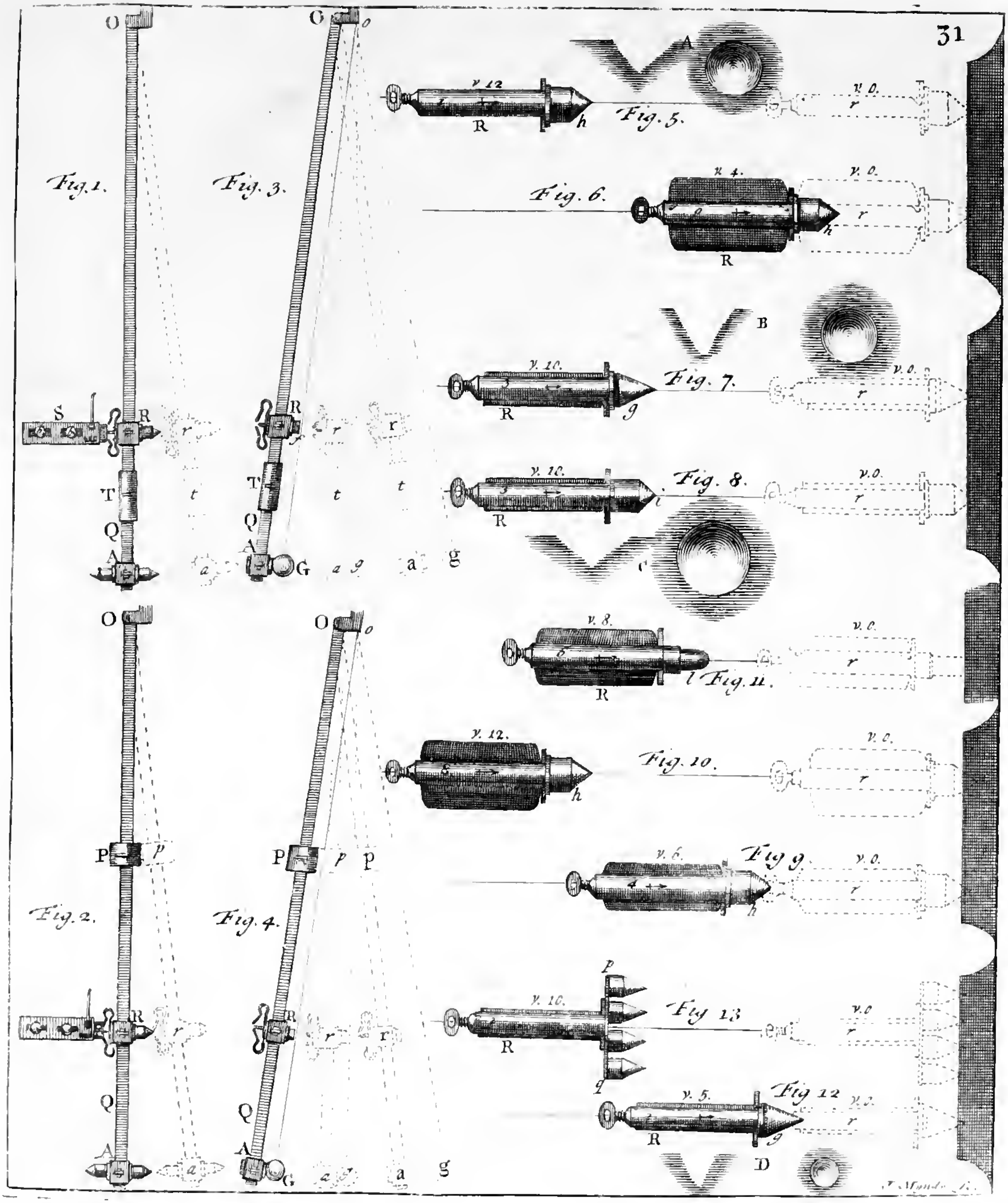
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig 9.
Fig. 10.
Fig. 11.
Fig 12
Fig 13
A
B
C
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p
h
v. 0.
v. 12
v. 4.
v. 10.
v. 8.
v. 6.
v. 5.

We have demonſtrated in general, and the Experiments fully confirm it, that we muſt not regard the Time, that we may determine the Effect, which a Body produces, whilſt it loſes its Force. But I will add ſomething about determining the Time, and comparing different Times with one another, and I will give the Demonſtrations in the Scholia. 858.

In my Machine, *if a Body*, cylindrically terminated, ſtriking againſt the Clay with the Velocity ten, *penetrates into* it, (viz. *the Clay)* the Depth of one Inch, *the Time of the Action upon the Clay* will be the tenth Part of one Second; and this Time, *the Cylinder being changed, or the Reſiſtance of the ſoft Body being varied, as long as the Striking is made with the ſame Velocity, follows the Ratio of the Depth.* 859.

If the Bodies are different, and the Cylinders have different Diameters, the Time follows the direct Ratio of the Product of the Maſs by the Velocity, and the inverſe one of the Baſe of the Cylinder. 860.

When the Cavities are ſimilar, but howſoever unequal, the Cubes of the Times follow the direct Ratio of the Maſſes, the inverſe one of the Velocities of the Bodies. 861.

If various Bodies be terminated by a Figure formed by the Revolution of the ſame Parabola round its Axis, and theſe be carried according to the Direction of the Axis of the Parabola, the Squares of the Times, are as the Maſſes; therefore the ſame Body, with whatſoever Velocity it is carried, the Circumſtances mentioned being laid down, loſes its Motion in the ſame Time. 862.

If with the Velocity ten, in my Machine, the Cavity be an Inch deep, the Time in this Caſe, and in others, in which the Velocity only is changed, will be a Twelfth of a Second. But *if, the Velocity remaining, the Depth be chang'd, the Time will be as the Depth.*

SCHOLIUM I.

The comparing the Segments of a Sphere.

IN ſome Experiments * of this Chapter we took notice of a Table to be exhibited in this Scholium. 863.

* 852. 855.

In this Table we compare ſimilar Cones with one another, or other ſimilar Bodies; for all theſe follow the ſame triplicate Ratio of the correſponding Lines, ſuch as are the Diameters of the Baſes of ſimilar Cones.

We

864. We alſo compare with one another the Segments of the ſame Sphere, from the Diameters of the Segments being given; which we meaſure by Parts, an hundred of which are contained in the Diameter of a Sphere.

We ſuppoſe the Hemiſphere, which is the greateſt of the ſmaller Segments, to contain a thouſand Parts, and by theſe Parts we expreſs the other Segments.

865. And that we may compare Cones with Segments, we meaſure the Cones themſelves by theſe ſame Parts; if the Diameters are determined in hundredth Parts of the Diameter of the Ball; Cones being given, whoſe Sections through the Axes give in the Vertex Angles of 85 Degrees.

866. Segments, whoſe Diameters differ little from the Diameter of the Ball, are not put in the Table; becauſe the leaſt Difference in the Diameters, anſwers to a very great Difference in the Cavity: we have alſo neglected

* 824. the ſmaller, becauſe theſe are of no uſe in the Experiments *.

867. *A* Table, *whereby the Segments of a Sphere and Cone are compared, from the Diameters given, an Hemiſphere being divided into a thouſand Parts, and the Diameter of the Cone into an hundred.*

Diam.	Segment of the Depth.	Segment of the	Cone.
35.			23.
36.			25.
37.			27.
38.			30.
39.			32.
40.			35.
41.			38.
42.			40.
43.			43.
44.			46.
45.			49.
46.			52.
47.			56.
48.			60.
49.			64.
50.	7.	26.	68.
51.	7.	28.	72.
52.	7.	30.	77.
53.	8.	33.	81.
54.	8.	36.	86.
55.	8.	39.	91.

Diam.

Diam.	Segment. of the Depth.	Segment of the Cone.	
56.	9.	42.	96.
57.	9.	45.	101.
58.	9.	48.	106.
59.	10.	52.	112.
60.	10.	56.	118.
61.	10.	60.	124.
62.	11.	64.	130.
63.	11.	69.	136.
64.	12.	74.	143.
65.	12.	80.	150.
66.	12.	85.	157.
67.	13.	91.	164.
68.	13.	97.	172.
69.	14.	104.	179.
70.	14.	111.	187.
71.	15.	118.	195.
72.	15.	126.	203.
73.	16.	134.	212.
74.	16.	134.	221.
75.	17.	152.	230.
76.	17.	162.	239.
77.	18.	173.	249.
78.	19.	184.	259.
79.	19.	196.	269.
80.	20.	208.	279.
81.	21.	221.	290.
82.	21.	235.	301.
83.	22.	250.	312.
84.	23.	266.	323.
85.	24.	283.	335.
86.	24.	301.	347.
87.	25.	320.	359.
88.	26.	341.	372.
89.	27.	363.	385.
90.	28.	387.	398.
91.	29.	414.	411.
92.	30.	442.	425.
93.	32.	473.	439.
94.	33.	508.	453.
95.	34.	547.	468.
96.			483.
97.			498.
98.			514.
99.			530.
100.	50.	1000.	546.

SCHOLIUM.

SCHOLIUM II.

Of the Time, in which the Cavities are made in general.

868. THAT we may determine the Times, in which Bodies running againſt ſoft Bodies, make Cavities; and that we may compare the Times in which different Parts of the ſame Cavity are form'd, me muſt attend to the Maſs, Velocity, and Figure of the Body ſtruck; and firſt, the Reſiſtance from the Coheſion of the Parts of the ſoft Body will be to be determin'd by Experiments.

869. Therefore the Cavity being given, which was made by the ſtriking of the Body whoſe Maſs and Velocity are given, in all others we are to ſuppoſe the ſame ſoft Body to be conſidered.

We ſuppoſe the Surface of this Body plane; that the Striking is direct; and that the ſoft Body makes an immovable Obſtacle.

From the Cavity given in one Experiment, the Cavity in another Caſe
* 841. alſo, if the Force of the Body be given, is determin'd *; therefore we ſuppoſe the Depth of the Cavity known.

870. Let this Depth be A B; let A I C be a Curve, by whoſe Revolution
Pl. 32. F. 5. round its Axis A B, the Figure of the Body was determin'd; we call this Curve *the Line of the Figure.*

871. We conceive a ſecond Line A L D, which has the ſame Axis A B; but whoſe Ordinates, as H L, follow the duplicate Ratio of the correſponding Ordinates, as H I, in the firſt Curve; that is, H L is as the Square of the Line H I; and, from the firſt Curve being known, it is diſcover'd in the Second. If the Body itſelf be cut by a Plane, perpendicular to the Axis, the Section will be as the Square of H I, that is as H L, and the Curve A L D will repreſent the Solidity of the Body, which agrees with the Cavity. We call this Line *the Line of the Cavity.*

872. The Surface A B D repreſents the whole Cavity; and the Surface A H L is proportional to a Portion of the Cavity made, when the Body has penetrated into the ſoft Body to the Depth A H. The Surface H L B D repreſents that, which remains of the Cavity to be made, that the whole Force may be deſtroy'd; that is, this Surface H L B D is proportional to
873. that part of the Force, which the Body has left when it is immers'd to the
* 841. Depth H A *: therefore this Surface is proportional to the Square of the Velocity, in this very Moment.

874. We now conceive a third Line E M B, which we call *the Line of the Velocity.* The Axis is A B again; the Baſe A E repreſents the Velocity, with which the Body projected comes to the Surface of the ſoft Body; the Velocity decreaſes, and when the Depth of the Cavity is H A, the Velocity is proportional to the Ordinate H M. This is a Property of this Curve, that the Square of the Ordinate, as H M, follows the Proportion
* 873. of the Surface H B D L *; wherefore, the Quadratures of the Figures being granted, from the given Curve A L D, we determide E M B itſelf.

From

From the known Line of Velocity, we deduce a fourth, which we call *the Line of Time*, of which this is a Property. The Base B F expresses the whole Time, in which the Cavity is impressed, and the whole Force is consum'd; the Ordinate H G denotes the Time, in which the Body has penetrated to the Depth A H. 875.

The Difference between this Ordinate H G, and the following *hg*, namely *ng*, represents the Moment of Time, in which the small Space H *h* was run thro', with the Velocity H M, that is, H M x *ng* follows the Ratio of H *h* *; which small Space, if it be conceiv'd constant, will give the constant Product H M x *n g*. * 121.

Therefore, the constant small Space H *h* is to *n g*, as the Ordinate H M is to some constant Line; which Property determines the Curve of the Time. But the Line B F touches this in the Vertex F, by reason of the evanescent Velocity in B. 876. 877.

If the Body, keeping the first Velocity A E, with an uniform Motion, shou'd run thro' A B, the Time would increase uniformly, and all the small Lines *n g*, being put equal to H *h*, would be equal, and the Curve would be turn'd into a Right Line, which at A would coincide with it, that is, it would touch it. Let this Tangent be A N; then B N will be to B F, as the Time, in which the Body wou'd run thro' the Space A B, with the Velocity with which it runs against the soft Body, is to the Time, in which it loses its Force by making the Cavity. 878. 879.

If for the various Strikings we form similar Figures, and equal Quantities in all, be denoted by equal Lines, by this we compare, what relates to these different Cases. 880.

SCHOLIUM III.

The Demonstrations of N. 859. 860. 862.

IN the Application of the Theory, explained in the forgoeing Scholium, which is very universal, there often occurs a great Difficulty; because we have to do with the Curves, called Mechanical Curves; and then, if we tend to the Arithmetical Expressions by Algebra, we must for the most part have recourse to the infinite Series. Yet in some Cases all is performed by Geometrical Lines, as will appear by the following Examples. 881.

Let us consider the right Cylinder, moved according to the Direction of its Axis, running perpendicularly against the Surface of a soft Body. 882.

The Line of the Figure is a right one parallel to the Axis; such also is the Line of the Cavity *; we represent both these by the same *a* C. We suppose A E to represent the Velocity, with which a Body projected comes to the Surface of the soft Body, and that the Line of the Velocity is B M E, whose Ordinate, as H M, follows the subduplicate Ratio of the Rectangle I H C B *: But this Rectangle is every where as the correspond- * 871. Pl. 32. F. 6. 873.

 ing

ing Abſciſs; therefore the Square of the Ordinate is as the Abſciſs, which is a Property of the Conick Parabola *.

* *La Hire Sect. Con.* lib. 3. prop. 1.

That we may now diſcover the Line of the Time, we ſuppoſe B N proportional to the Time, in which A B may be run thro' by a Body, mov'd with that Velocity, with which the ſoft Body is ſtruck; and A N will be a Tangent to the Curve in A *.

* 878.

Let the Curve be A F; whoſe Axis is F Q.

If G R be drawn perpendicular to the Curve in G, the rectangular Triangle G R O will be ſimilar to the rectangular Triangle G *n g*; by reaſon of the equal Angles R G O and *n* G *g*; for *g* G O is the Complement of both to a right Angle. Therefore G *n*, or H *h*, is to *n g*, as G O is to O R; that is, G O is to O R, as H M is to a conſtant Line *; but as we have ſeen, in this Figure, H M follows the Ratio of the ſquare Root of the Abſciſs B H, or F O; therefore G O is to O R, as the ſquare Root of the Abſciſs F O to that conſtant Line; whence it follows that the Curve A G F alſo is a conick Parabola; for in this G O follows the ſubduplicate Ratio of F O * and O R is conſtant †.

* 876.

* *La Hire Sect. Con.* lib. 3. prop 1.
† *Ibid.* lib. 3. prop 19.
* *Ibid.* lib. 2. Prop. 20.
† 879.

Hence it follows that F P and F Q, or A B, are equal *; and that F N, B N are alſo equal. Wherefore the Time in which the Cavity is impreſſed, which is proportional to B F, is double that, in which the Body, with the Velocity with which it was ſtruck, could run thro' a Space equal to the Depth of the Cavity, which Time was denoted by the Line B N †.

883. If we would apply theſe Things to the Example propoſed in N. 859, we ſuppoſe theſe Things known otherwiſe. That the Body, applied to my Machine, deſcends to the Depth of one Inch, the Velocity being ſuppos'd 14,6. That it was alſo diſcover'd by Experiments, made with Pendulums *, that a Body, by falling from the Height of ten *Rhinland* Feet, acquires a Velocity, with which it would run thro' twenty-five ſuch Feet in one Second.

* 415. 470.

This Height is to the Height of one Inch, as 120 to 1; therefore the Body, in falling from the Height of one Inch, acquires a Velocity, with which in one Second a Space of 27,4 Inches * is run thro'; and this is in my Machine the Velocity, which we call 14,6, which is to 10, as 27,4 is to

* 374.

884. 18,7. Therefore *the Space run thro' in one Second, with the Velocity called ten in my Machine, is* 18,7 *Inches*; and the Time, in which one Inch is run thro', is 0″,053; the double of which, namely the Time in which a Cavity, one Inch deep, is made, is 0″,106; which Time ſcarce exceeds that mention'd in N. 859.

885. If we conſider another Caſe, it is manifeſt that *the Time* is changed as B F, which follows the Ratio of B N. But this, *if the Velocity remains, follows the Ratio of the Depth* A B, as we ſaid in the ſaid N. 859.

886. *If the Velocity be changed, the Time*, in which a Line as A B is run thro', is diminiſh'd, as the Velocity is increaſed, and B N, and therefore B F, *is inverſly as the Velocity.*

In general the Time is directly as the Depth and inverſly as the Velocity.

From

From these Things we easily deduce, what relates to different particular Cases as long as we consider cylindrick Bodies. 888.

Let p be the Depth; d the Diameter; M the Mass; v the Velocity; the Cavity will be ppd *, which is equal to the Force Mvv †. Whence we deduce $\frac{p}{v} = \frac{Mv}{dd}$. But $\frac{v}{p}$ is as the Time *; therefore this is as $\frac{Mv}{dd}$; as we said in N. 860. 889. * 11. 14. El. 12. † 757. 845. * 887.

When a Body, by making a cylindrick Cavity, loses its Force; it continually loses of this according to the Ratio of the Space run thro' *, in the same manner as a Body projected upwards †; therefore it is subject to the same Law of Retardation, with this, that is, *the Velocity is diminish'd equably, in equal Times* *. 890. * 840. † 370. 377. 378. 754. * 377.

Let us now suppose the Body to be terminated by a Figure, which a Parabola, revolving about its Axis, makes. 891.

Let A I C be this Parabola; which is the Line of the Figure, whose Axis is A B, which also is the Axis of the Cavity. The Line of the Cavity is a right one, drawn from the Vertex A L D; for the Square of the Ordinate A I is as A H *, the Ratio of which A L also follows †; wherefore H L is as the Square of H I, which is the Nature of the Line of the Cavity ‖. Pl. 32. F. 7. * *La Hire Sect. Con.* lib. 3. prop. 1. † 4. El. 6. ‖ 871.

Let the Line of Velocity be E M B; the Square of the Ordinate follows the Ratio of the Surface L H B D *, which is the Difference of the Triangles A D B, A L H; these Triangles are similar as the Squares of the Sides A B, A H *; therefore the Square of the Ordinate H M, which is as the Difference of the Triangles, is also as the Difference of these Squares. Whence it appears that the Line B M E, is a Circle, or Ellipse; for if with the Center A, and Radius A B, the Quadrant of a Circle be described B M E; the Square H M will be equal to the Difference of the Squares A B and A H. If instead of a Circle an Ellipse be used, we shall not have this Equality, but the Ordinates will be in the same Ratio *. * 874. * 19. El. 6. 892. * *La Hire Sect. Con.* lib. 2. prop. 3.

The Line of Velocity, determines the Line of Time *; but in this Case we have to do with a Mechanical Line; but if we make use of a Circle for the Line of Velocity, we don't want another Line of Time: For we have seen that the Line of Velocity of a Pendulum, moved in a Cycloid, is also a Circle, and that the Time is determin'd by the Circumference of it *; which will take place here also. The Time in which a Body penetrates to the Depth A H, is to the Time in which it makes a whole Cavity, as the Arc E M, to the Quadrant of the Circle E M B 893. * 876. * 468.

The infinitely small Space A a, is run thro' with a Velocity, with which the striking is made; and the Arc E e, equal to A a, represents the Time, in which it is run thro'; therefore if a Body, moved with the same Velocity, runs thro' the Quadrant of a Circle E M B, it will do this in *the Time, in which the Cavity* itself *is made.* Therefore this Time *is to the Time in which the Body, with the Velocity, with which the Striking is made, would run thro' the Depth of the Cavity, as the Quadrant of a Circle to the Semidiameter.* If A B be one Inch, E M B will be equal to 1,57. Inch. 894.

 With

With the Velocity, which in my Machine is called ten, a Body in one
* 884. Second can run thro' 18,7 Inches*; in a twelfth Part of this Time it runs thro' almost 1,57 Inches, and in this Time the Cavity is made; as we have shewn in N°. 862.

895. *If, the Body remaining the same, the Velocity be chang'd,* the Cavity,
* 845. which the Surface ADB represents *, is changed as the Square of the Ve-
† 871. locity †; this Surface follows the Ratio of the Square of the Line AB ||, re-
|| 19 El. VI. presenting the Depth of the Cavity; therefore the Velocity is as the Depth;
* 119. and this being varied, yet is run thro' in equal Time *: whence it follows
† 894. that *the Cavity is always made in equal Time* †, as we have also observed in N°. 862.

896. *If, the Velocity remaining the same, the Depth of the Cavity be changed, for any reason whatsoever, the Time is varied in the Ratio of the Depth*
* 120. *changed,* in which this might be run over with that same Velocity *; but
† 894. that has a constant Ratio to the Time, *in which the Cavity is made* †; which therefore is also varied in the same Ratio.

897. By *the* following Rule we also determine this very *Time, in which the Cavity is impressed*; for it *is to the Time, in which a Body by falling acquires the Velocity, with which the striking is made, in a Ratio compounded of the Depth of the Cavity to the Height, from which the Body fell, and the Quadrant of the Circumference of a Circle to the Diameter.*

898. These Times are to one another, in a Ratio compounded of the Time, in which the Cavity is impressed, to the Time in which the Body with the Velocity, with which the striking is made, can run thro' the Depth of the Cavity, and the Ratio of this last Time to the Time of the Fall thro' the said Height. The first Ratio is that, which is given between the
* 894. Quadrant of a Circle and its Semi-diameter *. The second Ratio coincides with the Ratio of the Depth of the Cavity to the Height shewn,
† 120. 376. duplicate †. The compound Ratio is not changed, if one Consequent being square, we reduce the other to half; if instead of the Semi-diameter we put the whole, and instead of the duplicate Height we make use of the simple Height, we have what was to be demonstrated.

SCHOLIUM IV.

Of comparing the Times, in which the Cavities are made, some peculiar Figures being given.

899. WE shall call those analogous Curves, whose Ordinates are proportional, which answer to proportional Absciffes.

Plate XXXII. Fig. 8. The Curves FAG, OHP, whose Axes are AC, HL, are analogous, because if we take at pleasure AB : AC : : HI : HL, this other Proportion is given DE : FG : : MN : OP.

Let us conceive these Curves to turn about their Axes, and to determine the Figures of the Bodies. If such Bodies, moved according to the Direction of the Axes, strike against the Surface of a soft Body perpendicularly,

cularly, the Times in which they lose their Forces may be compared with one another by an easy Rule; but by it the Times can't be compared with the Time in which a Body, with a known Velocity, runs thro' a given Space, as we have done in the foregoing Scholium *. * 884. 894. 898.

Let us conceive the Bodies terminated by these Curves, to strike upon a soft Body in such manner, that they may penetrate to F G and O P. Let us further suppose these Bodies to be divided, by Planes perpendicular to the Axis, into small Orbs, but so, that in every Body all the Orbs may be of the same Thickness, and each of the Bodies may contain the same Number of Orbs. 900.

The Axes A C, and A L, are divided into an equal Number of Parts; therefore if there be the same Number of Parts in A B, and H I; it will be F G : D E :: O P : M N *; and by altern. F G : O P :: D E : M N †. And the Orbs are also proportional, of which these are the Diameters; and there is given the same Ratio between any corresponding Orbs whatsoever, as between the last; that is, the correspondent Orbs are every where in the same Ratio; and the Sum of all, is to the Sum of all, that is, *one Cavity is to the other*, as any Orb whatsoever to its corresponding one; *or as the Sum of any Orbs whatsoever to the Sum of the corresponding * ones.* * 899. † 16. El. 5. 901. * 12. El. 5.

Whilst the Bodies come to the Surface of the soft Body, they have Forces, proportional to the Cavities F A G, O H P *; and they are moved with Velocities which are in a determinate Ratio. * 841.

When the Parts D A E and M H N, have penetrated into the soft Body, the Forces destroyed are in the Ratio of those which the Bodies had in the Beginning *, therefore the remaining Forces also are as the first †; as also the Velocities in the same Ratio as in the Beginning ‖. 902. * 901. † 17. El. 5. ‖ 753.

The Time in which the Depth of the Cavity D A E is increased by the Quantity B b, is to the Time, in which the other Body runs thro' I i, directly as B b to I i, and inversly as the Velocities, with which the Bodies are moved in these Moments *. But B b is to I i, as A C is to H L; by reason of the equal Number of Parts in both Lines: and the Velocities in this Moment as in the Beginning †. Therefore the Time in which any Orb is immerg'd, is to the Time in which the corresponding Orb penetrates into the soft Body, *directly as the Depths of the Cavities, which the Bodies impress in consuming their whole Forces, and inversly as the Velocities, with which the Bodies strike against the soft Body*; and in the same Ratio is the Sum of all the Moments, in which all the Orbs successively, in F A G, are immerg'd, to the Sum of the Moments, in which this same Thing happened in O H P *. But these *are the Times, in which the whole Cavities are impressed.* 903. * 111. † 902. 904. * 12. El. 5.

This is an universal Rule, which takes place, whatsoever the Cavities be; and the Demonstration of this Rule may be applied to any Figures whatsoever of Bodies, when the immersed Parts are similar; wherefore *this Rule may be applied to all similar Cavities.* 905.

In this latter Part I will first explain the Use of the Rule. Let the Depth of the Cavity be x; the Cavity will be x^3 *; therefore, if M be the 906. * 8. 12. El.

Mass

* 845. Mafs of the Body, and v the Velocity, it will be $x^3 = Mvv$ *: if T
† 904. 905. be the Time; it will be $T = \frac{x}{v}$ †; and $T^3 = \frac{x^3}{v^3} = \frac{Mvv}{v^3} = \frac{M}{v}$; as we faid in N°. 861.

907. Let us return to the analogous Curves, DE : FG : : MN : OP; be-
PlateXXXII. caufe AB : AC : : HI : HL*: let us now put AX : AZ : : HI : HL,
Fig. 8. it will be RS : TV : : MN : OP; therefore putting in the fame Curve
* 899. AB : AC : : AX : HZ, it will be DE : FG : : RS : TV. Whence it follows that the Change of the Ordinate depends upon the Change of the Abfcifs, according to fome conftant Law; and that the Nature of all analogous Curves poffible may be expreffed by a general Equation.

908. If x be the Abfcifs as AB; y the duplicate Ordinate correfponding to it; the Equation will be $x = y^{\frac{n}{m}}$ or $x^m = y^n$, m and n denoting any whole Numbers and Fractions.

If one Number or the other be negative, the Curve will be of no fervice in the prefent Affair.

909. But that we may be able to compare different Curves, which are expreffed by the fame particular Equation, we fupply the Dimenfions which are deficient in one part by making ufe of the Line a, by which we diftinguifh fuch Curves from one another, and the general Equation becomes $a^{n-m} x^m = y^n$.

910. If the Abfcifs x expreffes the Depth of the Cavity, the Cavity itfelf will be equal to a Spheroid, of which x will be the Height, which Solid is
* 845. proportional to Mvv*, and is equal to $\frac{n}{2m+n} a^{\frac{2n-2m}{n}} x^{\frac{2m+n}{n}}$; as they know, who are not ignorant of the firft Elements of the Quadrature of Curves.

We may neglect the conftant Multiplier $\frac{n}{2m+n}$, for we do not therefore change the Proportion, whatfoever the peculiar Equation be: yet a is kept, that a Comparifon of different Lines of the fame Equation be made.

Therefore we put $a^{\frac{2n-m}{n}} x^{\frac{2m+n}{n}} = Mvv$, that is, $x^{2m+n} = a^{2m-2n}$
* 904. $M^n v^{2n}$: but $T = \frac{x}{v}$*; therefore $T^{2m+n} = \frac{x^{2m+n}}{v^{2m+n}}$.

911. If now inftead of x we fubftitute the Value, we difcover the Equation $T^{2m+n} = a^{2m-2n} M^n v^{n-2m}$, which fupplies an univerfal Rule, for comparing the Times in all analogous Curves, as will appear by Examples.

912. Let us put m=o, n=1; the Equation of the Figure $a^{n-m} x^m = y^n$* is
* 909. changed into this $a = y$, and the Figure is cylindrick. Now $T = a^{-2} Mv$†
† 911. or

or $T = \frac{Mv}{aa}$, that is, the Time is directly as the Product of the Mass by the Velocity, inversly as the Square of the Diameter, or as the Base *, as we had it above †. * 2 El. 12. † 889.

If m = 1, and n = 1; the Equation of the Line * gives $x=y$, and we have to do with the Cone; but a vanished: and that we may compare different Cones with one another, we ought to make a new Computation. 913. * 909.

Let us suppose d to be the Diameter, when c is the Height, and it will be, $d : c :: y : x$; or $dx = cy$. The Solidity of the Cone is as yyx *; that is, as $\frac{dd}{cc}x^3 = Mvv$ †, but $T = \frac{x}{v}$ ‖; therefore $T^3 = \frac{ccM}{ddv}$, gives a general Rule for any Cones whatsoever. 914. * 11. 14. El. 12. † 845. ‖ 904.

Let m = 1, and n = 2; the Equation of the Figure * is $ax = yy$ and the Figure is a Parabola, whose Parameter is a. A Substitution of Numbers being made in the Equation, $T^{2m+n} = a^{2m-2n} M^n v^{n-2m}$ which exhibits the Value of the Time *, we have $T^4 = a^{-2} M^2$, or $T^2 = \frac{M}{a}$, and the Square of the Time is directly as the Mass, inversly as the Parameter. 915. * 909. * 911.

Neither must we proceed otherwise in other Curves. 916.

$$\text{The Curve } a^2x = y^3, \text{ gives } T^4 = \frac{M^3 v}{a^4};$$

$$ax^2 = y^3, \text{ gives } T^7 = \frac{M^3}{aav}.$$

$$x^3 = aay, \text{ gives } T^7 = \frac{a^4 M}{v^5}.$$

$$x^3 = ayy, \text{ gives } T^4 = \frac{aM}{v^2}, \text{ \&c.}$$

In the two last Cases the Solid is formed by the Conversion of the Curve round the Tangent in the Vertex.

BOOK

BOOK II.

Part II. Of the simple Collision of Bodies, direct, and oblique.

CHAP. IV.

Of the simple Collision of Bodies, direct.

Definition 1.

917. *THE Celerity, with which two Bodies come towards one another mutually, or are separated, is called the respective Celerity.*

918. *When both the Bodies tend to the same Part, they come towards one another, or are separated, with a Velocity, which is equal to the Difference of the absolute Velocities.*

919. *But the respective Velocity is the Sum of the absolute Velocities, if the Directions of the Motions are contrary.*

Definition 2.

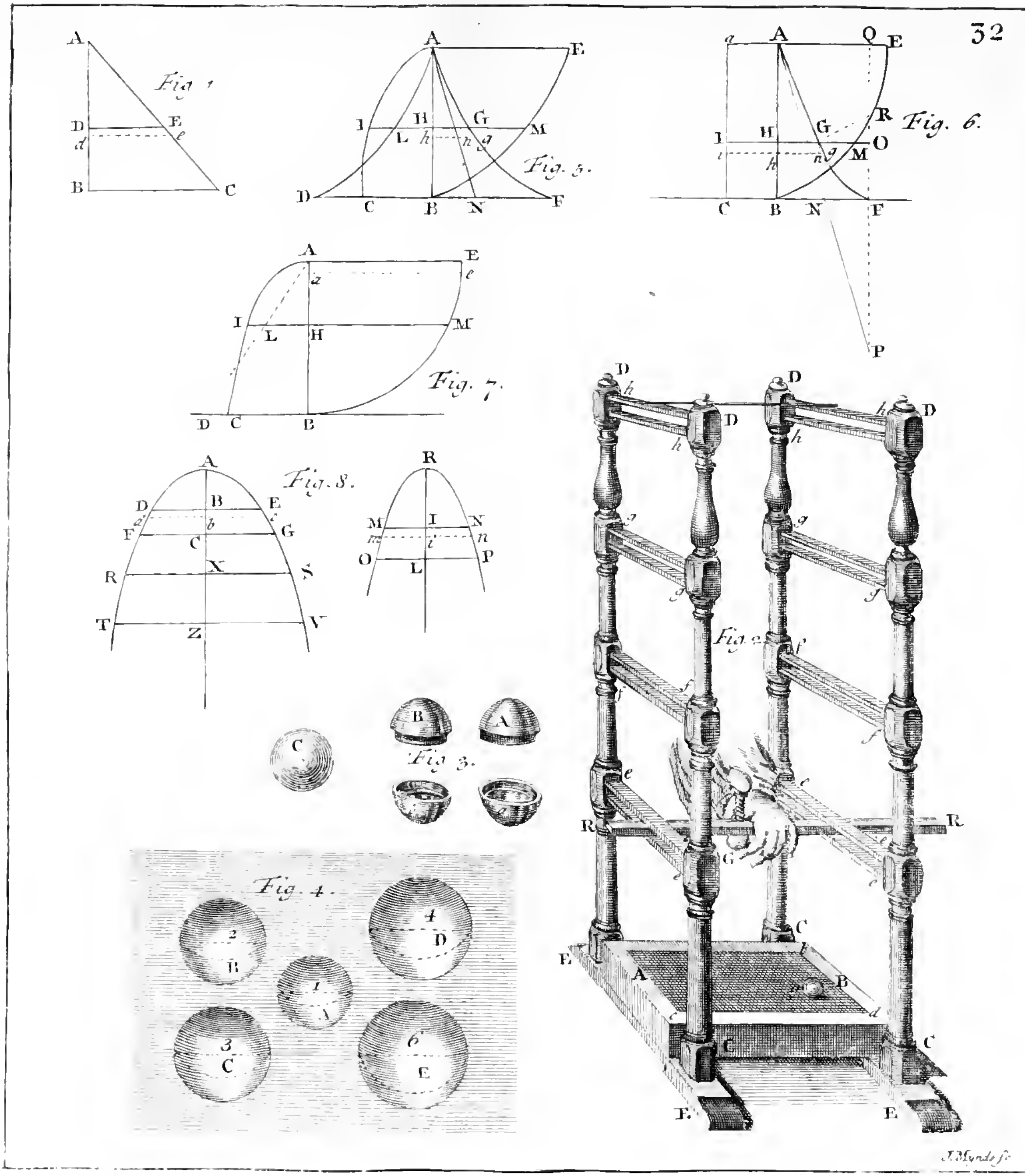
920. *The striking is said to be direct, when they so meet, that there is no reason, why they should turn towards one Part, rather than towards another; so that, before and after the Concourse, the Motion is in the same Line, if it is not all destroy'd.*

In such a striking, these three Things should concur. The Direction of the Motion, or Motions, when both Bodies are moved, should pass thro' the Centers of Gravity of each; this same Line, which passes thro' both Centers, should cut the Parts of the Surfaces, which run against one another mutually; lastly, these Surfaces, which mutually run against one another, should be perpendicular to the Line, which passes thro' the Centers of Gravity.

Definition 3.

922. *In every other Case the Stroke is said to be oblique.*

923. The Bodies, in which Collision takes place, are either hard or
* 66. soft; we know of none perfectly hard *; all that are said to be hard


32
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6.
Fig. 7.
Fig. 8.
J. Mynde sc.

hard by us are really elaſtick; therefore we muſt ſpeak of theſe and of ſoft ones: we ſhall briefly ſhew what would happen in every Caſe to perfectly hard Bodies.

Not every Concourſe of Bodies belongs to the ſtriking; Bodies may ſo come together, (and of this Caſe we ſhall ſpeak afterwards), that the Acceſs of the Surfaces may be with a Velocity infinitely ſmall, and this Action may be continued during Time, but no ſtriking is given here. 924.

This takes place, when a Surface comes to a Surface, upon which it acts immediately, with a finite Velocity, ſo that the Action is given from the innate Force. 925.

All Bodies, known to us, conſiſt of Parts cohering together by a Force, whoſe Effect we know, but whoſe Cauſe we are ignorant of: but no one will call it in queſtion, that the Parts do cohere together *by* a true *Preſſure*, to what Cauſe ſoever we attribute it. 926.

There is no Preſſure, which cannot be overcome by the ſmalleſt innate Force *; therefore *there is no Colliſion of Bodies without ſome Introceſſion of their Parts.* 927. * 703.

If there were Bodies perfectly hard, they would be broke by the ſmalleſt Colliſion; for in theſe the ſmalleſt Motion of the Parts is not without their Separation *. 928. * 65.

I ſhall ſpeak of the Colliſion of Bodies in general in this Chapter; therefore I muſt explain *what obtains in Bodies non-elaſtick*; for this alſo *takes place in thoſe that are elaſtick, in the Moment in which the Bodies concur*, before the Parts preſſed inwards return to their former Figure. 929.

By this reſtoring themſelves to their Figure, *elaſtick Bodies* mutually repel one another; therefore they *are ſeparated after the Stroke.* But there is no ſuch Action given, *if they are deſtitute of all Spring*; therefore *after a direct ſtriking they are not ſeparated*; for in this ſtriking, the Direction can't be changed *, *and* therefore if the Blow does not make them both reſt, *they both continue their Motion in the ſame Line, in which they mov'd before the Stroke*, and in which they are not mutually repell'd from one another. 930. 931. * 920.

Whilſt the Parts of Bodies are preſſed inwards, the Force * is deſtroyed, which exceeds the Preſſure, by which they cohere †; therefore *one Body cannot run againſt another, or two againſt one another mutually, without a Diminution of the Sum of the Forces* *; *if there is given a Colliſion of the Bodies.* * 708. † 926. 932. * 927.

In elaftick Bodies the Parts ftruck return to their former Figure, and returning prefs the Body, by whofe Action they were prefs'd inwards; by this Preffure a new Force is generated: but
933. we don't fpeak of this yet, *in elaftick Bodies* themfelves *there is a Diminution of the Forces, before the Figure is reftor'd, of which we fpeak here.*

934. *No Force is deftroy'd in the Collifion of Bodies, befides that by which the Parts are preffed inwards.*

935. Let us firft fuppofe the Bodies to tend towards the fame Part. That which goes foremoft neceffarily moves flower than the other, and is accelerated by the Stroke; but the hinder one, becaufe it acts on the other, lofes fomething of its Force. The Effect of the Force loft is the Increafe of the Force in the foremoft Body, as is alfo the Introceffion of the Parts; this Effect is equal to the
* 709. Force loft by the hinder Body *; but that, which the foremoft acquir'd, is not the Force deftroyed; therefore this only is deftroyed, by which the Parts yield inwards.

936. Secondly, let the Bodies tend towards contrary Parts. The Body, which runs againft a foft, and fixed Obftacle, lofes its whole Force by preffing the Parts inwards; for it produces no other Effect: and therefore lofes its whole Force by preffing the Parts inwards, becaufe the Obftacle has a fufficient Refiftance.

The Refiftance is not lefs, when the Obftacle is not fixed, but approaches towards the Body with a contrary Motion: wherefore the Body, in this Cafe, does not exert a lefs Effect by preffing the Parts inwards, and likewife confumes its whole Force by this Action.

But two Bodies being given, which are carried towards contrary Parts, each is an Obftacle in refpect of the other, and each confumes its Force by preffing the Parts inwards. But if one lofes its whole Motion before the other, in that Moment we have the Cafe now examin'd, and the Demonftration is univerfal.

937. But by Experiment we prove this Propofition, which is a Paradox, *viz.* that *Force never immediately deftroys Force.*

Experiment I.

938. We make ufe of the fame Machine *, by which Experiment 2. of
Plate XXVII. Fig. 1. the foregoing Chapter †, is demonftrated. But we apply to it
* 760. both the Rectangles *r*, and *s*; the Hooks, which fuftain the
† 827. Threads, being difpos'd, as is explain'd before ‖.
‖ 766.

To

To the Rectangle *r* we join the Cone H (*Plate* XXVIII. *Fig.* 7.), as in the Experiment mentioned *. * 829.

939. To the Rectangle *s* is added the Cylindrick wooden Box M (*Plate* XXVIII. *Fig.* 10.); this, for this Purpose, has the Screw *n*, which is put into the Cavity in the anteriour Surface of the Rectangle *. The anteriour Cavity *o* of this Box is filled with Clay; which is so scraped off by a thin Plate, or wooden Knife, whose Edge is rubbed over with Oil, that the Surface may be plain, and even; then the Box is weighed, and a little Clay is thrust into its hinder Cavity, as much as is necessary, to make the Weight of the Cylinder equal to the Weight of the Cone, applied to the Rectangle *r*. Many such Cylinders, thus prepared, are required, three or four at least. * 770.

We put the Cylinders into the Rectangles, and there fasten them, whereby their Weight is increased *; and indeed such, that each of the Masses may be equal to three. * 774.

940. The Rectangles are disposed in such manner, as was said before of one of them *; but Care must be taken that they are exactly in a horizontal Situation, at the same Height, and at the same Distance from the Board BC; then, the Rectangles being at rest, in the Situation which they acquire of their own Accord, the Threads joined to the Hooks *g* and *f*, are parallel, as those also, which pass over the Hooks *h* and *i*; the Vertex of the Cone, joined to the Rectangle *r*, answers to the Center of the Surface of the Clay in *s*, and there touches it. * 769.

The Ruler VX is fastened, and the Rectangle *r* is raised, as is said in the Experiment often mentioned *; the Index O is so applied, that it may answer to the tenth greater Division of the Ruler *. * 830. * 775.

941. The Rectangle is let down from this Division, and runs against the Rectangle *s* which is at rest with the Velocity ten, and carries it along with it, and impresses a Cavity in the Clay.

942. The Threads which were parallel when the Rectangles were suspended are not so now, because the Vertex of the Cone does not touch the Clay, as before, but has penetrated into it. They are to be reduced to a Parallelism. The Hooks *g*, *g*, *g*, and *h*, *h*, *h*, remaining, the Plates are turned, which determine the Distances of the intermediate Hooks *g*, *f*, and *h*, *i* *; and *f*, *f*, *f*, as likewise *i*, *i*, *i*, are moved towards the Middle, as much as is necessary, that the Parallelism may be restored, in which there is no Difficulty; and we fasten the Ruler YZ in such manner, that the End Y * 764.

may anſwer to the hinder Threads of the Rectangle *s*, when the
* 830. Bodies joined, hanging freely, are at reſt *.

943. The Cylindrick Box is taken away, which was join'd to *s*; and we ſubſtituted another, filled with Clay in the ſame manner. We place the Index ſo, that it may agree to the fifth greater Diviſion of the Ruler Y Z.

We raiſe the Rectangle *r* again, and it is let go againſt the quieſcent Body *s* with the Velocity ten; they aſcend together to the Index *q*, to which the hinder Threads of the Rectangle *s* come, but don't run againſt it.

The Cavity, which is impreſſed in the Clay, and in general the Effect of this Percuſſion, does not differ from the Effect in
* 941. the firſt Trial *; becauſe, in both Caſes, the ſame Force was acquired by the Body *r* in its Deſcent, which was the ſame; but, in the firſt Caſe, by reaſon of the Parallelism of the Threads being deſtroyed, we could not exactly meaſure the Velocity, after the Percuſſion, before that was reſtored.

944. We had the Body R, whoſe Maſs is three, that ſtruck, with
Pl. XXXIII. the Velocity ten, againſt the quieſcent Body S, whoſe Maſs is
Fig. 1. alſo three; after the Percuſſion they were moved together with the Velocity five, and a Cavity was made which we repreſent at A.

945. The wooden Cylinder *m* being taken away, which is joined to
Pl. XXXIII. S, we again ſubſtitute another, filled with Clay; the firſt Situa-
Fig. 2. tion of the Hooks is reſtored that the Bodies may be ſuſpended,
* 940. as they were at firſt *, the Situation of the Ruler Y Z *(Plate* XXVII.) is a little changed alſo, that the End Y may agree again to the hinder Threads of the Rectangle S. The Indices being ſo applied, that, on either Side, they may anſwer to the
* 775. fifth * greater Diviſion; from them the Bodies R, and S, are let down together, that they may meet in the middle, where they are at reſt. The Motion of both was deſtroyed; and the Cavity impreſſed was exactly equal to A.

946. In the firſt Caſe the Velocity of the Body R was ten, the
* 757. Maſs three; the Force therefore was 300 *. After the Stroke the
† 757. Maſs was ſix, the Velocity five, and Force 150 †. This Force only remained, and an equal Force was deſtroyed; which
* 934. could not be conſumed, without preſſing the Parts inwards *; for there was no other Effect.

* 757. In the ſecond Caſe, the Force of each Body was 75 *, and the whole Force deſtroyed alſo 150; but by reaſon of the equal Cavity,

Cavity, ſimilar to the former, this very Force was conſumed in making the Cavity *. * 934.

It plainly appears, that the Force 150 is required in every Caſe, that ſuch a Cavity may be made, Circumſtances being changed.

The Rectangle R, with its Cone *b*, is kept; the Maſs only is 947. changed, which is now ſix; this, with the Velocity five, ſtrikes Pl. XXXIII. againſt a fixed Obſtacle, as, in many Experiments of the forego- Fig. 3. ing Chapter *, Bodies were ſtruck againſt ſuch an Obſtacle; the * 827. 843. Cavity again is exactly equal to the foregoing, and the Force deſtroyed is alſo 150 *. * 757.

We ought alſo here to conſider that, which we took Notice of on another Occaſion concerning the Threads extended *, * 831. when the Weight was encreaſed.

Theſe Bodies cannot act mutually upon one another with a Motion common to the two Bodies; therefore the Stroke depends upon *the reſpective Velocity*, which *remaining the ſame, the Inten-* 948. *ſity of the Stroke will be the ſame, howſoever the abſolute Celerities vary*; *the Introceſſion of the Parts* depends upon this Intenſity, 949. which therefore *will be always the ſame, if two Bodies, run againſt each other mutually, with the ſame reſpective Velocity, with what Velocities ſoever they are moved.*

Experiment 2.

This Experiment is performed as the foregoing.

To the Rectangle R the ſame Cone *b* is join'd, but the Maſs is 950. equal to four *. The Maſs of the Rectangle S is three, and Pl. XXXIII. to it is join'd the Box with Clay *m* †. The Bodies are ſuſpended, Fig. 4. * 774. as is ſaid above ‖. † 939.

The Body R ſtrikes againſt the quieſcent Body S, with the ‖ 940. Velocity ſeven; a Cavity is made in the Clay. The Box *m* is taken away, and another is ſubſtituted. The reſpective Velocity was ſeven *. * 918. 919.

The Index *o* is ſo placed, that *r* by deſcending may acquire 951. the Velocity nine; this remaining, the Situation of the Ruler V X Pl. XXVII. is changed in ſuch manner, that the End X may anſwer to the Fig. 1. anteriour Threads of the Rectangle *s*, this being at reſt. Then *s* is raiſed towards V, to ſuch an Height that the ſame Threads may anſwer to the ſecond greater Diviſion of the Ruler X V, and the Index *p* is ſo placed, that its Point may agree to the inmoſt of the ſaid Threads. This Index, which is repreſented by itſelf

at

* 777. at P, and p, (*Plate* XXVIII. *Fig.* 6 *.), conſiſts of a Screw, that, its Place being kept, it may approach towards, and be removed from the Thread, to which it anſwers; but this Screw is ſo turned, that, the Index remaining, the Bodies may paſs near it freely, the inmoſt Threads paſſing at a ſmall Diſtance only from the Point. If s be now raiſed, that the anteriour Threads may anſwer to the Index p, it will by deſcending acquire two Degrees of Velocity towards Z.

952. Pl. XXXIII. Fig. 5. The Bodies R and S being raiſed together, that the Threads may reſpectively anſwer to their Indices; let them be let down at the ſame Time, and they will come to the loweſt Place, that

* 407. is, the middle of the Machine, at the ſame Time alſo *; and they will meet there, whilſt they are carried towards the ſame Part, the foremoſt with two Degrees of Velocity, the hindermoſt

* 918. with the Velocity nine: the reſpective Velocity was ſeven again *. The Box m is removed, and we ſubſtitute another Box like it.

Pl. XXVII. Fig. 1. The Situation of the Ruler X V is reſtored, that the End X may anſwer to the hinder Threads of the Rectangle r. The End Y, of the other Ruler, ſhould anſwer to the hinder Threads of the Rectangle s in the ſame Manner. We ſuppoſe the Rectangles to be at reſt.

953. Pl. XXXIII. Fig. 6. The two Bodies R and S are let down in ſuch manner, that being carried towards contrary Parts, S acquires two Degrees of Velocity in deſcending, R five; with theſe Velocities they meet

* 407. in the Middle of the Machine *, and in the Percuſſion the re-

† 919. ſpective Velocity is ſeven †.

954. In theſe three Percuſſions the reſpective Velocity was the ſame, namely ſeven; the three Cavities made likewiſe are exactly equal.

955. Now I think I have ſufficiently explained how Bodies may with any Velocities whatſoever mutually ſtrike againſt one another, whether we conſider conſpiring, or contrary Motions; alſo how the Velocity, common to both, may be meaſured after the Stroke: for this Reaſon it will be unneceſſary in what follows, to explain farther in like Experiments, what relates to the Diſpoſitions of the Machines.

* 949. Let us now ſee what follows from the laſt Propoſition *. Equal

† 841. Forces are conſumed in making equal Cavities †; no Force is loſt

‖ 934. beſides that, which is conſumed in making the Cavities ‖; therefore *howſoever two Bodies are moved, if the reſpective Velocity is*

* 949. *the ſame, the ſame Force will be deſtroyed by the Stroke* *.

Expe-

EXPERIMENT 3.

This Experiment is made as the foregoing one; all Things are performed in the same Manner; but every Thing must be repeated, of the three Trials *, those Things being observed which were explained in N°. 942. that we may determine the Velocity after the Percussion each Time; but we discover the following. 957. Pl. XXXIII. Fig. 4 5. 6. * 950. 952. 953.

In the first Case * the Bodies are carried by a common Motion with the Velocity four. In the second † with the Velocity six. In the last ‖ they have only two Degrees of Velocity. * 950. † 952. ‖ 953.

In the first Case, before the Percussion, the Body R only was moved. The Mass was 4, Velocity 7; therefore the Force 196*. After the Percussion the Bodies were joined, and the Mass was equal to 7, the Velocity 4; the Force was 112; therefore the Force 84 was lost by pressing the Parts inwards. 958. * 757.

In the second Case the Force of the Body R, was $4 \times 81 = 324$; the Force of the Body S was $3 \times 4 = 12$; therefore the Sum of the Forces was 336. After the Stroke the Force was $7 \times 36 = 252$. Therefore the Force destroyed was also equal to 84.

In the last Case the Forces before the Percussion were 100 and 12; the Sum of which is 112. 84 also exceeded the Force 28 remaining after the Stroke.

Since the Force destroyed by the Stroke, the same Bodies remaining, and the same respective Velocity, is always the same, it will be sufficient to determine this in one Case; and it will be given in all the rest. 959.

If two Bodies, whether equal, or unequal any how, carried towards contrary Parts, run against each other mutually, their Motion may be so compounded, their respective Velocity being given, that either of them may carry the other along with it after the Stroke; whence it follows, that the Case is given, in which they *are at rest after the Stroke.* 960.

In this Case *the Sum of the* absolute *Forces is* equal to the Force destroyed in every Case, the same respective Velocity being given *. In this same Case this Sum is *the smallest of all, the respective Velocity being kept:* for if a less Sum were given, a less Force would be lost by the Stroke, which is impossible*. * 956. * 956.

But we shew in the first Scholium following, that *this Sum is the smallest of all, if, the Directions being put contrary, the Celerities are inversely as the Masses,* and that it is least in this Case only. 961.

Whence

962. Whence therefore it follows, in this Caſe only, that *Bodies carried towards contrary Parts, and running againſt one another, are at reſt after the Stroke, if the Velocities are inverſely as the*
* 960. *Maſſes* *.

963. But in this Caſe the Forces themſelves are as the Velocities,
* 791. that is, they are unequal *, if the Bodies are unequal; which ſeems to be a great Paradox. For this Reaſon I will demonſtrate the Propoſition itſelf alſo directly, that it may appear from the Nature of Percuſſion, that this Inequality is altogether neceſſary that there may be Reſt given, the Bodies being ſuppoſed unequal.

964. Let us ſuppoſe two Bodies, carried towards contrary Parts, and running againſt one another directly; they conſume their Forces in ſuch manner by preſſing the Parts inwards, whilſt they either become flat; or one penetrates into the other, that, after the firſt Contact, the Bodies run through a certain Space; the Parts in the mean time receding between thoſe next to them.

An equable Coheſion is not to be overcome through this whole Space; but if we ſuppoſe this Space divided into very ſmall Spaces, the Reſiſtance to be overcome in each of them may be looked upon as equable through the whole ſmall Space; and each Body will overcome ſome of this Reſiſtance, by moving its Particles between thoſe next to them, according to the Ratio of the Part of ſmall Space run through by it: but two Bodies, carried towards contrary Parts, do indeed together run through a whole ſmall Space; but the Parts of it, run through by each, are as the Velo-
* 119. cities *; in which ſame Ratio are the Reſiſtances of the Coheſion
† 361. overcome; which are as the Actions of the Bodies †; or as the
|| 709. Forces loſt ||.

965. Therefore *in every Colliſion of two Bodies, running againſt each other with contrary Motions, the Decrements of the Forces, in each of the infinitely ſmall Moments, are as the Velocities of the Bodies, in theſe very Moments.*

Which Rule takes place 'till one of the Bodies loſes its whole Force; which is then repelled by the other, and acquires new Force. But if both Bodies loſe their Forces, at the ſame Time, they are at reſt at the ſame Moment; and this is the Caſe, which we muſt examine.

966. Let us ſuppoſe *two Bodies, carried towards contrary Parts, and running againſt each other* with Velocities, which are inverſely as
791. the Maſſes; the Forces will be as the Velocities *.

In the first Moment, after the Surfaces have mutually touched each other, the Decrements of the Forces, which are as the Velocities *, are as the Forces themselves; and the remaining Forces as * 965.
the first Forces †; in which same Ratio are the remaining Velo- † 19. El. 5.
cities ||. || 791.

The same Way of Reasoning may be applied to the second, and following Moments; and, in each, the Decrements of the Forces are as the Forces themselves; which therefore are consumed in the same Time: wherefore the Bodies *are at rest at the same Moment*; which the same Demonstration evinces to obtain *in this Case only, in which the Velocities are opposite as the Forces.*

From this also it appears, that the Decrements of the Velocities, 967.
in every Collision, each Moment, are inversely as the Masses. For if, the respective Velocity remaining, the Motions be any how changed, those Things are not altered, which immediately depend upon the Stroke *; and what has been demonstrated in a peculiar * 948.
Case concerning the Decrements of the Velocities, may be referred in general to *the Changes of the Velocities*, in any Collision whatsoever.

From the foregoing Demonstration it follows, that *unequal Bo-* 968.
dies, carried towards contrary Parts, are not at rest by a mutual Concourse, except they have unequal Forces; about which Inequality of Forces, I will addhere some Experiments, worthy to be taken Notice of.

Experiment 4.

The Body R with it's Cone *b*, which we made use of in the 961.
foregoing Experiments, whose Mass is nine, and Velocity two, Pl. XXXIII.
strikes against the Body S, join'd to the Box *m* having Clay in Fig. 7.
it, and whose Mass is equal to two; whilst this is carried towards the contrary Part with the Velocity nine. The Bodies, with the said Velocities, which are inversely as the Masses, meet in the middle of the Board, and are at rest; and make a Cavity which we represent at B.

By this Experiment we immediately confirm the Proposition 970.
itself; what relates to the Inequality of the Forces will appear, if we compare this Experiment with the following one.

Experiment 5.

971. The Maſſes of both Bodies R, and S, are equal to two, the
Pl. XXXIII. Velocity of each is nine, being carried towards contrary Parts they
Fig. 8. meet, and are at reſt, and make the Cavity C.

972. The Maſſes being changed, that they may be nine, if each Bo-
Pl. XXXIII. dy runs againſt the oppoſite one, with two Degrees of Velocity,
Fig. 9. they will be at reſt again; and the Cavity will be D.

973. It is manifeſt, that the Bodies, in theſe Circumſtances, ought to be at reſt, but it will ſeem ſtrange to him, who did not well
* 966. underſtand the foregoing Demonſtration *, that the Cavities are unequal; for this Reaſon I will add ſome other Experiments.

974. The Force, which is conſumed in the foregoing Experiment, whereby the Cavity B was made, is equal to half of the whole Force, which was deſtroy'd in both Trials of this Experiment, and whereby the Cavities C, and D, were made: for this Rea-
* 841. ſon it follows from what has been demonſtrated before *, that the Cavity B is equal to half of the Sum of the other Cavities C, and D; ſo that it equally differs from each of theſe: which agrees with the Experiment itſelf, as is diſcovered by meaſuring the Cavities.

975. But theſe are eaſily meaſured by applying the Sector, whereby ſimilar Solids, whoſe homologous Sides are the Diameters of the Cavities, are compared with one another; if we have not ſuch a Sector at hand, the following Experiment will be ſufficient.

Experiment 6.

976. The Body R, whoſe Maſs is nine, ſtrikes againſt a fixed Ob-
Pl. XXXIII. ſtacle with two Degrees of Velocity, it makes the Cavity E.
Fig. 10. The ſame Body, the Maſs being changed, that it may be two,
Fig. 11. ſtriking a fixed Obſtacle with the Velocity nine, made the Cavity F.

977. The Inequality of theſe Cavities demonſtrates the Inequality of
* 969. the Forces in Experiment 4 *; theſe are inverſely as the Maſſes †,
† 791. and the Cavities are in the ſame Ratio ‖; the Sum of theſe is equal
‖ 841. to the Cavity B, made in the ſaid fourth Experiment, as appears by meaſuring the Cavities; but it is alſo evinced by Experiment.

Ex-

EXPERIMENT 7.

Let the Body R run twice againft the fame Place of the Surface of the Clay in fuch manner, that it may the fecond Time enlarge the Cavity made the firft Time, if in one Cafe the Mafs is nine, the Velocity two; in the other the Mafs two, and Velocity nine; the whole Cavity will be equal to the Cavity B of Experiment fourth *. 978. Pl. XXXIII. Fig. 12.

* 850.

979. Pl. XXXIII. Fig. 13. In this fourth Experiment the Force of the Body R was $9 \times 2 \times 2 = 36$; the Force of the Body S was equal to $2 \times 9 \times 9 = 162$*; the Sum is 198. If a Body, whofe Mafs is nine, and that has the fame Cone *b*, as before, runs againft a fixed Obftacle with the Velocity four and feven tenths, it makes a Cavity, which is alfo equal to the faid Cavity B. The Force, deftroy'd by this Stroke, is alfo equal to 198, at leaft it fcarce differs any thing from it; which demonftrates that fuch alfo was the Force deftroy'd in the fourth Experiment *; which again puts out of doubt the Inequality of the Forces, in that Experiment.

* 841.

If a Cafe being given, in which the Bodies are at reft after the Stroke, the lefs Force be increafed, but fo as not yet to equal the Force of the other Body, *the Body, whofe Force fhall be leaft, fhall compel the Body moved with the greater Force to go back.* 980.

The Body that moves fafteft, although it has the greater Force, confumes its Force in a fhorter Time, by preffing the Parts inwards, and is repelled by the other, which has a Force remaining.

EXPERIMENT 8.

Let us fuppofe the Bodies R, whofe Mafs is equal to two, and S, whofe Mafs is nine, carried towards contrary Parts, to run againft each other, this with the Velocity four, that with the Velocity twelve; R is repelled by the Stroke, and S continues in motion, carrying R along with it, with a Velocity, which exceeds one Degree. 981. Pl. XXXIV. Fig. 1.

S being removed, the Body R, its Cone *b* being kept, ftrikes againft a fixed Obftacle, with the fame Velocity twelve; it impreffes the Cavity A. 982. Pl. XXXIV. Fig. 2.

The Mafs is changed, that it may be equal to two, and it ftrikes againft another Part of the fixed Obftacle with the Velocity four, with which S was moved in the laft Collifion, and the Cavity 983. Pl. XXXIV. Fig. 3.

vity is B, which is very much exceeded by the other; tho' this Body does not loſe all its Motion by the Stroke, and carries the other along with it.

984. *When two Bodies run againſt each other, there are given two actions, and two Re-actions, each Action is equal to it's Re-action.* That the Bodies may be at reſt after the Stroke, it is requiſite, that each Body may ſuffer ſuch a Reſiſtance, by which being given, it may conſume its Force by acting, which cannot happen, when the Bodies are unequal, except the Forces are unequal.

From what has been demonſtrated we deduce, that Bodies being
985. given, and their reſpective Velocity, that *the Force deſtroy'd by the Stroke is determined*, if the Sum of the Forces be determined, ſuppoſing the ſame reſpective Velocity, contrary Motions, and the Ve-
* 959. 960. locities in an inverſe Ratio of the Maſſes *. But we demonſtrate in the Scholia that this Sum is given, *if the Product of the Maſſes be multiplied by the Square of the reſpective Velocity, and divided by the Sum of the Maſſes.*

EXPERIMENT 9.

986. The Body R, whoſe Maſs is four, ſtrikes againſt the quieſcent
Pl. XXXIV. Body S, whoſe Maſs is equal to two, with the Velocity nine; it
Fig. 4. makes the Cavity C; to which we have one exactly equal, if a Body, whoſe Maſs is three, runs againſt a fixed Obſtacle, with the
Fig. 5. Velocity ſix.

* 757. In this laſt Caſe the Force deſtroy'd is $3 \times 36 = 108$ *. An e-
† 826. qual Force was deſtroy'd in the firſt Caſe †. But we diſcover this by multiplying the Product of the Maſſes 8 by 81, the Square of the reſpective Velocity nine; and dividing the Product 648 by the Sum of the Maſſes 6.

From what has been demonſtrated concerning Bodies that are at reſt after the Stroke, we deduce Rules, by which, the Velocities of Bodies after the Stroke, are determined, in every Caſe.

987. *Let the Bodies be moved, either towards the ſame Part (Fig. 1.),*
Pl. XXXVII. *or towards contrary Parts (Fig. 2.)*, and let the Maſſes be as AB
Fig. 1. 2. and BC; let the Velocity of this be BE; of that BN: the re-
* 918. 919. ſpective Velocity will be EN *. Let this be divided in I in ſuch manner, that IN may be to IE, as BC is to BA; and BI will be the Velocity, with which both Bodies are carried *after the Stroke*; for *the Changes in the Velocities are in an inverſe Ratio of*
† 967. *the Maſſes* †, BC acquires EI, whilſt AB loſes NI. If we

con-

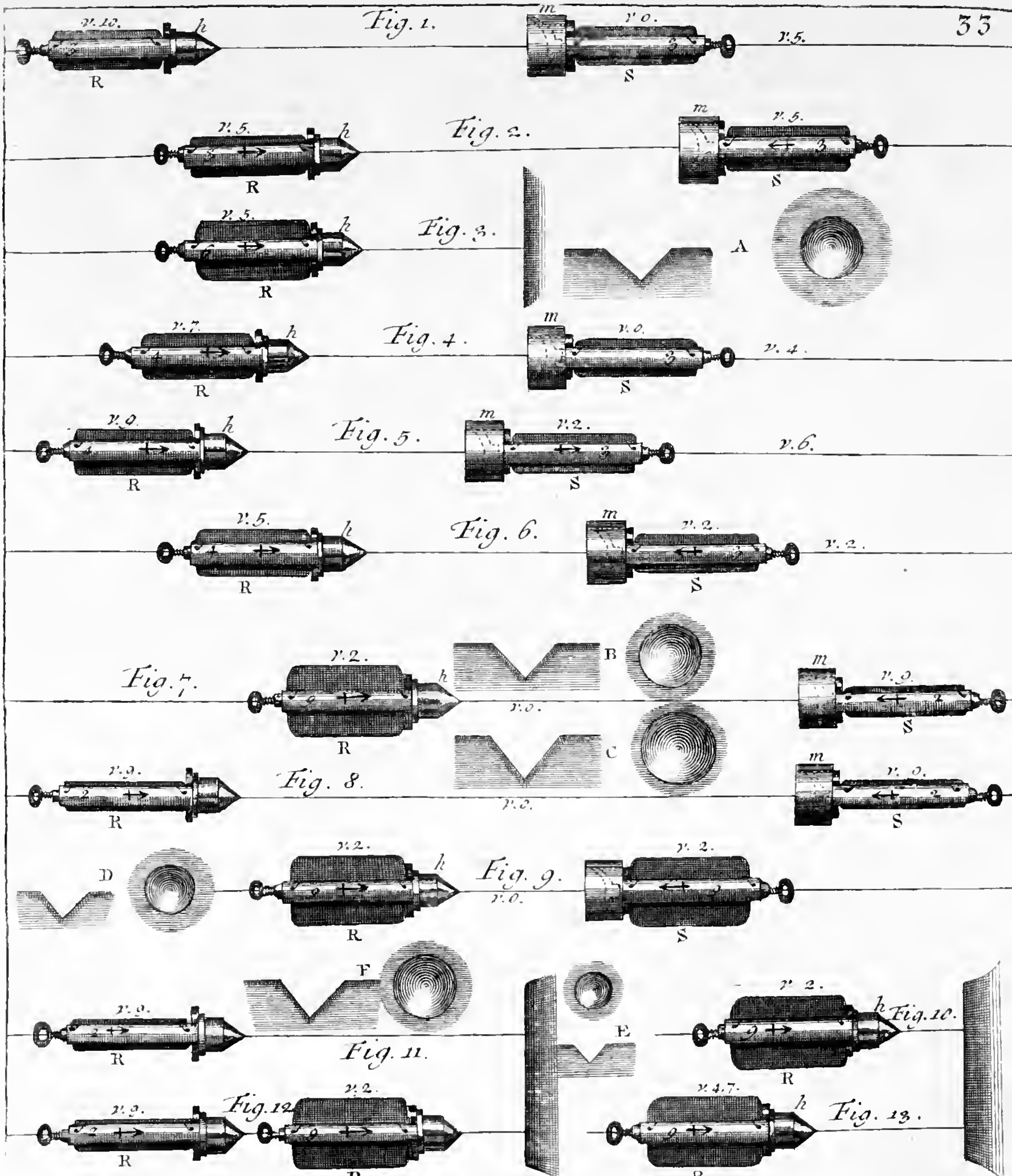


Fig. 1.
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Fig. 9.
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Fig. 11.
Fig. 12
Fig. 13.
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conceive a Ship to be carried with the Velocity B I, and the Body B C be moved in it, with the Velocity I E, from the Prow to the Stern, it has the absolute Velocity B E; and the Body A B be carried from the Stern to the Prow with the Velocity I N, this will have the absolute Velocity B N; these Bodies, when they are carried in the Ship with contrary Directions and Velocities, which are inversely as the Masses, will be at rest in the Ship after the Stroke ||; || 962.
that is, they will be carried with the same Velocity with the Ship.

B I is determined by an easy Rule, which that we may discover, let the Rectangles B M, B F, be the Products of the Masses by their Celerities, and let the Parallelograms A O and C D be compleated. D O being drawn, this cuts B N in I; for the Triangles D I E and I N O are similar; and I N is to I E, as N O, or B C, is to D E, or A B. Through I let K L be drawn, parallel to A B, and the Complements I M, I F, will be equal *; * 43. El. 1.
988. therefore *in Bodies tending towards the same Part*, *if* from *the Sum of the Products* B M, and B F, *of the Masses by their Velocities* we Fig. 1.
substract M I, and substitute I F in its Place, the foresaid Sum will be equal to the Rectangle A L; which if it *be divided by* A C, *the Sum of the Masses*, *the Quotient of the Division will give* A H, or B I, *the Velocity common to the Bodies after the Stroke.*

EXPEIMENT 10.

989. Against the Body S, whose Mass is three, and Velocity three, the Body R runs, whose Mass is equal to two, and which tends Pl. XXXIV. towards the same Part with the first with the Velocity thirteen. Fig. 6.
After the Percussion the Velocity common to both is seven. We discover this by multiplying 3×3 and 2×13. The Sum, of the Products 9, and 26, is 35. This being divided by the Sum of the Masses 5, we have seven.

We have before explained how any Velocities whatsoever may be impressed on Bodies tending towards the same Part *; we have * 951. 952.
seen also how the Velocity after the Stroke may be measured †. † 779. 942.

990. *If the Bodies tend towards contrary Parts, and* we substract M I from the greater Product BM, and substitute I F, we have B M Pl. XXXVII.
equal to the Figure A H L F E B; from which if we substract the Fig. 2.
Product B F, we have H C *the Difference of the Products of the Masses by their Velocities*; but *if this be divided by the Sum of the Masses* A C, *the Quotient will be the Velocity sought* B I; *which*

is

is directed towards the same Part with B N : that is, both Bodies, the Velocity being discovered, are carried towards the same Part, *with the Body, whose Product of the Mass by the Velocity exceeds the similar Product of the other.*

Experiment 11.

991. Pl. XXXIV. Fig. 7. The same Bodies, which were made use of in the foregoing Experiment, run against each other in this also, but with contrary Motions; R with the Velocity five; S with ten Degrees of Velocity, and both Bodies, after the Percussion, have the common Velocity four; which is directed towards the same Part with the Motion of the Body S before the Percussion.

The Products of the Velocities by the Masses are 30 and 10, the Difference 20, divided by the Sum of the Masses five, gives four.

992. *If one Body is at rest*, it follows from both Rules, that *the Product of the Velocity by the Mass of the Body moved must be divided by the Sum of the Masses.*

Experiment 12.

993. Pl. XXXIV. Fig. 8. The same Bodies being given again; let R run against S which is at rest with the Velocity ten, and the Velocity of both after the Stroke will be four.

The Product of the Velocity by the Mass is 20; this being divided, by the Sum of the Masses five, gives four.

994. In these Demonstrations we have considered the respective Velocities, and have applied the Conclusions to the absolute Velocities, and in N. 956, to determine the Force destroy'd in Collision, we considered the respective Action only. This Way of Reasoning takes place by considering both these Velocities, because the respective Velocity can't be changed, without the same Change being made in the absolute Velocities. The Force also which is consumed by pressing the Parts inwards is a Diminution of the absolute Force, though it depends upon the respective Action, and follows the Ratio of this Action.

995. In the others *the respective Action must be distinguished from the absolute*; for the same respective Change gives different Changes of the Forces, according to the different absolute Forces before the Concourse; and a less respective Action of the same Body, moved in

in the same Manner, upon another determined Body, can communicate a greater Force to it.

EXPERIMENT 13.

Let the Mass of the Body R be two, the Velocity ten; let this run against the quiescent Body S, whose Mass is eight; after the Stroke both Bodies are moved with two Degrees of Velocity: which agrees with the foregoing Rule *. 996. Pl. XXXIV. Fig. 9. * 992.

The same Body R, its Mass being kept, strikes with the same Velocity ten against the Body S, whose Mass is eight, as before, but it is carried with the Velocity five towards the same Part. The Velocity common to both after the Percussion is six; which again agrees with what is said before *. 997. Pl. XXXIV. Fig. 10. * 988.

In the first Case the quiescent Body S, acquired two Degrees of Velocity, and therefore the Force 32 *; by the Action of the Body R. 998. * 757.

In the second Case S had the Force $25 \times 8 =$ † 200. After the Stroke it had $36 \times 8 = 288$; and, by the Action of the Body R, it acquired the Force 88. Which might be easily confirmed by Experiments, if it had not been abundantly confirmed by Experiments in the foregoing Chapter, that the Effects of Forces are as the Products of the Masses by the Squares of the Velocities. † 757.

The Motion of the Body R, in both Cases was the same; and although, in the first Case, it acted upon the Body S, with a greater respective Action, yet in the second Case it communicated to it almost a triple Force: nevertheless we demonstrate that all Things agree well together. 999.

The respective Velocity was double in the first Case, and the Velocity communicated double; for in this Case S acquired two Degrees of Velocity, and in the second Case only one. 1000.

The respective Velocity doubled gives the respective Action, which is as the Square of the respective Velocity, that is, quadruple; in the first Case also such a Cavity is discovered, if it be compared with the Cavity in the second Case. 1001.

These Things relate to the respective Motion, let us now consider the Motions themselves of the Bodies.

In both Case, before the Stroke, R had the Force 200 *. In the first Case, after the Stroke, it had the Force 8 remaining, therefore it lost 192. 1002. * 757.

In

In the ſecond Caſe, after the Colliſion, it had the Force 72 remaining; and it loſt 128.

* 934. 985. In the firſt Caſe the Force, deſtroy'd in making the Cavity, is 160 *; and the Body R communicated thirty-two Degrees of Force to S. In the ſecond Caſe R indeed loſt leſs Force, only 40 Degrees were deſtroy'd in making the Cavity †; therefore it communicated the Force 88 to S.

† 954, 985.

* 712. The whole Effects are proportional to the Forces, deſtroy'd by acting *. For this Reaſon when a Body by acting produces many different Effects, all together muſt be conſidered, if we would from theſe determine the Force, which the Body has loſt by acting.

1003. *A Body in motion, may communicate motion to another Body,* without ſtriking, *by acting upon it by Preſſure only; in which Caſe,* if the Preſſure, whereby the Parts cohere together, overcomes the mutual Preſſure of the Bodies, *there is no* Introceſſion of the Parts, and no *Force deſtroy'd* * *:* And therefore the Sum of the Forces before and after the Action is the ſame.

* 934.

1004. Pl. XXXV. Fig. 1. But that we may demonſtrate how Bodies in motion, may communicate Motion to other Bodies, by a Preſſure upon them, without ſtriking, we muſt conceive a Body Q, which is formed by the Revolution of the Figure *a b c d*, which is terminated by a Semi-circle and two Quadrants, round the Axis *a c*.

Let this be at reſt, tho' the Demonſtration may be applied to a Body in motion alſo; let us farther conceive the two Bodies P, P; we conceive two, that the Action on the Body Q may be direct; the Reaſoning is the ſame, as if we had to do with one only; let theſe be moved with equal Velocities, in Directions parallel to each other, and to the Axis of the Body Q; let them alſo be mov'd ſo, that when they come to Q, the Surface of the Body Q may touch the Bodies P, P, in Points, in which this Surface is parallel to the Direction of the Motion. Therefore the Bodies P, P, exert no Action on the Body Q, in the Moment in which they come to it: whilſt they continue their Motion along the hollow Surfaces, *a d*, *a b*, they preſs the Body Q, which not being retained yields, and whilſt the Preſſure is continued, Q is accelerated, as long as the Bodies P, P, remain applied to Q *; but they forſake it, when the Bodies P, P, come to the Points *b* and *d*, in which the Directions of the Motions of the Bodies P, P, are perpendicular to the firſt Direction, in which they came to the Body Q. I ſhall explain in the laſt

* 697.

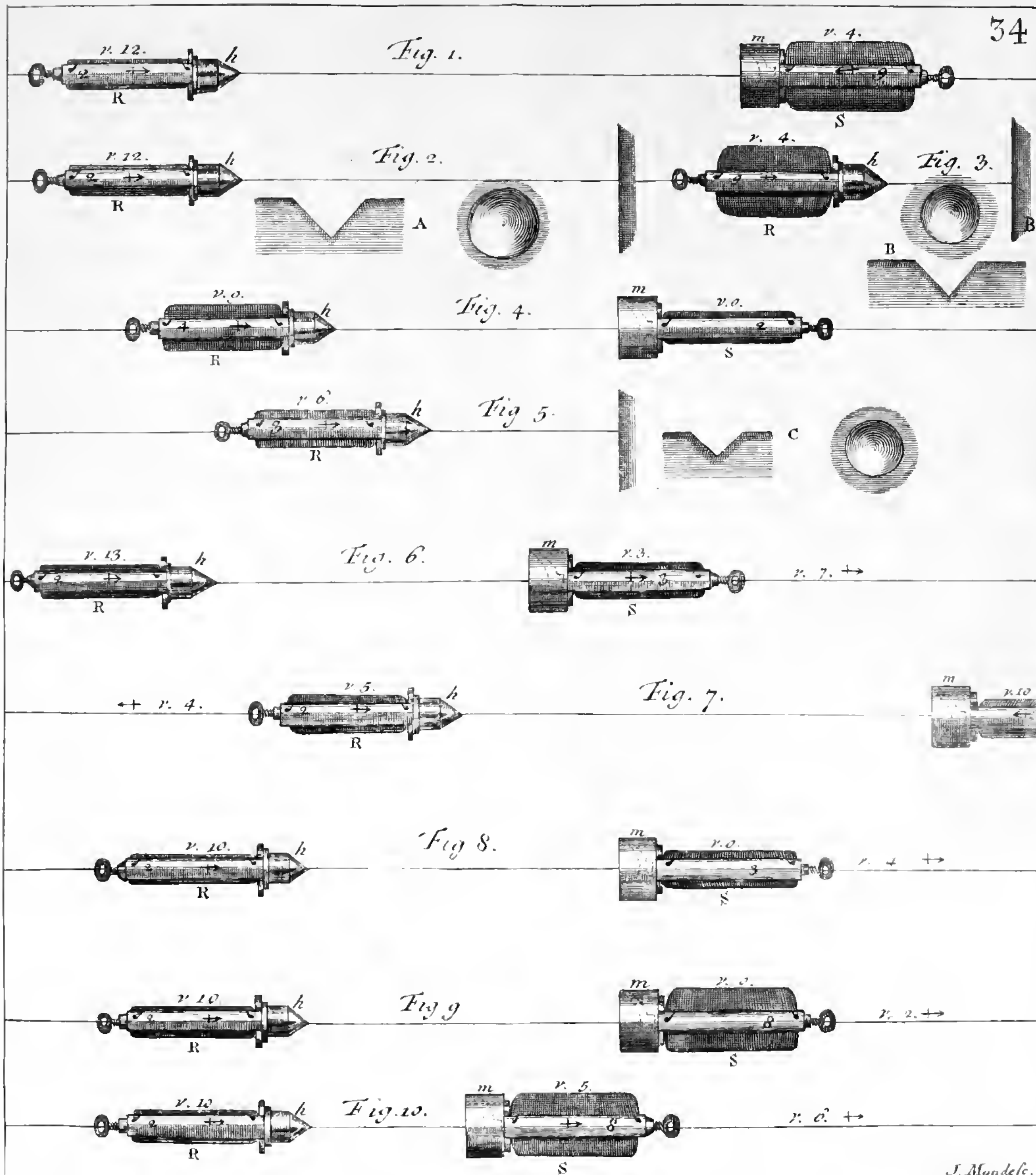
Fig. 1.
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laſt Scholium of the tenth Chapter of this Book, how we may determine the Velocities of theſe Bodies.

This Preſſure exerts no Effect beſides the Motion, which it communicates to the Body Q; and therefore the Bodies P, P, loſe ſo much of their Forces, as the Body Q acquires *. In theſe we ſet aſide the Friction, which cannot be given without ſome Introceſſion of the Parts; and therefore without a Deſtruction of the Forces. But in Scholium 3, of Chapter 10, of this Book, we determine theſe very Motions after the Concourſe. * 709.

If a Body as P, preſſes an Obſtacle, with a ſimilar Action, which is not moved by this Preſſure, as A B C, and whoſe Parts cohere together with a ſufficient Stiffneſs, ſo as not to yield to this Action, the Velocity of the Body will not be changed; in this Caſe the Preſſure of the Body upon the Obſtacle is indeed deſtroy'd, by the Reſiſtance of the Obſtacle; but as there is no Introceſſion of the Parts, nor Force communicated, the Force of the Body P is not diminiſhed; thus a Body, which deſcends along an inclined Plane, is accelerated in the ſame Manner, as a Body that falls freely, if they both deſcend to the ſame Depth *; tho' the firſt preſſes the Plane. In theſe Caſes, that which retains the Obſtacle in its Place, deſtroys the Action of the Body, and communicates a Force to the Body equal to that, which the Body loſes by its own Action; wherefore the Force of the Body is not changed, with reſpect to the Quantity. 1005. Pl. XXXV. Fig. 2. * 393.

But if we conſider the Force itſelf, it is really changed, whilſt the Direction is varied; for the Motion in a certain Direction is not a Motion in another Direction. Whilſt the Body P runs through the Curve A B C, in each of the Points it loſes a ſmall Part of its Force, and acquires an equal Force in another Direction; but when, by a continual Inflexion, the Direction changed, makes a right Angle with the firſt, the Motion of the Body has nothing common with the firſt Motion, and has loſt its whole Force, and acquired a new Force, equal to the former. 1006.

Hence it appears, that *by the Action of a Body its Force, and therefore Velocity, is not diminiſhed, without a Motion of the Obſtacle itſelf, or the Parts that conſtitute it, ariſing from this Action.* 1007.

Mechanics ſhould have regard to this Propoſition, that they may hinder all tremulous Motion in Machines, and Agitation of the Parts ariſing from thence; for by theſe the Labour in the Uſe of a Machine is encreaſed, and the Machine itſelf, in a ſhort time, becomes unfit for the Uſe for which it is deſigned. 1008.

SCHOLIUM I.

The Demonstrations of N. 961. 985.

1009. LET there be two Bodies A and B; let the Velocity of this be b; the Celerity of that a; the respective Velocity, if they are carried towards contrary Parts, is $a+b$ *; we call this d. The Sum of the Forces is $Aaa+Bbb$, which, the respective Velocity remaining, we said was the smallest of all, putting $A:B::b:a$ †, that is, $Aa=Bb$.

* 919.

† 961.

For such Velocities being given, let a be increased by any Quantity e; the Force of the Body A will now be $Aaa+2Aae+ee$. The Velocity of the Body B, because the respective Velocity remains $d=a+b$, will be $b-e$; for $a+e+b-e=a+b$; therefore the Force of the Body B will be $Bbb-2Bbe+Bee$, and the Sum of the Forces is $Aaa+Bbb+Aee+Bee+2Aae-2Bbe$.

But by reason of $Aa+Bb$ the two last Terms mutually destroy each other, and the Sum is equal to $Aaa+Bbb+Aee+Bee$, which exceeds the first. It is a like Demonstration, if the Velocity b be encreased, the same Quantity being diminished, by the Velocity a; whence appears the Demonstration N. 961.

1010. We put $A:B::b:a$; by Composition $A+B:B::b+a=d:a$; therefore $a=\frac{Bd}{A+B}$, in like Manner $b=\frac{Ad}{A+B}$; therefore the Sum of the Forces $Aaa+Bbb=\frac{ABBdd+BAAdd}{\overline{B+A}^q}$ by dividing the Numerator and Denominator by $B+A$ this Quantity is equal to $\frac{ABdd}{B+A}$ as we observed in N. 985.

SCHOLIUM II.

The Algebraick Demonstrations of N. 988. 990.

WE have demonstrated geometrically the Rules of N. 988. 990. these are very easily deduced algebraically from the Proposition of Number 987.

1011. Let the Body A be moved with the Velocity a; the Body B moved with the Velocity b: the respective Velocity is $a-b$, if the Bodies tend towards the same Part *; this is destroy'd by the Stroke †, and is the Sum of the Changes in the Velocities of the Bodies after the Stroke. B is to A, as the Change of the Velocity in A is to the Change of the Velocity in B ‖; and by Composition, the Sum of the Masses $A+B$ is to A, as the Sum of the Changes $a-b$ is to the Change of Velocity of the Body

* 918.

† 931.

‖ 987.

Body B, which Change therefore is $\frac{Aa - Ab}{A + B}$. When the Velocity b is less than the Velocity a, it is encreased in the Percussion: therefore the Velocity of the Body B, that is, the Velocity of each Body * after the Stroke, is * 931.

$$b + \frac{Aa - Ab}{A + B} = \frac{Bb + Aa}{A + B}$$ as is had in N. 988.

The respective Velocity being put $a + b$, namely the Bodies 1012.
tending towards contrary Parts *, by a like Way of reasoning the Rule of * 919.
N. 990. is discovered.

Both these Rules concerning the Collision of Bodies, may be also deduced from what has been demonstrated, about the Quantity of the Force lost *; which Demonstration I will here subjoin, that the Strength * 985, 1010.
of those Things, which are demonstrated above about innate Forces, may appear more clearly; whilst from them, by Ways intirely different, we deduce the Rules confirmed by Experiments.

Let there be again the Bodies A and B; the Velocity of this b, of that a; 1013.
let them tend towards the same Part, and the respective Velocity will be $a - b$.

The Sum of the Forces before the Stroke is $Aaa + Bbb$*; the Force de- * 757.
stroy'd by the Stroke is $\frac{ABaa - 2ABab + BBbb}{A + B}$ †; by subtracting this † 985. 1010.
from the Sum of the Forces, we have the Force remaining after the Stroke $\frac{AAaa + 2ABab + BBbb}{A + B}$; the Bodies are not separated after the * 931.
Stroke*, and the Mass is A+B, by which if we divide the Force remaining after the Stroke, we have the Square of the Velocity after the Collision; which Square therefore is $\frac{AAaa + 2ABab + BBbb}{\overline{A + B}^q} = \frac{\overline{Aa + Bb}^q}{\overline{A + B}^q}$; whose Root $\frac{Aa + Bb}{A + B}$ gives the Velocity sought.

If the Computation be made, the respective Velocity $a + b$ being applied, 1014.
the Rule of N. 990. is discovered.

The Quantity of Motion, which is supposed to follow the Proportion of 1015.
the innate Force, is commonly determined by multiplying the Mass, not by the Square of the Velocity, but by the Velocity itself; from this Principle, Philosophers have deduced those very Rules of N. 988. 990. which we, by various Methods, have deduced from our Principles. Something strange happened here; Error was the Destruction of Error, and a double Error led to the Truth; they followed a false Principle concerning the Measure of Forces, and which is also little agreeable to Truth, they supposed the Bodies to lose no Force, by pressing the Parts inwards, and overcoming their Cohesion.

SCHOLIUM III.

A geometrical Demonſtration of the Changes, which happen in the Forces of Bodies, during the Colliſion.

1016. Pl.XXXVII. Fig. 3, 4. LET there be a Line A F, and one perpendicular to it A D, in a Point A taken at pleaſure.

In this Perpendicular, I put A D, and A C, which are to one another as the Velocity of two Bodies concurring, which are called M and N: there are two Caſes given; 1. Of Bodies tending towards the ſame Part. 2. Of Bodies carried towards contrary Parts: in the firſt Caſe I ſuppoſe C and D, at the ſame Part of the Point A (Fig. 3.); in the ſecond Caſe the contrary (Fig. 4.).

In the firſt Line A F, I alſo mark two Parts, A B, A *g*, which are as the Maſſes of the ſame Bodies M, N; in the ſecond Caſe at the ſame Part of A (Fig. 1.), in the firſt the contrary (Fig. 2.), as the Figures ſhew. Through the Points B and C, I draw a Line, which I produce indefinitely at the Part of *g*. I alſo join together, the Line being drawn, the Points C and *g*, in the firſt Caſe; in the ſecond I apply the Line *c g*, A C, and A *c*, being ſuppoſed equal; D F is drawn parallel to C *g*, or *c g*, through D; which alſo muſt often be produced beyond F.

1017. By ſuch a Figure we determine all Things, which happen in any peculiar Colliſion whatſoever.

We have the Triangles B A C, D A F, which are to one another, as the Forces of the Bodies M and N in the Moment of the Concourſe. For theſe Triangles are to one another in a Ratio compounded of the Baſes
* 23. El. 6. B A and A F, and the Heights A C, A D *; the Baſes B A, A F, are
41 El. 1. in a Ratio compounded of the Ratios of B A to A *g*, and A *g* to A F; the firſt is the Ratio of the Maſſes M and N; the ſecond is the Ratio of the Velocities A C, A D; therefore the Baſes are, as the Products of every Maſs by its Velocity: if every Product be multiplied again by the ſame Velocity, we have the Ratio of the Triangles, which will be as the
† 757. Forces †.

1018. The Change, which happens in the Velocity, anſwers, in this Figure, to the correſponding Change of the Force itſelf. Let the Velocity be *a c*, this being drawn parallel to A C; the Force will be as B *c a*; for the ſimilar Triangles B C A, B *c a*, follow the duplicate Ratio of the homologous
* 19. El. 6 Sides A C, *a c* *; and the Forces of the ſame Body M follow the dupli-
† 753. cate Ratio of the Velocities A C, *a c* †.

1019. The Velocities of both Bodies are chang'd during the Colliſion; in that Moment, in which the Velocity of the Body M is *c a*, the Velocity of the Body N is *d a*; for the Changes of the Velocities, which happen in the ſame Time, are to one another in an inverſe Ratio of the Maſſes *, which
* 967. takes place in this Figure *.

Theſe

These Changes are co, dp; Co, Dp, being drawn parallel to AB. By reason of the similar Triangles Coc, ABC; and also the similar ones Dpd, ACg;

$$co : oC :: CA : AB;$$
$$Dp = oC : dp :: Ag : CA;$$

Therefore *ex æquo, perturb.**, $co : dp :: Ag : AB$; that is, as N is to M, or inversely as the Masses. * 23 El. 5.

What remains, belonging to this Collision, now also easily appears. The Force which the Body M acquired, or lost, is $ACca$; the Force which N lost is $ADda$; the Force destroy'd, by the mutual Action, in making the Cavity, is $CDdc$; to which the Cavity itself hitherto made is proportional *. 1020. * 841. 934.

All these Things are so, wheresoever the Line pa is drawn, between A and e; for the mutual Action of the Bodies ceases in E, by the Intersection of the Lines AB and DF, and Ee determines the Velocity, with which both Bodies are carried together after the Stroke. 1021.

The Force of the Body M is then, BEe; the Force of the Body N is EeF; the Force destroy'd, which is proportional to the whole Cavity *, is represented by the Surface DEC. 1022. * 841. 934.

It appears also, the Line fl being drawn a little distant from pa, that *the Changes of the Forces, during the mutual Action, in any Moment infinitely small, are to one another, as the Velocities of the Bodies are*; and that *the Force destroy'd is to the Change of Force in one of those Bodies, in the same Moment, as the respective Velocity is to the Velocity of the same Body, in that very instant.* 1023. 1024.

SCHOLIUM IV.

Of comparing together the Times, in which Percussions are made, and the Changes of the Forces, and Velocities, which happen in certain Times.

WHAT was demonstrated in the Scholia of the foregoing Chapter, concerning the Times, in which Cavities are made, may be applied to Collision, if the Surface of one Body be plane, and soft, but the other be made up of Parts, which don't yield in Collision, as in the Experiment of this Chapter; and this Body has any one of those Figures, which we treated of in the foregoing Chapter. 1025.

That this Application may be made, we should attend to this; that the Laws, which relate to the Formation of Cavities are not changed, when the Velocity and Magnitude of the Cavity are changed; by these the Times indeed are varied, but the Decrements of the Velocities are subject to the same Rules. 1026.

In Collision the respective Velocity is that, whereby the Cavity is made; but this may be varied according to the Diversity of the Masses, the respective Velocity remaining; but there are no other Differences, that can alter the Time. Whence we conclude, That *what is demonstrated concerning*
1027. *striking against a fixed Obstacle, may be referred to Collision, if instead of the Velocity of the Body struck against the fixed Obstacle, we make use of the respective Velocity in the Collision*; *and instead of* the Cavity, which is made in the fixed Obstacle, we put the Cavity made in the Collision itself.

This first Cavity is as the Product of the Square of the Velocity by the
* 845. Mass *; the second is as the Product of the Square of the respective Velo-
† 841. 934. city by the Product of the Masses, divided by the Sum of them †; there-
985. fore, as instead of the Velocity itself we make use of the respective Velocity, so also instead of *the Mass we* must *make use of the Product of the Masses divided by their Sum.*

1028. I am persuaded that this general Demonstration illustrates the Thing sufficiently; but if any one shall apply each of the peculiar Demonstrations, given in the Scholia of the foregoing Chapter, to Collision, he will find, that each of the peculiar Solutions always leads to this general Rule.

1029. I will add something farther, which relates to Collision only, and indeed those Cases only, in which the Part of a hard Body, which runs against a soft Body, is cylindric; and we suppose the Cylinder to be a right one, and the Direction of its Motion to agree with its Axis, and the striking to be
1030. direct, as before, and the Surface of the soft Body to be plane. Let M be the Mass of the first Body; N of the second; let the respective Velocity be called r: as long as we consider the same Cylinder, the Time in which
* 860. 889. 1027. the Cavity is made, is as $\frac{M\,N\,r}{M+N}$ *. During this Time the Velocity
† 890. decreases uniformly †; that is, *in each of the Moments, infinitely small, and*
1031. *equal, the Diminutions of the respective Velocity are equal*; *the Circumstances*
‖ 1029. *mentioned being laid down* ‖. But this Diminution is the Sum of *the Changes*
1032. *of the Velocities of both Bodies concurring*, and these Changes are in a con-
‡ 967. stant inverse Ratio of the Masses ‡; therefore these Changes also, *in equal Times, are equal, each Body being considered separately.*

1033. We have the whole Changes in the Velocities of the Bodies M and N,
* 987. by dividing r in the inverse Ratio of the Masses *; that is, the Change for M is $\frac{N\,r}{M+N}$. The Time, in which this happens, is the very Time, in
† 1030. which the whole respective Velocity is destroy'd, which is as $M \times \frac{N\,r}{M+N}$ †; therefore the Time, in which the Velocity of the Body M is changed, follows the Ratio of the Change itself of this Velocity, if the Mass M remains, the rest N and r being varied at pleasure; and therefore in equal Times, in different Collisions, the Changes of the Velocities are equal.

1034. Therefore, *if a Body, cylindrically terminated, being moved according to the Direction of the Axis of the Cylinder, strikes directly against a soft, and plane*

plane Surface, of any moveable Obstacle whatsoever, what Magnitude soever this has, and with whatsoever Velocity it be moved, if the Cohesion of the Parts be the same, the Body struck, wil. lose an equal Velocity, in an equal Time, with whatsoever Velocity it is projected.

In this same Case, the Change of the Velocity, which the Obstacle undergoes, is the same always, in equal Time. If the same Obstacle remains; the Mass of the Body struck, and the Velocities, whether of the Body, or Obstacle, being varied any how, but the same Cylinder being kept; for the Demonstration N. 1033, may be referred to either Mass. 1035.

But we may consider the Thing more generally, the Cylinder itself being also varied, whose Diameter we call *d*. *If the Body* struck be M, and v its Velocity, and it loses its Force by acting upon an immoveable Obstacle, the Time in which it loses it is as $\frac{Mv}{dd}$ *; if, the Body *strikes against a moveable Obstacle* N, and the respective Velocity be r, the Time will be, as $\frac{MNr}{\overline{M+N}\times dd}$ †. The Change in the Velocity of M, which happens in such a Time is $\frac{Nr}{M+M}$ ‖. If this Quantity be given, it may be expressed by Unity; then $\frac{MNr}{\overline{M+N}\times dd}$ is changed, and $\frac{M}{dd}$; and the Time, in which any determinate Change whatsoever happens in the said Velocity of the Body M, is as $\frac{M}{dd}$; and in this very Ratio, but inverse, is *the Change of the Velocity in a determinate Time*, namely as $\frac{dd}{M}$; that is, it *is directly* as the Base of the Cylinder, or *as the Surface, in which there is given a mutual Application of the Bodies, and inversely as the Mass of the Body itself.* But the Change of Velocity of the Obstacle itself is as $\frac{dd}{N}$; which is evinced by a like way of Reasoning, and also follows from N. 987. 1036.

* 889.

† 1027.

‖ 1033.

If we suppose the Moments infinitely small, and equal, we may compare the Changes of the Forces, and Increases of the Cavities in any one determined Moment of these; any different Collisions whatsoever being given. 1037.

The Increase of the Cavity is as the Base of the Cylinder, and as the respective Velocity in that determined Moment, as is manifest; therefore this Increase is as ddr; which same Proportion the Force destroy'd in this very Moment follows *; but this is to the Change of Force, which any one of the concurring Bodies undergoes in the mean Time, as the respective Velocity is to the Velocity of this Body †. If we call this v, we have r is to v, as ddr is to the Change of which we are speaking; which is equal to

* 840.

† 1024.

1038. to $d\,d\,v$. Therefore in general it appears, that *in all Collision, the Change of the Force of a Body, in a determinate and infinitely small Moment, follows the Ratio of the Surface, in which there is given a mutual Application, as also of the Velocity of the Body, whatsoever Mass this has; the Magnitude and Velocity of the Obstacle, being also varied at pleasure.*

1039. But if we have regard neither to the Collision, nor to the Time, we demonstrate an universal Proposition, concerning the infinitely small Change of the Force of a Body.

Plate XXXII. Fig. 1. Let there be a Triangle A D E; its Surface is changed, a Parallel to D E being drawn, as the Square of the Line A D, or the Line D E *; therefore, if either of these Lines represents the Velocity of the Body, the Surface will represent the Force, as long as the Mass is the same *; if this differs, the Surface will be to be multiplied by the Mass of the Body.

* 19 El. VI.

† 753.

Now let $d\,e$ be parallel to D E, removed to a Distance from this infinitely small; the Surface D E $d\,e$, multiplied by the Mass, represents the

1040. Change of Force, when the Change of Velocity is D d; and *the infinitely small Change of the Force, follows the Ratio of the Mass of the Body, of the Velocity* D E, *and Change of Velocity, namely* D d.

CHAP. V.

Of the Collision of Bodies, which are made of various Bodies joined together: where we treat of the Center of Percussion.

1041. ALL Bodies consist of Particles joined together; and all Bodies may be resolved into smaller Bodies. But we call that one Body, whose Parts are moved together, their respective Situation being so kept, that it is not disturbed without applying an external Force.

1042. In this Sense a compound Pendulum is one Body only; and the Percussion of two Pendulums is to be referred to the simple Collision of two Bodies.

1043. So also we refer to this the Percussion of Bodies, joined by an inflexible Right Line, and moved horizontally round a Center.

Plate XXXV. Fig. 3. Let us suppose the Bodies A and C, joined by such a Line, moveable about the Point H; let the Bodies B and D also be joined in the same manner, and moveable about I. These Bodies being moved, a different Percussion will be given, according as the concurring Places differ, tho' they are moved in the same manner, and

and always concur directly. We now suppose the Concourse of the Bodies A and B to be direct. These Bodies after the Percussion are moved with the same Velocity *, and are separated for this * 931. Reason only, because the Lines are moveable about different Centers: but this Velocity is determined by a Rule, which I shall demonstrate in the first Scholium following.

DEFINITION I.

1044. *I multiply every Body by the Square of its Distance from its Center of Motion; I collect into one Sum the Products of all the Bodies, applied to the same Line, and I multiply this Sum by the Square of the Distance, between the other Center and the Point in which the Percussion is made. I shall call this Product the Number of the Center about which the Line is moved.*

I multiply the Body A by the Square of the Distance A H; C by the Square of the Distance C H; I multiply the Sum of these Products by the Square of the Distance I B. This Product gives the Number of the Center H. After the same manner the Number of the Center I is determined.

1045. *I multiply the Number of the Center* H *by the Velocity of the Point, in which the Percussion is made in the Line of the Center* H, that is, by the Velocity of the Body A; *and I multiply the Number of the Center* I *by the Velocity of the Point, in which the Percussion is made in the Line of this Center*, namely by the Velocity of the Body B; *I collect the Products into one Sum, if the Bodies tend towards the same Part; but I subtract the less Product from the greater, if the Motions are contrary; and I divide this Sum, or Difference, by the Sum of the Numbers of the Centers* H and I, *and the Quotient gives the Velocity sought of the Point in which the Percussion is made.* The rest also, that belongs to this Percussion, we illustrate in the Scholium above-mentioned.

1046. In these Agitations it is to be observed, that we suppose a free Motion about the Center; but that the Line is so retained there, as not to have the least Motion communicated to the Pin, or Catch, whilst the Bodies turn about it; lest some Force be destroy'd by this Action *. * 1007.

1047. It is manifest that such an Action is always given, at least by the centrifugal Force, that Case only being excepted, in which the Bodies are turned round a common Center of Gravity *. But * 613. besides this, in many Cases, there is also given another Action a-againſt the Catch in the very Moment of Percuſſion, which we ſhall explain more diſtinctly in the compound Pendulum.

Plate XXXV. Fig. 4. Let there be ſuch a Pendulum A D I, made up of the Bodies A, and D, joined by an inflexible Line, turning about I: what we ſay of two Bodies may be applied to more.

1048. *If* the ſaid *compound Pendulum* be rais'd, and left to itſelf, and when it has come to a vertical Situation, it *runs againſt an Obſtacle*, which we ſuppoſe to be *immoveable*, *the Bodies act differently according as the Point, which ſtrikes againſt the Obſtacle, is at a leſs, or greater Diſtance from* the Center of Motion I.

1049. But this Difference is to be ſought in the Preſſure itſelf, which the Pendulum exerts upon the Catch I, whilſt the Percuſſion continues; which Preſſure is directed towards the one, or the other Part, according to the different Diſtance of the Point in which the Percuſſion is made. But the Point H may be ſo determin'd in a Pendulum, which acts, that there may be an Æquilibrium given between the Actions; and that the Percuſſion may produce no Preſſure againſt the Spring or Catch in I. In this Caſe, the Bodies loſe their whole Forces by Percuſſion, and the Pendulum is at reſt by the Stroke, though it is not retained in I, and is moveable about this Point.

DEFINITION 2.

1050. *The Point, in a Pendulum, about which ſuch an Æquilibrium is given is called the Center of Percuſſion.*

1051. We demonſtrate in the third Scholium, annexed to this Chapter, that *the Center of Percuſſion coincides with the Center of Oſcillation* *.

* 425.

1052. If the Percuſſion be made in ſome other Point, and the Pendulum be not retained at I, the Stroke will be leſs, and there will be a Motion of the Pendulum about the Point, in which the Percuſſion is given.

1053. Nevertheleſs, in this very Caſe we keep the Magnitude of the Stroke, if the Pendulum be ſo retained at I, that it cannot communicate any Motion at all to the Catch itſelf, as we have obſerved above concerning another Motion *. For in this Caſe no Force is loſt, by the Action againſt I †; yet the Pendulum is at reſt; therefore all the Force is deſtroy'd, by acting againſt the Obſtacle; wherefore this Action is equal to that which is performed in the Percuſſion of the Center of Oſcillation.

* 1046.
† 1007.

Whence

Whence we deduce, that *a Pendulum, which turns freely about its Point of Suspension, but is retained at it, and can communicate no Motion at all to the Catch, has no Center of Percussion; or rather,* that *all its Points are such Centers.* 1054.

This Proposition is confirmed by some Experiments explained before, but more directly by this following one.

Experiment.

This Experiment differs from Experiment 4, of Chapter III *, 1055.
only in some Circumstances, which are to be observed in this, and * 837.
are neglected in that. Namely, we fix the middle Cursor at the Center of Oscillation; that is, at the Point which is the Center of Percussion, when the Axis is not retained in the Holes. At the End of Scholium II. Chapter II. we demonstrated †, that this † 816.
very thing obtains; if in our Machine the Distances of the middle Points of the Cursors from the Point of Suspension be 12, 24, 29 Inches; the other Things are performed, as in the Experiment mentioned; the Success is the same; that is, the Cavities are equal to each other, when there is the same Agitation of the Pendulum, which soever of the Cursors strikes against the Clay. Therefore there is the same Action of the Pendulum ||, in these Cases, and in || 826.
this respect the Center of Percussion is not distinguished from the rest of the Points.

SCHOLIUM I.

A Demonstration of those Things, which were mentioned in N. 1045, *concerning the Percussion of Bodies, cohering together by stiff Lines, and moved round Centers.*

LET the Bodies A *and* C *be joined together by an inflexible Line, and* 1056.
moved round a Center H; *let there be also other Bodies* B *and* D, Pl. XXXV.
joined in the same Manner, and moved round I. Fig. 3.

Let us suppose the Percussion of these Bodies to be direct. This happens, if there be a Stroke made by one of the Bodies sticking to one Line with any of those Bodies which stick to the other Line, as A and B. But the Stroke will be direct if those Bodies run directly against one another; which cannot be, unless in the Moment of the Concourse, the Lines with which the Bodies cohere, are parallel to one another.

If, in the Moment of their running against one another, in which both Bodies are moved in the same Line, they be carried with a certain common Motion, they will not act upon one another by this Motion; therefore the striking will depend upon *the respective Velocity*, which *remaining*, *there is*

* 949. † 956. *given* the fame Introceffion of the Parts *, and *the fame Force loft* †, *with whatfoever Velocities the Bodies are moved.*

1057. It is manifeft, that there is given a *Cafe, in which Bodies,* carried towards contrary Parts, *are at reft after the Stroke*; and in this Cafe it is alfo plain that, *the refpective Velocity being given, the Sum of the Forces is the fmalleft of all*; for the whole Force is deftroyed, and a lefs Quantity can never be deftroyed *: but I will fhew what is the Ratio of the Velocities in this Cafe. * 1056.

Let a be the Diftance of the Body A from the Center H, about which it is turned; and c the Diftance of the Body C from the fame Center. In the fame Manner let b be the Diftance of the Body B, and d the Diftance of the Body D, from the Center I, about which thefe Bodies are moved. Further let m be the Velocity of the Body A; and n the Velocity of the Body B.

1058. *In the Cafe, in which Bodies are at reft after the Stroke, the Motions being fuppofed contrary, we have* $m : n :: \overline{Bbb + Ddd} \times aa : \overline{Aaa + Ccc} \times bb$; that is, $\overline{Aaa + Ccc} \times bbm = \overline{Bbb + Ddd} \times aan$.

1059. For *in this Cafe the Sum of the Forces, the refpective Velocity remaining* $m + n$, *is the fmalleft of all.*

1060. The Sum of the Forces is $Amm + \frac{Cccmm}{aa} + Bnn + \frac{Dddnn}{bb}$ *; * 757.

for $a : c :: m : \frac{mc}{a} =$ the Velocity of the Body C; and $b : d :: n : \frac{dn}{b}$ $=$ the Velocity of the Body D.

Let us now fuppofe the Velocity m to be increafed by the Quantity e, and the Velocity n to be diminifhed by the fame Quantity, that the refpective Velocity may remain; we fhall fee that the Sum is greater.

The Velocity of the Body A is now $m + e$; of the Body C is $\frac{mc + ec}{a}$; of the Body B is $n - e$; and laftly the Celerity of the Body D is $\frac{nd - ed}{b}$.

The Sum of the Forces will now be $Amm + 2Ame + Aee + \frac{Cccmm + 2Cccme + Cccee}{aa} + Bnn - 2Bne + Bee + \frac{Dddnn - 2Dddne + Dddee}{bb}$. But $\overline{Aaa + Ccc} \times bbm = \overline{Bbb + Ddd} \times aan$; for we fuppofe we have to do with this Cafe. By dividing this Equation by $aabb$, we have $Am + \frac{Ccm}{aa} = Bn + \frac{Ddn}{bb}$; therefore in the laft Sum of the Forces thefe deftroy each other, namely $+ 2Ame + \frac{2Cccme}{aa}$ and $- 2Bne - \frac{2Dddne}{bb}$, and the Sum is reduced

duced to this $A\,mm + A\,ee + \frac{C\,ccmm + C\,ccee}{aa} + B\,nn + B\,ee + \frac{D\,ddnn + D\,ddee}{bb}$ which is greater than the first Sum mentioned. Q. D. E.

Neither is the Demonstration different if n be increased, the Velocity m being diminished.

We demonstrate in a shorter, and more direct Way, this very Proposition, that Bodies are at rest, if the Velocities have the Ratio mentioned *; for we deduce in general from what is said before †, that Bodies carried towards contrary Parts, and concurring, lose their Forces in equal Times, and therefore are at rest by the Stroke, if the Velocities in the Points, in which they exert Actions, that is, in the Points in which they concur, are to one another as the Forces to be destroy'd. Therefore in this Case it is $m : n :: A\,mm + \frac{C\,ccmm}{aa} : B\,nn + \frac{D\,ddnn}{bb}$. 1061. * 1058. † 966.

Whence follows $\overline{A\,aa + C\,cc} \times bbm = \overline{B\,bb + D\,dd} \times aan$. Q. D. E*. *1058.

The Force destroyed in any Collision whatsoever may be determined, the respective Velocity being given, for it is equal to the Sum of the Forces in the Case in which this is smallest *. Let it now be $m + n = r$. 1062. * 1056.

The Ratio between m and n is given †, and by Composition $\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa : \overline{A\,aa + C\,cc} \times bb :: m + n = r : n$; † 1058.

therefore $n = \frac{\overline{A\,aa + C\,cc} \times bbr}{\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa}$. After the same Manner we discover $m = \frac{\overline{B\,bb + D\,dd} \times aar}{\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa}$: the Sum of the Forces is $\frac{\overline{A\,aa + C\,cc} \times mm}{aa} + \frac{\overline{B\,bb + D\,dd} \times nn}{bb}$*, by substituting instead of m and n, the Values, this Sum will be * 1060

$$\frac{\overline{A\,aa + C\,cc} \times \overline{B\,bb + D\,dd}^{q} \times aarr + \overline{B\,bb + D\,dd} \times \overline{A\,aa + C\,cc}^{q} \times bbrr}{\overline{\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa}^{q}}.$$

By dividing the Numerator and Denominator by $\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa$; *we have* $\frac{\overline{A\,aa + C\,cc} \times \overline{B\,bb + D\,dd} \times rr}{\overline{A\,aa + C\,cc} \times bb + \overline{B\,bb + D\,dd} \times aa}$, *the Force lost, the respective Velocity* r *being given.* 1063.

That we may now demonstrate the Rule delivered in N°. 1045, we suppose a Point to be given, which is moved with the same Velocity, with which the Bodies are carried after the Stroke, before the Separation, and according to the same Direction. 1064.

With

With respect to this Point the Bodies are at rest after the Stroke; therefore in respect of that, before the Stroke, they were moved with con-
* 1058. trary Velocities in the Ratio of $\overline{Bbb + Ddd} \times aa$ to $\overline{Aaa + Ccc} \times bb$ *, and they lose these Velocities, when they are at rest after the Stroke in respect of this Point; wherefore these very Velocities are the Changes, which happen in the Velocities from the Stroke, which Changes therefore are in the said Ratio, and by composition $\overline{Aaa + Ccc} \times bb + \overline{Bbb + Ddd} \times aa$ is to $\overline{Aaa + Ccc} \times bb$ as the Sum of the Changes, that is, as the respective Velocity, is to the Change in the Velocity of the Body B.

Now if the Velocity of the Body A be called p; and q the Velocity of the Body B, this being put less; the respective Velocity will be $p - q$, if the Motions be directed towards the same Part; and the Change of the Velocity of the Body B, is discovered

$$\frac{\overline{Aaa + Ccc} \times bbp - \overline{Aaa + Ccc} \times bbq}{\overline{Aaa + Ccc} \times bb + \overline{Bbb + Ddd} \times aa},$$ which Change is the Velocity acquired; because the less Velocity is increased in conspiring Motions: wherefore if it be added to the Velocity q we have the Velocity of both Bodies after the Stroke; which therefore *is*

1065. $$\frac{\overline{Aaa + Ccc} \times bbp + \overline{Bbb + Ddd} \times aaq}{\overline{Aaa + Ccc} \times bb + \overline{Bbb + Ddd} \times aa}.$$

1066. If the Motions be directed towards contrary Parts, the respective Velocity is $p + q$, and the Velocity after the Stroke is discovered by a like way of reasoning $\frac{\overline{Aaa + Ccc} \times bbp - \overline{Bbb + Ddd} \times aaq}{\overline{Aaa + Ccc} \times bb + \overline{Bbb + Ddd} \times aa}$, namely the less Product in the Numerator being subtracted from the greater.

1067. It plainly appears that it matters not whether in this Collision the Bodies, which are joined to the same Line, are given at the same Part of the Center, about which the Line is moved, or at different Parts; for the Body is moved in the same Manner, on whatsoever Part of the Center it be given, if the Distance from it be the same: it is also manifest enough, that the centrifugal Force, whereby Bodies endeavour to recede from the Center, and the Actions which they exert upon the Catches, whilst they con-
* 1007. cur, should not be consider'd here *.

1068. These Demonstrations may be applied to any Number whatsoever of Bodies, and universal Rules may be easily drawn from what has been demonstrated.

1069. We see also what obtains, *if a Body, moved in a Right Line, runs against another directly, which together with others adheres to a Right Line, moveable about a Center*; for that Body, moved in a Right Line, acts as if it adhered to a Right Line, moveable about any Point whatsoever.

Let the Bodies A and C be at rest in a and c. whilst they are moveable about H as before. Let us suppose B, or b, moved in a Right Line, to run against a, with the Velocity q, directly, and perpendicularly to a H;

a H; *we discover the Velocity after the Stroke* by the foregoing Rule. For I suppose B to be joined to the Line, and to be moved about the Center at any Distance b. In this Collision p, and D, are equal to nothing; therefore the Quantities vanish, which are multiplied *by* these, wherefore the Rule mentioned *, is changed into this $\frac{Bbbaaq}{\overline{Aaa+Ccc}\times bb+Bbbaa} =$ * 1065.

$\frac{Baaq}{Aaa+Ccc+Baa}$: from whence we deduce *this Rule. A Body, which strikes against another, is multiplied by the Square of the Distance of the Point, against which it runs from the Center, and by its Velocity; and this Product is divided by the Sum of all the Bodies, of every one multiplied by the Squares of their Distances from the Center.*

The Propositions of N°. 962. 985. 987. 988. 990. are peculiar Cases 1070.
of the Propositions demonstrated in this Scholium in N°. 1058. 1063. 1064. 1065. 1066; as appears, if we suppose two Bodies, which are joined to Lines, moveable about any Centers whatsoever.

SCHOLIUM II.

An Examination of the Experiment made on Bodies striking against a Scale, or Arm of a Balance.

MEersennus, *de Lanis*, and others, have given an Experiment made 1071.
on falling Bodies; and have observed that a Body, striking against the Arm of a Balance, raises another Body a little, whose Weight is greater, that is put in the opposite Scale; and that Bodies are thus raised to a small, but equal Height, (which Circumstance nevertheless *Mersennus* does not observe) if the Body, which strikes against the Beam with a Motion acquired in falling, falls from Heights, which are as the Squares of the Weights raised.

Yet *Mersennus* takes notice, that the Experiment did not answer in certain Circumstances; which happened to me also, who made the Experiment in a somewhat different Manner; I attributed this to the Defect of the Machine, and I thought the Defects, which I found in the Machine, of less detriment in those Heights only, in which I found the Rule to hold good with sufficient Exactness. But when I examined the Affair, I found I had quite mistaken the Matter; and that it was repugnant to those very Principles of Mechanicks, about which all are agreed, that there is given the Proportion mentioned between the Squares of the Weights raised, which is given between the Heights, from which the Body falls, which strikes against the Scale, or opposite Arm; and I could not doubt but it must be attributed to the defect of the Machine, if this Proportion were sometimes discovered within certain Limits, as it had always happened to me. I confess there would be no sensible Error, if the Weight of the

Body falling, and that of the whole Balance, that is, of the Beams and Scales, were very ſmall in reſpect of the Weights raiſed: but in this Caſe the Experiment could not be made; for a great Weight cannot be put upon a ſmall Beam.

But that, what relates to this Experiment, might appear more clearly, I took care to have a Machine made, whereby the Experiment is made, as exactly as can be, and altogether without any ſenſible Error; and after the Experiment was performed, I made my Computation about it.

A BALANCE, *whereby the Heights are compared, from which a Body falling, raiſes Weights a little.*

1072. Plate XXXV. Fig. 4. A B is a Beam of a Balance; it is ſuſtained by a Foot, whilſt it turns about a Center, as in other Balances: the Scale L is of Iron; the oppoſite one M of Wood, and round, made hollow one Inch deep. This, when the Experiments are to be made, is filled with ſoft Clay, which is ſcraped with a wooden Knife, in ſuch manner, that its Surface may be free from Inequalities, and horizontal; for which reaſon this Scale may be eaſily taken off, and ſuſpended again in its Place. The Diſtance B M exceeds three Feet, wherefore the Machine muſt be placed on the End of a Table.

The Ball G hangs by a Thread, and is faſtened to a Hook joined to the Plate D.

The Weight Q is put into the Scale L, that there may be an Equilibrium. Which Things being given, the Weight P to be raiſed by the Stroke is added; and that the Beam may remain horizontal, the Arm A, which is now loaded moſt, is ſuſtained by an Iron Gnomon, joined to the Foot. We may eaſily ſee that before putting on the Weight, P, the Machine muſt be ſuſtained by the Foot without the Gnomon, in order to make an Equilibrium.

To the Gnomon there is joined at *f* a thin elaſtick Plate *f g*, which being extended reaches to *i*, where the End *g* is retained, by help of the ſmall Plate *i*, joined to the Arm A. When the Arm is raiſed a little *g* is let looſe; whence it may appear that it is equally raiſed in various Tryals; if, namely, the Stroke being diminiſhed only a little, by which the Beam is moved, the Spring be not let looſe.

EXPERIMENT.

1073. All Things being ordered, as has been ſaid, the Weight P of four Ounces being applied, I ſuſpended the Ball G ſo, that its Height, namely the Diſtance between the lower Part of the Ball and the Surface of the Clay, might be $6\frac{7}{8}$ Inches; the Thread being cut, the Plate *f g* was let looſe, by the ſtriking of the Ball: and the Experiment being repeated many Times, it ſucceeded in the ſame Manner; but the Height being diminiſhed, a quarter of an Inch, or even leſs, the Spring was never let looſe, by which ſame Method the following Heights were determined.

The

The Weight P being doubled, the Height of the Ball was $14\frac{1}{8}$ Inches.

Laftly the Weight P being tripled, that is, being twelve Ounces, the Height was $23\frac{1}{2}$ Inches.

To all thefe Heights the Depths of the Cavities, made in the Clay by the Strokes, muft be added, and the Heights were, fmall Fractions being neglected, 7. $14\frac{5}{16}$. $23\frac{3}{4}$.

If, with this fame Machine, the fame Experiments be made, with other kind of Clay, the Heights may be varied a little. If the Clay be lefs foft, the Cavities will be fmaller, and the Heights above the plane Surface of the Clay greater, but the whole Heights the fame. But if the Clay be more or lefs heavy, there will be a Difference, for, tho' by that the Matter to be raifed be not changed, yet the Matter to be moved is changed, whence a Difference neceffarily follows, as will appear more clearly by the following Computation.

The Beam of the Balance is the Figure reprefented at A B, it is made hollow in the Parts A and B, as may be feen in *Fig.* 4; as to what remains, it is every where of the fame Thicknefs. 1074. Plate XXXV. Fig. 5.

By reafon of this irregular Figure, the Computation would be very difficult; therefore, the Weight of the Balance being kept, we conceive the Figure changed, fome Parts being removed from the Center, and others moved towards it: we fuppofe the Figure to be fuch as is reprefented in Fig. 6, whofe whole Length reprefents that, which is given in the Balance between the Points of Sufpenfion; from which Change there can be only a fmall Error in the Computation.

The Surface of this Figure, as the Balance is every where of the fame Thicknefs, may reprefent the Weight of the Balance, in all Parts. This Figure A B is made up of a Parallelogram, and two Triangles: the Triangles being joined, the Figure is reduced to that, which is reprefented in Fig. 7. which being affumed, I will make the Computation.

This Computation will be of this Service, that it will thence appear, that my Experiments agree with what is demonftrated about Percuffion. But the Foundation of the Computation itfelf is had in N. 1069.

First of all, all the Points of the Surface A D F E B, reprefenting the Weight of the Balance, muft be multiplied by the Squares of their Diftances from the Center of Motion refpectively. This will be done without a fenfible Error, if inftead of the Diftances from the Center, the Diftances from the Line C F be taken, whereby the Computation becomes eafier. Plate XXXV. Fig. 7.

If now the Operation for the Parallelogram be made, all the parallel Lines, and equal to the Line D A, muft be multiplied by the Squares of their Diftances from C F; that is, all thefe Squares muft be multiplied by the fame Quantity A D, or C G; that is, the Sum of the Squares muft be multiplied by C G: but the Sum of the Squares is a Pyramid, whofe Bafe is the Square of the Line A C, and Height the fame A C; which Pyramid is equal to $\frac{1}{3}$ A C^{c}*. This being multiplied by C G, we have $\frac{1}{3}$ CG x AC x AC^{q}, the Sum of the Products of all the Points of the Paral-

* 7. El. 12.

Parallelogram D C, multiplied by the Squares of their Diſtances from C G.

A like Sum, for all the Points of the Triangle D F G, is equal to $\frac{1}{12}$ of $GF \times AC \times AC^q$. Thoſe who are ſkilled in the more ſubtle Part of Geometry will eaſily diſcover this, and it would be of no Service to endeavour to explain it to others. By doubling theſe Products, we ſhall have a like Sum for the whole Figure A D F E B; and it is this $\frac{2}{3} CG \times AC \times AC^q + \frac{1}{6} GF \times AC \times AC^q = b \times AC^q$; by putting $b = \frac{2}{3} CG \times AC + \frac{1}{6} GF \times AC$.

1075. Theſe Things being laid down, let a be called the Height, from which the Ball is let down; and the Velocity, acquired by falling, with which the Ball runs againſt the Scale M, and which is proportional to the Square

* 374. Root of this Height *, may be denoted by $\sqrt{a}$.

By multiplying this Velocity by the Ball G (*Fig.* 4.) and by the Square of the Diſtance A C, and by dividing this Product by the Sum of all the Bodies, moved in the Experiment, reſpectively multiplied by the Squares of their Diſtances from the Center of Motion, we have the Velocity of the

* 1069. Point A after the Stroke *.

Part of this Sum we have already determined, namely with reſpect to the Beam, we have what remains by multiplying the Weights of the Scales L, and M, as alſo P, Q, and G (*Fig.* 4.) by the Square of the Diſtance A C; for all theſe Bodies may be conſidered as if given in the Points of

* 183. Suſpenſion A and B *. We call the Sum of the Weights of the Scales, as alſo P, Q, and G, c, and the Velocity of the Point A after the Stroke

* 1069. will be $\frac{AC^q \times G\sqrt{a}}{b \times AC^q + c \times AC^q} = \frac{G\sqrt{a}}{b+c}$ *.

That, this Velocity being given, we may compare the Height to which the Point A is raiſed with the Height a, the Center of Oſcillation muſt be

* 425. determined, which is moved, as a Body on which Gravity only acts *; but the Diſtance of the Center of Oſcillation from the Center of Mo-

* 474. tion is $\frac{b \times AC^q + c \times AC^q}{P \times AC}$ * $= \frac{b \times AC + c \times AC}{P}$.

But the Diſtance A C is to this Diſtance of the Center of Oſcillation, that is, (by multiplying Diſtances by P, and dividing them by A C), P is to $b+c$, as the Velocity of the Point A is to the Velocity of the Center of Oſcillation; and in the ſame Ratio is the Height to which A aſcends, which we call d, to the Height to which the Center of Oſcillation aſcends; therefore $P : b+c :: \frac{G\sqrt{a}}{b+c} : \frac{G\sqrt{a}}{P}$ = the Velocity of the Center of Oſcillation. And $P : b+c :: d : \frac{db+dc}{P}$ = the Height, to which the Center of Oſcillation aſcends.

This Height is alſo expreſſed by the Square of the Velocity of this Center, when a expreſſes the Height, to which a Body comes with the Velocity

city $\surd a$ *. Therefore we have this Equation $\frac{G^q \times a}{P^q} = \frac{db + dc}{P}$ that is G^q * 374. 380.

$\times a = db \times P + dc \times P$: and $a = \frac{\overline{b+c} \times d \times P}{G^q}$.

That Numbers may be substituted instead of Letters, we must consider, that b is equal to $\frac{2}{3}$ G C × A C + $\frac{1}{6}$ G F × A C, whilst the Figure A D F E B, that is, 2 G C × A C + G F × A C *, represents the Weight of the Beam; wherefore this Weight of the Beam is to b, as 2 G C + G F is to $\frac{2}{3}$ G C + $\frac{1}{6}$ G F. 1076. * 34. El. 1.

In my Machine G C is to G F, as 3 is to 4; that is, 2 G C + G F is to $\frac{2}{3}$ G C + $\frac{1}{6}$ G F, as 15 is to 4. The Weight of the Beam is nineteen Ounces, two Drachms and one Scruple, that is 463 Scruples. Therefore $15 : 4 :: 463 : b = 123\frac{1}{2}$ Scruples.

The Weights of the Scales, Q and G being added, namely $c - P$ are equal to 1320 Scruples, that is, $c = 1320 + P$; the Ball G, weighs 67 Scruples; the Height d is equal to 0,21 Inches, that is, it somewhat exceeds the fifth Part of an Inch.

And the foregoing Equation is changed into this

$$a = \frac{\overline{b+c} \times d \times P}{G^q} = \frac{\overline{123\frac{1}{2} + 1320 + P} \times 21\,P}{67^q \times 100} = \frac{\overline{1443\frac{1}{2} + P} \times 21\,P}{448900}.$$

By substituting successively, instead of P four, eight, and twelve Ounces, that is, 96,192,288 Scruples, we discover $a = 6{,}91$. $a = 14{,}68$; and $a = 23{,}32$.

Which Heights differ very little from the Heights discovered by the Experiment; but the Difference is to be attributed to the Change of the Figure of the Beam in the Computation *. * 1074.

In this Computation we have neglected the Consideration of the Distance between the Center of the Balance and the Center of Gravity of the Beam; because the Error arising thence cannot by any means be perceived.

SCHOLIUM III.

Of the Center of Oscillation, and Percussion.

We have above * deduced a Method of determining the Center of Oscillation, from what has been demonstrated about Pressure; the same Rule delivered there we very easily deduce from what is demonstrated about Forces. 1077. * 474. 475.

A Body acquires the same Velocity, and therefore Force, in falling from a certain Height, whatsoever Way it follows in its Descent *; and the Force acquired is proportional to this Height †. Whilst Bodies, joined to a compound Pendulum, descend, the Force acquired in descending is destroy'd by no Action; there is also nothing given whereby it can be increased; therefore the Sum of the Forces is equal to the Sum of the Forces, * 393. † 754.

 which

which the Bodies could have acquired, in falling from their Heights separately.

Let the Bodies be A, B, C, D; the Diftances from the Point of Sufpenfion a, b, c, and d. The Heights, from which thefe Bodies defcend, are as a, b, c, and d, and the Velocities are in the fame Ratio. Let the Diftance of the Center of Ofcillation from the Point of Sufpenfion be called x, and the Velocity acquired in defcending from the Height, from which this Center defcends, $\surd x$; therefore the Velocity of the Body A, if it had
* 374. fallen freely, would have been $\surd a$ *; and its Force A a †; and the Sum of
† 757. the Forces, if all the Bodies had fallen freely, $Aa + Bb + Cc + Dd$. If there be given fome Bodies at the oppofite Part of the Point of Sufpenfion, thefe afcend, and their Forces are negative.

In Bodies joined to a Pendulum, the Velocity of the Body A is difcovered by this Rule, $x : a :: \surd x : \frac{a}{\surd x}$, and the Velocities of the reft of the Bodies $\frac{b}{\surd x}$, $\frac{c}{\surd x}$, and $\frac{d}{\surd x}$; and the Sum of the Forces is
* 757. $\frac{Aaa}{x} + \frac{Bbb}{x} + \frac{Ccc}{x} + \frac{Ddd}{x}$ *; which as it is equal to the Sum mentioned, we difcover $x = \frac{Aaa + Bbb + Ccc + Ddd}{Aa + Bb + Cc + Dd}$ according to the Rule N. 474.

We have above obferved that the Center of Percuffion coincides with
* 1033. the Center of Ofcillation *; we will now demonftrate it.

1078. *This is a Property of the Center of Percuffion, that there is an Æquilibrium between the Actions, whereby Bodies, on both fides of this Center, act upon a Pendulum.*

Therefore we may confider a Pendulum as if it were a Lever, whofe Fulcrum is the Obftacle againft which it runs, placed in the Center of Percuffion, and that there is an Æquilibrium given, when Bodies run againft this Lever, with the Velocities, with which they are moved in the Pendulum.

1079. The Center of Percuffion of the Pendulum A I, to which the Bodies A
Plate XXXV. and D, joined by an inflexible Line, fufpended at I, are applied, will be
Fig. 8. H, if, fuppofing a Lever, whofe Fulcrum is H, and the Bodies A and D running againft it, in the Points A and D, with Velocities, which they have in the Pendulum, there be an Æquilibrium between thefe Actions; for then the Point I of the Pendulum cannot be affected by any Motion, or, if it be retained, exert any Preffure upon the Catch.

That we may determine this Cafe of the Æquilibrium, various Bodies being given, the Actions of all of them muft be determined; that is, thefe Actions muft be compared together.

1080. The Confideration of the Pendulum being now fetafide, and having regard only to the Lever, let the Velocity *of the Body* A be m, and a the Diftance

tance A H; the Velocity of the Body D n, and d the Distance H D. The Body A acts upon the Lever in the same manner, whether it runs against it with the same Velocity, in A at the Part M, or in L at the Part N, H A and H L being put equal. For the Action will be the same, if, the Velocities of the Bodies being kept, we suppose them to be turned about the Center H, and thus moved to run *against the Lever* along e L and f D: Let H f be continued to G, that H G and H e, or H A, may be equal; an Æquilibrium will be given, if the Velocity of the Point e is to the Velocity of the Point G, as D $d\,d$ is to A $a\,a$ *, that is D $d\,d$: A $a\,a$ * 1058.

$: : m : \frac{a\,n}{d}$, for this is the Velocity of the Point G; therefore A $a\,m =$

D $d\,n$; whence it appears that *the Action follows the Ratio of the Product of the Mass of the Body by the Velocity, and by the Distance from the Fulcrum.*

In the Pendulum the Velocity follows the Ratio of the Distance from the Point of Suspension; and the Distance from the Fulcrum, is the Distance from the Center of Percussion; therefore the Action of the Body follows the Ratio of the Product of the Body by its Distances from the Centers of Suspension and Percussion; and there is given an Æquilibrium between the Actions on both Sides of the Center of Percussion, when these Products on both Sides of this Center are equal; and as *the Center* 1081.
of Oscillation has the same Property *, it follows, that it *coincides with the* * 472.
Center of Percussion.

CHAP. VI.

Of the Congress of elastick Bodies.

ELASTICK Bodies that concur, are separated after the 1082.
Stroke, as we have already observed *, but with a different * 930.
Force in similar Circumstances; for the Elasticity differs in different Bodies.

DEFINITION.

Elasticity is said to be perfect, when the Parts yielding inwards 1083.
return to their first Situation, with a Force equal to that, with which they were struck.

We speak of perfect Elasticity, tho' we are acquainted with no Bodies, that have such Elasticity; for general Rules can't be delivered except in respect of perfect Elasticity; the nearer Bodies approach to this, the more exactly their Motions agree with the Rules.

Im-

1084. Imperfect Elasticity may have innumerable Degrees; and we ought to discover by Experiments, what peculiar Bodies want of perfect Elasticity; that we may determine, how much the Motions of these Bodies recede from the Rules.

No Force is lost in the Collision of Bodies, except that, which is consumed by pressing the Parts inwards *; therefore, if the
* 934. Bodies are elastick, this whole Force is spent in the bending of the elastick Parts; but these return to their former Figure with an equal Force; therefore the Force destroy'd is again restored; and
1085. *the Sum of the Forces impressed on Bodies after the Stroke is equal to the Sum of the Forces before the Collision:* which Demonstration is very universal, and may be applied to any Collisions whatsoever.

1086. Hence it follows, that *an elastick Body, that strikes against a firm elastick Obstacle, returns with the same Celerity, with which it went. If the Direction be perpendicular to the Obstacle, it will also return in the same Line*; as it can't be turned more towards one Side than the other.

In what remains I treat of direct striking only in this Chapter. But the Action of the elastick Parts must be considered with more Accuracy.

1087. *A bent Spring, put between two Bodies at rest, whilst it expands itself, moves both Bodies.* If the Pressure whereby the Parts of a Body cohere, exceeds the Pressures which the Spring exerts on these Bodies, the whole Action of the Spring, as there is no Introcession of the Parts, is consumed in moving the Bodies, *and the Sum of the Forces, communicated to the Bodies, is equal to the Force, with which the Spring was bent.*

1088. This Spring, during the whole time in which it expands itself,
* 361. continually presses equally both ways *; that is, exerts Pressures, whose Intensities are equal; the Motions of the Obstacles, in every one of the small Moments, are inversely as the Bodies, which are
* 138. mov'd by these Pressures *; and the Velocities, communicated in
† 119. these Moments, are in the same Ratio †: in the same Ratio also
|| 723. are the Actions of the Spring both ways ||; as also the Forces im-
‡ 700. press'd on the Bodies ‡. But as this Ratio obtains in each of the small Moments, as long as the Actions of the Spring continue, *the whole Velocities communicated, and the whole Forces impress'd, are in*
* 12 El. V. *this very inverse Ratio of the Masses* *; which two agree together,
† 791. as we have demonstrated before †.

A

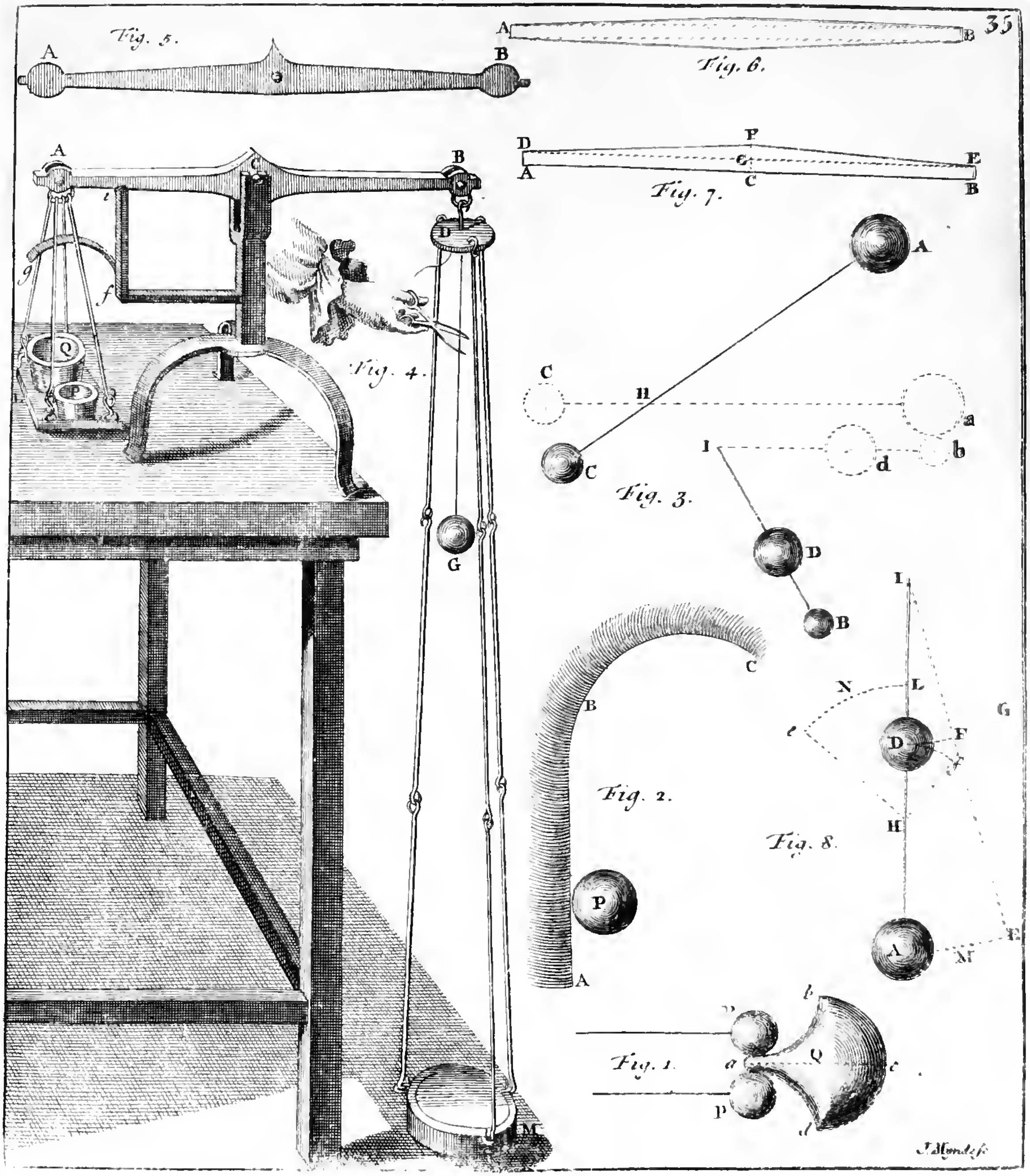


Fig. 5.
Fig. 6.
Fig. 7.
Fig. 4.
Fig. 3.
Fig. 2.
Fig. 8.
Fig. 1.
J. Mynde fc.

A Machine, *whereby a Spring, when bent, is relax'd between Bodies ſuſpended.*

This Machine is join'd to a Rectangle, altogether like that, of which we treated above *; in the ſame manner alſo Copper Cylinders, and leaden Weights, are join'd to it †; and the Rectangle itſelf, with the Machine join'd to it, weighs exactly the ſame, as the Rectangle mention'd, when one of thoſe Bodies, mention'd in the Explanation of that Rectangle, is join'd to it *. 1090. Plate XXVI. Fig. 6. * 769. † 774. * 771.

The Machine, of which I am now ſpeaking, conſiſts of a ſpiral Spring E, like thoſe, whereby Motion is communicated to Watches; in the ſame manner as in theſe, that is join'd to the Axis, round which it is wound, and to which is join'd a Wheel with Teeth; this is carried round, whilſt the Spring is relaxing, and by its Motion moves a ſecond Wheel, that, ſmaller Wheels being added, the Motion of the firſt may be regulated, by a Method commonly uſed in like Circumſtances. 1091.

But the Motion of the Spring, when it is wound up, is ſtopp'd by help of a ſmall Plate, which retains the laſt of the ſmall Wheels, which is eaſily let looſe, whereby the Motion of the Wheels is reſtor'd. All theſe things could not be repreſented in the Figure, but they have no Difficulty. 1092.

The Spring mention'd, together with all the Wheels, is contain'd between two Plates, into which the Ends of the Axes of the Wheels are put; this Part of the Machine does not cover half of the anteriour Surface of the Rectangle, and occupies one Side of its Surface, and is terminated within by the Plate V V. 1093.

The Axis of the greater Wheel *t*, to which the Spring E is join'd, goes thro' the Plate V V, and the Plate *i* is join'd to the prominent End; which is repreſented at I, ſeparately, and drawn in its true Bigneſs.

The Steel Plate F G, which covers the anterior Part of the whole Machine, is the ſame, which we explain'd above *, and which is repreſented in *Fig.* 3 and 4, of this Plate. * 740.

The Catches, which are to be ſeparated in the Experiments †, are ſeparated here alſo by the Hammer *m*, which, as that of which we ſpoke in N° 741. has a Tail alſo, which is join'd at right Angles in *o* to an Axis that turns about; the Separation of the Catches alſo is made in the ſame manner as there, by depreſſing the Hammer; which how it is done in this Caſe I will mention. † 740. 741.

1094. To the Tail of the Hammer, about the middle of it, is join'd the Hook *n*, which is applied to the Circumference of the Plate *i*, or I, by whose Circumvolution the Hammer is rais'd, whilst the Circumference *p l q* passes by along the Hook; but when the End *q* leaves the Hook *n*, the Hammer is suddenly depress'd. But that this may be done with a sufficient Force, and the Catches may be separated in the Experiments, we make use of a small, steel, elastick Point *s r*, which is fasten'd in the side Plate of the Machine at *s*, by help of a Screw surrounding the Point, near the Head *s*; the other end of the Point is free, and is applied to the Tail of the Hammer, which it depresses; but whilst this is rais'd, the Point is bent, and the Pressure from the Elasticity is increas'd; and for this reason the Hammer when freed descends with Force.

Experiment I.

1095. The Spring describ'd above * is applied to the Rectangle, as in
* 739. the second Experiment of the second Chapter of this Book †. We
† 773. make use of the same Machine, which we used in that Experi-
* 778. ment, and suspend the Rectangle in the same manner *. We suspend another Rectangle also; namely, that to which the Machine
† 1090. is join'd, which we explain'd last †.

We have now two Bodies suspended, as in many Experiments of the foregoing Chapter. They are dispos'd in the same manner; and then the little Tongue of the Spring answers to the Hole in the
* 778. middle of the Plate with Catches, as in the Experiment shewn *. The little Tongue is thrust into the Hole also in the same manner,
† 743. as has been already explain'd there, and elsewhere more distinctly †; but so, that the first Teeth of the little Tongue are of use here also,
* 745. 778. as in other Experiments *.

But that the corresponding Threads may be parallel, the Spring being now bent, we make use of a Method used elsewhere in a
* 942. peculiar Case *.

The Spring, whereby Motion is communicated to the Wheels,
† 1091. is wound up †; and its Motion hinder'd ‖.
‖ 1092. This is relax'd, when the suspended Bodies are at rest, whereby the Rest is a little disturb'd for some time; but the Motion continues some Moments, that the Bodies in the mean time may re-
* 1094. turn to rest; at length the Hammer falls of its own accord *, and the Bodies are separated by the Action of the Spring bent between them. And this Experiment is varied, the Masses of the Bodies suspended being chang'd.

In

In the firft Trial we put the Cylinders * into the Bodies, that each Mafs, R and S, may be equal to two. The Bodies are feparated with equal Velocities, and thefe are equal to 8,4. 1096. Pl. XXXVI. Fig. 1. * 774.

In the fecond Trial, one of thefe Maffes is kept; namely, that of the Body S; the other is chang'd, that it may be fixteen; and, the Spring being relax'd, the Velocity of this Body is equal to 1,4; whilft S is mov'd with the Velocity 11,2. 1097. Pl. XXXVI. Fig. 2.

If the Experiment be repeated the third time, putting the Maffes, R three, S four; the Velocities will be, of this 5,5; of that 7,3. 1098. Pl. XXXVI. Fig. 3.

In thefe three Cafes the Sum of the Forces is the fame, and the Velocities are inverfely as the Maffes. 1099.

In the firft Cafe the Maffes are equal, fo alfo are the Velocities. The Force of every Body is $2 \times 8,4 \times 8,4 = 141,12$. and the Sum is equal to 282.

In the fecond, the Mafs of the Body R exceeds the Mafs of S eight times, and the Velocity of that is exceeded by the Velocity of this in the fame manner. The Forces are $16 \times 1,4 \times 1,4 = 31,36$, and $2 \times 11,2 = 258,88$; and the Sum is again equal to 282. Laftly, in the laft Cafe the Maffes are as 3 to 4. The Velocities are in the fame Ratio, but inverfe, as 7,3 to 5,5: The Forces are $3 \times 7,3 \times 7,3 = 159,87$, and $4 \times 5,5 \times 5,5 = 121$; and the Sum which fcarce differs at all from 281, can leave no doubt of the Equality of the Forces.

We have this very Cafe which we have examin'd, *when two elaftick Bodies, run againft each other directly, with contrary Motions; with Velocities, which are inverfely as the Maffes:* for thefe being fuppofed not elaftick, they are at reft after the Stroke *; therefore in the very Moment of the Concourfe, before the Figure is reftor'd, the Spring is bent between the two Bodies at reft. Therefore they are feparated with Velocities which are inverfely as the Maffes †, that is, the Velocities after the Stroke are in the fame Ratio in which they were before the Stroke; whence it follows, that *each Body returns with the fame Velocity which it had before the Stroke;* for if it be diminifhed in one, the Ratio will not be kept, except it be diminifhed in the other alfo; wherefore the Sum of the Forces will be lefs, which is impoffible ‖. The Demonftration is the fame, if the Velocity of one Body is increafed. 1100. * 962. † 1089. ‖ 1083.

The Experiment, whereby we confirm this Propofition, and the reft alfo, which follow concerning the Collifion of elaftick Bo- 1101.

dies, are made by help of the ſame Machine, whereby the Expe-
* 760. riment of the Colliſion of Bodies non-elaſtick are demonſtrated *.

1102. Plate XXVI. Fig. 7. In thoſe we make uſe of ivory Bodies. The firſt is the Ball G, whoſe Diameter is an Inch and an half; we uſe ſix ſuch. The reſt of the Bodies are cylindrick, as B and C; at one end they are terminated by an Hemiſphere, but the other Baſe is flat; but the Diameter exceeds an Inch a little only. In making theſe Cylinders it muſt be obſerv'd, that the Axis of the Tooth ſhould coincide with the Axis of the Cylinder. The Weights of the Bodies G, B, and C, are to one another as one, two, and three.

1103. The Balls are ſuſpended by the two Hooks *v*, *v*; others are ſuſtain'd by four Threads, as the Rectangles, which are uſed in the
* 769. Experiments of the Colliſion of Bodies non-elaſtick *. There are ſix ſuch Bodies like G, as I ſaid, one like B, and two like C.

Plate XXVII. Fig. 1. * 764. That theſe Bodies may be ſuſpended, we faſten the middle Hooks *g*, *f*, and *h*, *i* *, by attending to what was ſaid in N° 765. then there is a Diſtance of an Inch and an half between *g* and *f*, as alſo between *h* and *i*. If then we ſuſpend two Balls, the correſponding Threads being reduc'd to the ſame Height, they will touch one another.

1104. Plate XXVI. Fig. 7. If we have to do with the Cylinders B and C, the next Hooks come to be uſed; which are placed at a Diſtance, equal to the Diſtance *v* V in the Body to be ſuſpended, from the middle Hooks already faſten'd; which Diſtance is immediately determin'd, the Body itſelf being mov'd to it.

Theſe Bodies, the Threads being reduc'd to the requir'd Length, touch each other in the ſame manner, as was ſaid of the Balls, when they hang freely, whether there be two Cylinders as C, or as B; or one as C, another as B; or laſtly, whether either of them be applied to the Machine together with a Ball; for the Diſtance of the Hooks *v*, *v*, or V, V, from the Ends of the Bodies is three Quarters of an Inch.

In other Circumſtances, the Experiments with the elaſtick Bodies don't differ from the Experiments, explain'd in Chapter 4; the Velocities requir'd are communicated to the Bodies, and the Velocities after the Percuſſion are meaſur'd in the ſame manner, as in theſe; but as in the Percuſſion the Paralleliſm is never diſturb'd, we don't regard what was deliver'd in N° 942.

EXPERIMENT 2.

The two Bodies P and Q, whoſe Maſſes are as one to three, run againſt each other, at p, q, this with the Velocity five, that with the Velocity fifteen; after the Stroke each of them returns almoſt to the ſame Height from which it fell at *p* and *q*. 1105. Pl. XXXVI. Fig. 4.

The Defect of Elaſticity is the reaſon why the Bodies don't return to the ſame Heights exactly, from which they fell.

1106. In this Experiment, as in the following alſo, the Velocity, generated by the reſtoring of the Figure, wants a twelfth Part of that, which would be produc'd, if it were perfect; but if the reſpective Velocity of the Bodies be great, the Defect of the Velocity is greater, the Proportion being kept.

What is demonſtrated of a Spring, expanding itſelf between Bodies at reſt, muſt be referr'd to a Spring between Bodies, carried with the ſame Velocity with them, and which is at reſt in reſpect of the Bodies: therefore *if in a Ship two elaſtick Bodies ſtrike againſt each other, with Velocities, which are inverſely as the Maſſes, they will return with the ſame Velocities in the Ship* *. 1107.

* 1100.

1108. Thoſe Things being laid down which are mention'd in N° 987. in a Ship, which is carried with the Velocity B I, Bodies non-elaſtick are at reſt after the Stroke, and the Changes of the Velocities are inverſely as the Maſſes, the Velocities being deſtroy'd, with which they came towards each other in the Ship: now if they be elaſtick, they recede from one another in the Ship with the ſame Velocities, with which they came towards one another in the Ship *; that is, there is given a ſecond Change in the Velocities equal to the firſt: wherefore each Body undergoes a double Change in the Velocity, and *the reſpective Velocity after the Stroke is equal to the reſpective Velocity before the Stroke.* In *Fig.* 1. the Body mov'd with the Velocity B N, had in the Ship the Velocity I N before the Stroke; it loſt this, and acquir'd a Velocity I G equal to this on the contrary Side; therefore it has the Velocity B G. The other Body, whoſe Velocity was B E, return'd in the Ship before the Stroke; that is, it was mov'd more ſlowly than the Ship itſelf, by the Quantity I E; after the Stroke, it is carried with an equal Velocity I P on the contrary Side; that is, faſter than the Ship itſelf, and the Velocity is B P. Pl. XXXVII. Fig. 1, 2. 1109.

* 1107.

After the ſame manner in *Fig.* 2. the Body which had the Velocity B N, loſt the Velocity I N, which it had in the Ship; and now returns in the Ship, with a Velocity I G, equal to it; that is, it is carried with the Velocity B G after the Stroke: the other Body whose Velocity was B E, return'd in the Ship with the Velocity I E; now, the Motion being chang'd, it is carried in the Ship from the Stern to the Prow, with the Velocity I P equal to it, and its absolute Velocity is B P.

Hence we deduce two Rules, by which we determine the Velocities of elastick Bodies after the Stroke.

Rule 1.

1110. *If the Velocity of a Body, supposing Bodies non-elastick striking against each other, be increas'd by the Stroke, a double Increase must be added to the first Velocity, that the Celerity after the striking may be determin'd, if the Bodies are elastick.*

Rule 2.

1111. *In two Bodies non-elastick running against each other, if a Body loses of its Velocity, the Part lost must be doubled, when the Bodies are elastick, and subtracted from the first Velocity, to determine the Celerity after the Percussion.*

1112. Concerning the second Rule it is to be observ'd, that the Body which returns, does not only lose its former Velocity, but besides that there is a Velocity instead of the Velocity lost, acquir'd towards the contrary Part; and in this Case, the Sum of both these Velocities must be doubled, and subtracted from the former Celerity. But when a greater Velocity is subtracted from a less, the Excess towards the contrary Part must be taken.

Experiment 3.

1113. Pl. XXXVI. Fig. 5. The Body P, whose Mass is two, and Celerity nine, strikes against the quiescent Body Q, whose Mass is one; if the Elasticity were perfect, after the Stroke Q would be carried with the Celerity twelve, and P would continue its Motion with the Velocity three; which is discover'd by Computation according to these Rules: for if the Bodies were non-elastick, the Celerity of both after their concurring would be six *; therefore the Body Q would acquire six degrees of Velocity, therefore by Rule 1 † it now acquires twelve degrees. The Body P, which setting aside the Elasticity, loses three degrees of Velocity, by Rule 2 * loses six, which if they be subtracted from nine, the former Velocity, there remain three degrees of Velocity. But the Elasticity is imperfect; and in that Experiment the Velocity of the Body Q, is eleven and an half; and

* 992. † 1110. * 1111.

P

P continues in motion with three Degrees and a Quarter of Velocity; which agrees with what we have observed *; that the Change arising from the Elasticity must be diminished a twelfth Part. * 1106.

EXPERIMENT 4.

1114. Pl. XXXVI. Fig. 6. The Body P strikes against another Q, which is at rest, and triple, with the Velocity twelve; and, if they were perfectly elastick, it would return with the Velocity six. In this Case, Bodies non-elastick would be moved with the Celerity three; therefore the Body P would have lost nine Degrees of Velocity, therefore it loses eighteen Degrees by the Rule 2 *; which if they be subtracted from the former Velocity twelve, there are given six Degrees towards the contrary Part †. The Body Q, which acquires three Degrees of Velocity, when the Bodies are non-elastick, should now acquire six. But this second Change must be diminished its twelfth Part, and the Velocity is five with three fourth Parts. By reason of the Defect of Elasticity, the second Change of Velocity of the Body P is only eight and a quarter; and the Velocity, to be subtracted from twelve, is equal to seventeen and a quarter, and the Body returns with five and a quarter; and we discover these by Experiment. In the same manner, what is discovered by the Rules, is confirmed by the following Experiments; if we consider the Defect of Elasticity.

* 1112.

† 1112.

EXPERIMENT 5.

1115. Pl. XXXVI. Fig. 7. The Body P, whose Mass is two, and Velocity eight, runs against the Body Q, whose Mass is one, and which is carried towards the same Part with the Velocity five; if the Elasticity were perfect, after their concurring, the Body Q would be moved with the Velocity nine, and P would have the Velocity six, as is determined by the foregoing Rules. For if the Bodies were non-elastick, both would be moved with the Celerity seven after the Stroke *: the Body Q would acquire two Degrees of Celerity, which must be doubled by the Rule 1, and added to the former Celerity five, whence we have nine: the Body P would lose one Degree of Velocity, by Rule 2 it loses two, therefore it has six remaining. By having regard to the Defect of Elasticity, P has the Velocity six and a twelfth Part, and Q is moved with the Velocity eight and five-sixths. This Experiment also demonstrates it.

* 988.

Ex-

Experiment 6.

1116. Pl. XXXVI. Fig. 8. The Body P, with the Velocity nineteen, is carried towards the ſame Part with the triple Body Q, moved with the Celerity three; after the Stroke the Body P goes back with the Velocity four, Q continues its Motion with the Velocity ten and two thirds.

Experiment 7.

1117. Pl. XXXVI. Fig. 9. The Body P with the Celerity five, and the triple Body Q with the Celerity eleven, are carried towards contrary Parts; after their Concourſe Q continues its Motion with a Celerity, equal to three and a third, and P goes back with the Velocity eighteen.

Experiment 8.

1118. Pl. XXXVI. Fig. 10. The ſame Bodies, P and Q, are carried towards contrary Parts, P with the Celerity ſixteen, Q with the Velocity eight; both go back after the Stroke; the Velocity of P is eighteen and a half, and Q has a Velocity, equal to three and a half.

Experiment 9.

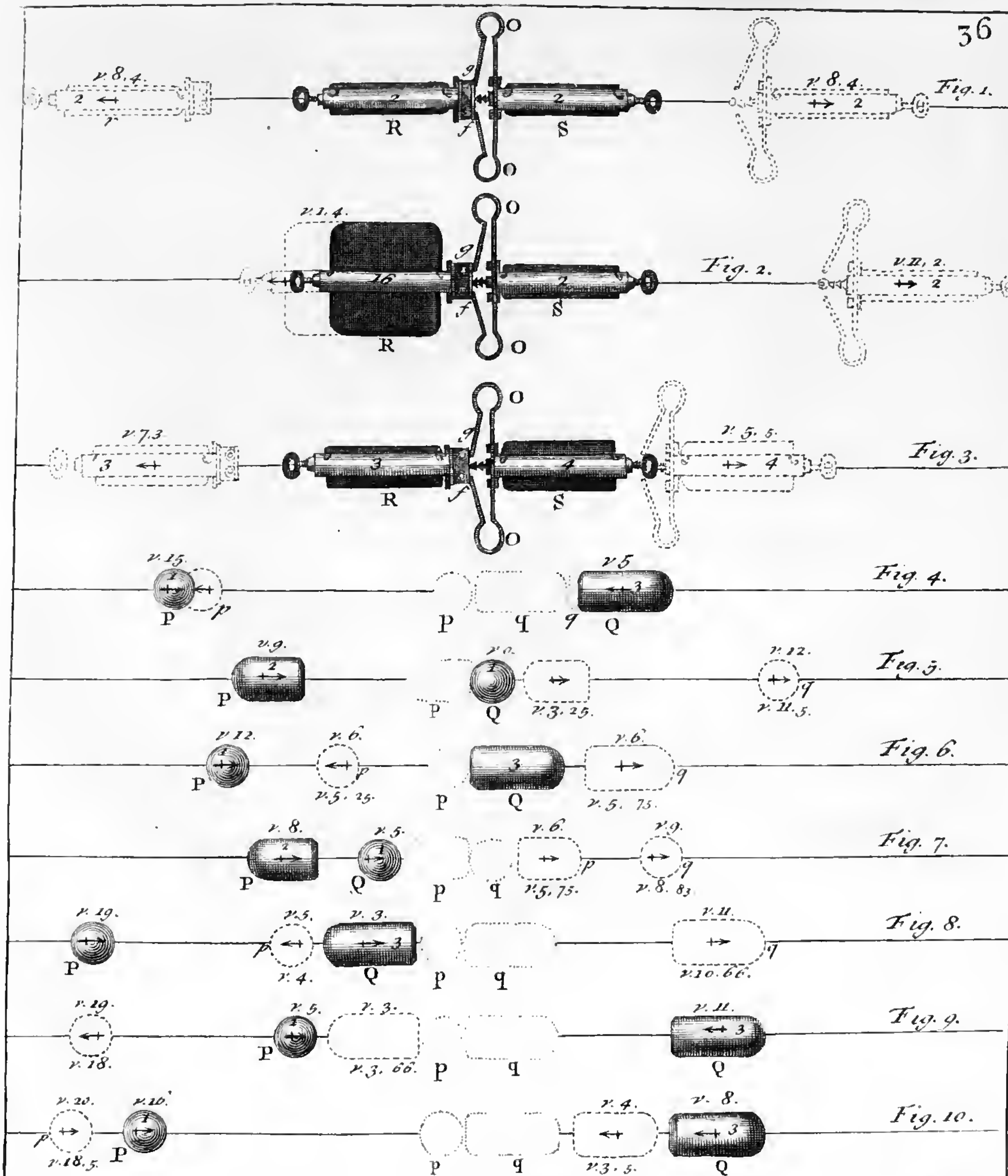
1119. If in the laſt Experiment the Velocity of the Body P be changed, and it be eight, the Bodies being ſuppoſed perfectly elaſtick, Q would loſe its whole Motion, and P would return with the Velocity ſixteen; but in the Experiment Q keeps a third Part of a Degree of Velocity, and the Velocity of the Body P is fifteen.

All the Caſes of the Percuſſions of elaſtick Bodies are determined by the Rules above-mentioned; we alſo deduce the following Propoſition from them.

1120. *When the Bodies are equal, and carried towards the ſame Part, they continue their Motion with changed Velocities; if they be carried towards contrary Parts, they go back with changed Velocities.*

1121. Pl. XXXVII. Fig. 5. *Caſe* I. Let the Bodies tend towards the ſame Part, and let A B be the Velocity of one Body, A C the Velocity of the other; by reaſon of their equal Maſſes, the Changes of the Velocities are equal *. Let B C be divided into two equal Parts in D, and A D will expreſs the Celerity of each Body after their Meeting, if they are non-elaſtick; the Celerity A B is increaſed by the Quantity B D, it ought to be increaſed by twice ſuch a Quantity by reaſon of its Elaſticity †; in which Caſe the Velocity A B is changed

* 987.



J. Mynde sc.

changed into A C. In the same manner the Celerity A C, in Bodies non-elastick, is diminished by the Quantity D B; it ought to be diminished by twice this Quantity ||, and become A B. || 1111.

EXPERIMENT 10.

Two equal Bodies are carried towards the same Part, the first with the Velocity ten, the other with the Velocity four; they would continue their Motion after the Stroke with the Velocities changed, if the Elasticity were perfect; which also is discovered by Computation from the foregoing Rules. But in the Experiment the Antecedent Body acquires less, the consequent one loses less, and each Difference is equal to a Quarter of a Degree of Velocity. 1122. Pl. XXXVII. Fig. 6.

Case 2. Let AC be the Celerity of one Body, A B the Celerity of the other; if B C be divided into two equal Parts, the Velocity of each after their Meeting, is towards the same Part A D *, when the same Bodies are non-elastick. Therefore the first Body lost the Velocity D C, the other Body lost the whole Velocity A B, and acquired A D towards the contrary Part; therefore the whole Quantity lost is D B †, equal to D C; this Quantity if it be doubled, B C will be the Quantity of the Celerity lost by each Body ||; which being subtracted from the Velocity of each Body, gives in each Case a Velocity towards the contrary Part ‡, equal to that, which the other Body had. 1123. Pl. XXXVII. Fig. 6.

* 990.
† 1112.
|| 1111.
‡ 1112.

EXPERIMENT 11.

If equal Bodies be carried towards contrary Parts, one with the Celerity twelve, the other with the Velocity six, they both go back after the Percussion with changed Velocities, each Velocity wanting three Quarters of a Degree of that, which the other Body had; which is discovered by Computation also, if we attend to the Defect of Elasticity. 1124.

A Body strikes against another quiescent Body equal to it, if the Velocities are changed, *the first Body will rest after the Stroke, but the other will be moved with the Velocity of the former.* 1125.

EXPERIMENT 12.

If a Body strikes against an equal Body at rest with the Velocity twelve; the first is moved with half a Degree of Velocity, the second acquires the Velocity eleven and an half, if the Elasticity were 1126.

were perfect, the first would be at rest after their meeting, and the other would be moved with the Celerity twelve.

1127. *In elastick Bodies the Action of the Spring is very sudden.* Therefore if various elastick Bodies are contiguous, and the last be struck, all the following are moved, as if they were separate; if likewise of many Bodies contiguous, moved with the same Velocity, the antecedent one runs against any Body, it acts as it

1128. would separated from the rest. Whence it follows that *a Body is moved only by the Action of the Body next to it, and* that *it acts upon the Body next to it only*, the elastick Parts returning to their Figure, before the Action can be communicated to the next Body.

Experiment 13.

1129. 1. Let there be many, equal Bodies, Q, R, S, T, V, disposed in

Pl. XXXVII. Fig. 7. the same Line, and touching one another; let a Body P, equal to the rest, strike against Q, with any Velocity whatsoever; after their meeting P, Q, R, S, and T, remain at rest; or rather are moved a very little, which follows from the Defect of Elasticity; and V only is moved.

Pl. XXXVII. Fig. 8. 2. Let the two contiguous Bodies P and Q be moved with equal Velocities, so that Q may strike against R, after the Stroke P, Q, R, and S, are at rest, but T and V are moved together.

Pl. XXXVII. Fig. 9. 3. If three be moved in the same Manner, three also are moved after their meeting.

Pl. XXXVII. Fig. 10. 4. If four be moved, four also are separated from the rest after the Percussion.

Pl. XXXVII. Fig. 11. 5. Lastly, if P, Q, R, S, and T, be moved together, and T strikes V, after the Percussion P only is at rest, Q, R, S, T, and V, are moved together. In general, whatsoever the Number of Balls be, as many as are moved before their meeting, so many also are moved afterwards.

1130. These Bodies act as if separated *. In the first Case P strikes

Pl. XXXVII. Fig 7. * 1128. † 1125. against Q, and is at rest †; Q, moved by the Stroke, strikes against R, and is at rest also; and so of the rest; 'till at last T strikes V, which only being retained by no Obstacle, continues in Motion.

Pl. XXXVII. Fig. 8. In the second Case, the Body Q drives the Body V in the same Manner; P immediately follows, running against Q, which is already at rest from the former Stroke; Motion is also communicated to the Body T in the same Manner, which can't strike V

 already

already in motion; and as the Motions of the Bodies P, and Q, are equally swift, and those Bodies follow each other very nearly, there is not given a sensible Time between those two Communications of Motion; whence also the Bodies V, and T, are moved equally swift and not separated.

It appears by another Experiment, that the Action of the Spring is so sudden as scarce can be conceived. 1131.

EXPERIMENT 14.

An hollow Ivory Ball, about two Inches Diameter, is made up of two Hemispheres A and B, which may be joined together very fast by a Screw. 1132 Pl XXXVII. Fig. 12.

The Hemisphere B is let down in such manner, from any Height, whatsoever, for Example, eighteen Inches, that it may strike against a marble Plane, made wet a little; and indeed that the Middle Point of the Surface may run against the Plane. This may be done without much Difficulty, if the Hemisphere be thicker in that Place than towards its Extremities. Let the Spot be measured, which the Body makes on the Marble by the Stroke.

Join the Hemisphere A, and let the Ball be let down from the same Height in such manner, that the same Point, of the Surface of the Hemisphere B, may run against the Marble Plane; which will be easily done, if the Hemisphere A be lighter than the other: the Spot will be exactly equal to the first Spot, and the Ball will rebound much less.

Lastly, let there be put into the Cavity of the Ball a Piece of Lead P, of the same Weight as the Ball itself; and let it be there fastened: the Ball being let down in the same manner, the Spot will be the same in this third Case also, and the Ball will scarce rebound at all.

But a solid Ivory Ball being let down from the same Height, equal to the Ball mentioned, the Spot will be greater, and the Ball returns almost to the same Height from which it fell.

In this Experiment we see, that the Parts struck of the Hemisphere B returned to their Figure, before the Action of the Hemisphere A, or the Lead in it, could be communicated to it, tho' these Bodies cohere closely enough.

In the last Chapter of this Book I shall treat about determining the Times themselves, in which the Inflections of elastick Bodies are made; and we shall find that in this Experiment the 1133.

Time, in which the Parts are preſſed inwards, is nine fifth Minutes; or $\frac{1}{23593}$ of one Second.

SCHOLIUM I.

In which what is demonſtrated in Scholium 3. *Chap.* 4. *of this Book, is applied to elaſtick Bodies.*

IN Scholium 3. Chap. 4. of this Book we have demonſtrated how what happens in Colliſion, during the mutual Action of the Bodies, when they are non-elaſtick, is determined geometrically; what is there demonſtrated, of the Diminutions of the Velocities, and Forces, may be referred
* 929. to elaſtick Bodies *; and it will be eaſy to determine, what happens in the reſtoring of the Figure.

1134. Thoſe Things being laid down which are explained in the Scholium
Pl.XXXVII. mentioned, E *e* is the Velocity with which both Bodies are moved, in the
Fig. 3. 4. very Moment of the whole Inflection of the Parts. Whilſt the Parts return to their former Figure, the Changes of the Velocities are continued,
* 967. and indeed according to the ſame Laws as in the preſſing inwards †; wherefore C E and D E being continued, and H R being drawn parallel to D A, the Lines R S, R H, will ſhew the Velocities of the Bodies; and the Triangle E S H will repreſent the Force reſtored. This Force, if we conſider the whole Action of the elaſtick Parts, is equal to that, which was
* 1085. deſtroyed by the Colliſion *; therefore the Triangles D E C, E S H, are equal. Therefore if *e* R be equal to *e* A, the Velocities of the Bodies after their Separation will be R S, R H. In the Caſe *Fig.* 3. the Body, which had the Velocity A D, returns with the Velocity R S, the other keeps its Direction; in *Fig.* 4. the Body, whoſe Velocity was A D, continues in motion with the Velocity R S, the other goes back with the Velocity R H.

Theſe Things are ſo, ſuppoſing the Elaſticity perfect; if there is a Defect in it, *r h*, muſt be drawn parallel to R H, ſo, that *e r* may be to *e* R, as the Velocity, which is really generated, by the reſtoring of the Parts, is to that, which would be produced, if the Elaſticity were perfect. Then *r s*, *r h*, are the Velocities ſought.

In the Figures, which would ſerve for the Experiments of this Chapter, *e r* would be to *e* R, or *e* A, as eleven to twelve.

SCHOLIUM II.

A fuller Demonſtration of N. 1085.

1135. WE have demonſtrated that in the Congreſs of elaſtick Bodies the
Pl.XXXVII. Sum of the Forces is the ſame before and after the Stroke *;
Fig. 1. 2. whence follows, laying down what is explained in N. 1108. 1109. $AB \times$
* 1085. $BN^q + BC \times BE^q = AB \times BG^q + BC \times BP^q$ †, of which we will
† 757. give a geometrical Demonſtration here alſo.

Firſt,

First, let the Bodies tend towards the same Part. Let the Squares of the Lines B E, B G, B N, and B P be formed; let the Diagonal of all B V be drawn. Let I S be drawn parallel to P V; and, let X S K be drawn parallel to P B, thro' S, the Point in which it cuts the Diagonal: let G R and E Q be continued to Z and K. Because I N and I G are equal, as also I P and I E, the Triangles Y S T, R S Z, are equal; also the Triangles S X V, S K Q. Therefore the Trapezium G R T N is equal to the Rectangle G Z Y N; and the Trapezium E Q V P equal to the Rectangle E K X P. Pl XXXVII. Fig. 1. 13.

The half difference of the Squares of the Lines B N, B G, is the Trapezium G R T N, or the Rectangle G Z Y N. In the same manner the half Difference of the Squares of the Lines B P, B E, is the Rectangle E K X P; but these Rectangles, by reason of the common Height I S, are as their Bases *; or as the halves of their Bases I N, I E; also, as are the half Differences of the Squares, so are the whole Differences: therefore $BN^q - BG^q : BP^q - BE^q :: IN : IE$; that is, as B C is to A B, from the Construction.

* 1 El. VI.

Therefore $AB \times BN^q - AB \times BG^q = BC \times BP^q - BC \times BE^q$; therefore $AB \times BN^q + BC \times BE^q = AB \times BG^q + BC \times BP^q$. Which was to be demonstrated.

1136. Let the Bodies now tend towards contrary Parts. Let the Squares of the Lines B P, B N, B E, or B *e*, and B G or B *g*, be formed again: Because I N, I G, and I P, I E, are equal, N P, E G or *e g* are equal; let us add on both sides *e* N, *e* P, *g* N, will be equal. The difference of the Squares B V and B Q, that is, of the Squares of the Lines B P, B E, is a Rectangle, whose Base is P V and *e* Q, that is, P E, and Height *e* P; the Difference of the Squares B T, B R, that is, of the Squares of the Lines B N, B *g* or B G, is a Rectangle, whose Base is N T and *g* R, that is, N G, and Height *g* N; by reason of the equal Heights these Rectangles are as the Bases P E, N G, or as their Halves I E, I N, which are as A B, B C; therefore $BP^q - BE^q : BN^q - BG^q :: AB : BC$. Pl. XXXVII. Fig. 2. 14.

Therefore $AB \times BN^q - AB \times BG^q = BC \times BP^q - BC \times BE^q$; whence we deduce $AB \times BN^q + BC \times BE^q = AB \times BG^q + BC \times BP^q$. Which was to be demonstrated.

SCHOLIUM III.

An Illustration of the mutual Action of elastick Bodies.

1137. ALthough there can be no doubt about the Equality of the Sum of the Forces, before and after the Stroke, as this follows from perfect Elasticity itself *; and also is deduced from the Rules of Computation, as we have done in the foregoing Scholium, I dont deny but there is notwithstanding some Difficulty in these Things; as it does not appear from what is demonstrated, how a Spring, which whilst it expands itself between Bodies, and communicates Forces towards opposite Parts, which are inversely

* 1083.

* 1098. versely as the Masses *, can often not only impress the whole Force, wherewith it expands itself, to one Body only, but besides as much as it takes away from the Force of the other Body: For example, the Body A, * 757. with two Degrees of Velocity, that is, with four Degrees of Force *, runs against the Body B, equal to it at rest, after the Stroke, setting aside the * 992. Elasticity, both have one degree of Velocity *; and each has one degree † 757. of Force †; that is, the Force of both is equal to two, and the Parts were ‖ 924. compressed with two remaining degrees of Force ‖; and, if the Bodies are elastick, the Spring was bent by this very Force, and expands it- * 1083. self with the same Force *: but after the Stroke the Body B has two de- † 1120. grees of Velocity †; that is, the Force four ‖; and A is at rest. Therefore ‖ 757. the Spring communicated three degrees of Force to B, and took one degree from A; tho' it was bent with two degrees of Force only; and tho' by reason of the equal Bodies, it exerted equal Impressions on both sides.

1138. To take away this Difficulty, we must distinguish between absolute and relative Force. The Spring placed between Bodies, communicates Forces to them, which are inversely as the Masses, if it be at rest between the Bo- * 1089. dies *; that is, if the Bodies being moved, it be carried with the same Motion with them; such as are the Motions communicated to Bodies in a Ship, which is carried with the same Velocity with the Bodies, and * 1107. in which therefore they are at rest with their Spring *. But this Proposition ought not to be referr'd to absolute Motions, of which one is accelerated, the other retarded, the Spring being already moved, before its Action.

With respect to absolute Forces we must observe, that those are often communicated to a Body, by a moving Cause which is moved itself, in 1139. which Case *not only the moving Cause acts upon the Body, but there is also given that Action upon the Body, which transfers the moving Cause itself;* * 706. *and a Force is communicated to the Body, equal to the Sum of these Actions* *: for this Force is the Effect of both Actions united; for we speak of the Case in which they produce no other Effect.

When a Spring stands against an Obstacle, which does not yield at the opposite Part, it communicates the whole Force, with which it was bent, to the Body which it repels from the Obstacle, as follows from Demonstration N. 1089. and is confirm'd by Experiment 2. Chap. 2. of this Book, compar'd with Experiment 1. of this Chapter.

But if the Spring which stands against the Obstacle on one side, which does not yield, exerts its whole Force at the opposite Part, much more *the* 1140. *Spring, which is moved towards that Part, at which it acts, will communicate the whole Force, with which it is relaxed, to the Body, on which it will impress the Force also, which is equal to the Action, which moves the Spring,* * 1139. *whilst it is relaxed* *.

1141. From whence it also follows, *when the Spring stands against an Obstacle, not altogether immoveable,* that *it exerts its whole Force at the opposite Part, taking away that, with which it can move the Obstacle.*

If

If we apply this to the Case mentioned, we easily see, that two degrees of Force are communicated to the Body B, whereby the elastick Parts were bent, and besides the Impression, with which the Spring was moved during its Expansion *, which Impression is the Action of the Body A upon the Spring, and is equal to the Force lost by the Body A in this Action †. But A lost one degree of Force, which therefore was communicated to B, besides the two mentioned; therefore B received three degrees of Force, which added to the one degree, which it had before the Action of the Spring, give four degrees of Force. Which was to be explained. 1142.

* 1140.
† 709.

The reasoning is entirely like this in other Cases, in which after the Separation the Bodies tend towards the same Part, or one is at rest; but if they are moved towards opposite Parts after the Stroke, the Difficulty is taken away by the same Principles, as will appear by this Example. 1143.

Let there be a Body A whose Mass is 1. which with the Velocity 6, that is, with the Force 36 *, runs against a double quiescent Body B; we suppose the Bodies perfectly elastick. After the Stroke B will have the Velocity 4, that is, the Force 32, and A will return with the Velocity 2, and will have the Force 4 *. This, which was demonstrated before, must now be illustrated. 1144.

* 757.
* 992. 1110. 1111.

Before the Figure is restored, both Bodies are moved with two degrees of Velocity *, and the Spring was bent with the Force 24 †. If we conceive a Ship in which the Bodies after the Stroke, before the Separation, are at rest; that is, whose Velocity is also two; the Bodies are separated in it, with Forces, and Velocities, which are inversely as the Masses *; B with the Velocity 2, and A with the Celerity 4; if the Spring had been less bent, the Forces would also have been inversely as the Masses †; therefore, the Spring being relaxed in part, the Forces, and therefore the Velocities, are in this Ratio ||; therefore when B has one degree of Velocity in the Ship, A has two, and the Action of the Spring is equal to six degrees of Force; for this is the Sum of the Forces communicated.

* 992.
† 985. 934. 938.
* 1089.
† 1088.
|| 791.

Then, considering the absolute Motions, B has the Velocity 3, and A is at rest; but the Spring exerts the Force 18 remaining, whilst in the Ship it communicates a second degree of Velocity to the Body B, and a third and fourth to the Body A. B has now, setting aside the Ship, the Velocity 4, and A returns with the Velocity 2.

The Body B, before the Spring is relaxed, has the Force 8 *; whilst the Spring exerts the first six degrees of Force, A does not yet return, and we have the Case of N. 1142. therefore these six Degrees are communicated to the Body B, and besides as much as A loses, as we have explained this in N. 1140, that is, 4; and B has, in this Instant, the Force 18, which answers to the Velocity 3 *; for the Mass is two.

* 757.
* 757

One side of the Spring now stands against a quiescent Body, the other against a Body moved, and the Spring itself, hitherto, was wholly moved; but now it is transferred in part only, and can repel the Body A, with the Force 4; therefore it can only impress the remaining Force 14 on B *. These

* 114[illegible]

added

added to the 18 Degrees, already communicated, give 32 Degrees, which it really has, as appears from what is before demonſtrated.

1145. In the ſame manner we illuſtrate an analogous Caſe. A Man projecting a Body impreſſes on it two Degrees of Velocity, therefore the Force four. If this were performed in a Ship carried with the Velocity eight, the Velocity of the Body would become ten, and the Force of the Body, which was equal to ſixty-four, as long as the Body had the ſame Velocity with the Ship, is now equal to an hundred, and the ſame Action, which in the Ship communicated four Degrees of Force to the Body, if we don't attend to the Ship, gave to the Body a Force, equal to thirty-ſix. If inſtead of the Action of a Man, to make it more regular, we ſuppoſe a Spring, which, communicates two Degrees of Velocity to the Body whilſt it is in the Ship, the Computation may be made, by a like Method with the foregoing, the Maſſes of the Body and Ship being determined at pleaſure; but we put an end to the Difficulty more eaſily, if we conſider, that the Body is at reſt in the Ship; and, if we do not attend to this, that it is moved; and that the Effect of the ſame Preſſure, whoſe Intenſity is determined, follows the Ratio of the Velocity of the Point to
* 723. which it is applied *. And that the Action, by not attending to the Ship, is ſo much the greater, as the Spring is moved with a greater Velocity together with the Ship.

SCHOLIUM IV.

An Explanation of a Paradox.

1146. FROM a Property of elaſtick Bodies mentioned in N. 1128, is deduced the Explanation of a Paradox, not leſs remarkable than common.

Silver-Smiths that live in the upper Part of any Houſe, put an Anvil upon a Cuſhion, whereby the Anvil reſiſts the Blows of the Hammer more, and the Houſe trembles leſs.

Let us ſuppoſe, the Cuſhion being taken away, that the Anvil is put upon a Beam, the Anvil is an elaſtick Body; and the Beam, whoſe Ends are fixed, is alſo an elaſtick Body. Let us ſuppoſe the Anvil to be ſtruck with the Hammer.

Let the Maſs of the Hammer be called M; the Maſs of the Anvil I; and the Maſs of the Beam, together with the Bodies joined to it, and which are moved together with it, T; let alſo the Velocity of the Hammer be v. By multiplying M by v, and dividing the Product by the Sum of the Maſſes of the Hammer and the Anvil, we have the
* 992. 1110. half of the Velocity communicated to the Anvil *. For, tho' the An-
† 1128. vil be put upon the Beam, it is moved as if it were ſingle †; and its Velocity is $\frac{2\,M \times v}{M + I}$; with which Velocity the Anvil ſtrikes the Beam and the

 Bodies

Bodies joined to it; by multiplying the Velocity of the Anvil by the Mass, we have $\frac{2\,M \times I \times v}{M + I}$; by dividing the double of this Product by the Sum of the Masses, we have $\frac{4\,M \times I \times v}{\overline{M+I} \times \overline{I+T}}$, the Velocity of the Beam *. The Denominator of this Fraction M × I + I × I + T × M + T × I, by reason of M being small in respect of I and T, scarce differs from this othe M × I + I × I + T × I; which being laid down, the Velocity discovered is changed into this $\frac{4\,M \times v}{M+I+T}$. * 992, 1128.

A soft Body, namely a Cushion, being put between the Beam and Anvil, these Bodies do as it were make one Mass; and therefore, as now the Hammer strikes a greater Body it suffers a greater Resistance, and the Velocity communicated to the Beam is had by dividing 2 M × v by the Sum of the Masses M + I + T *; and the Velocity is $\frac{2\,M \times v}{M + I + T}$, the half of the Velocity, the Cushion being taken away; wherefore the Agitation, this being taken away, can produce a quadruple Effect †. * 992. 1110. † 753. 712.

CHAP. VII.

Of Compound Motion.

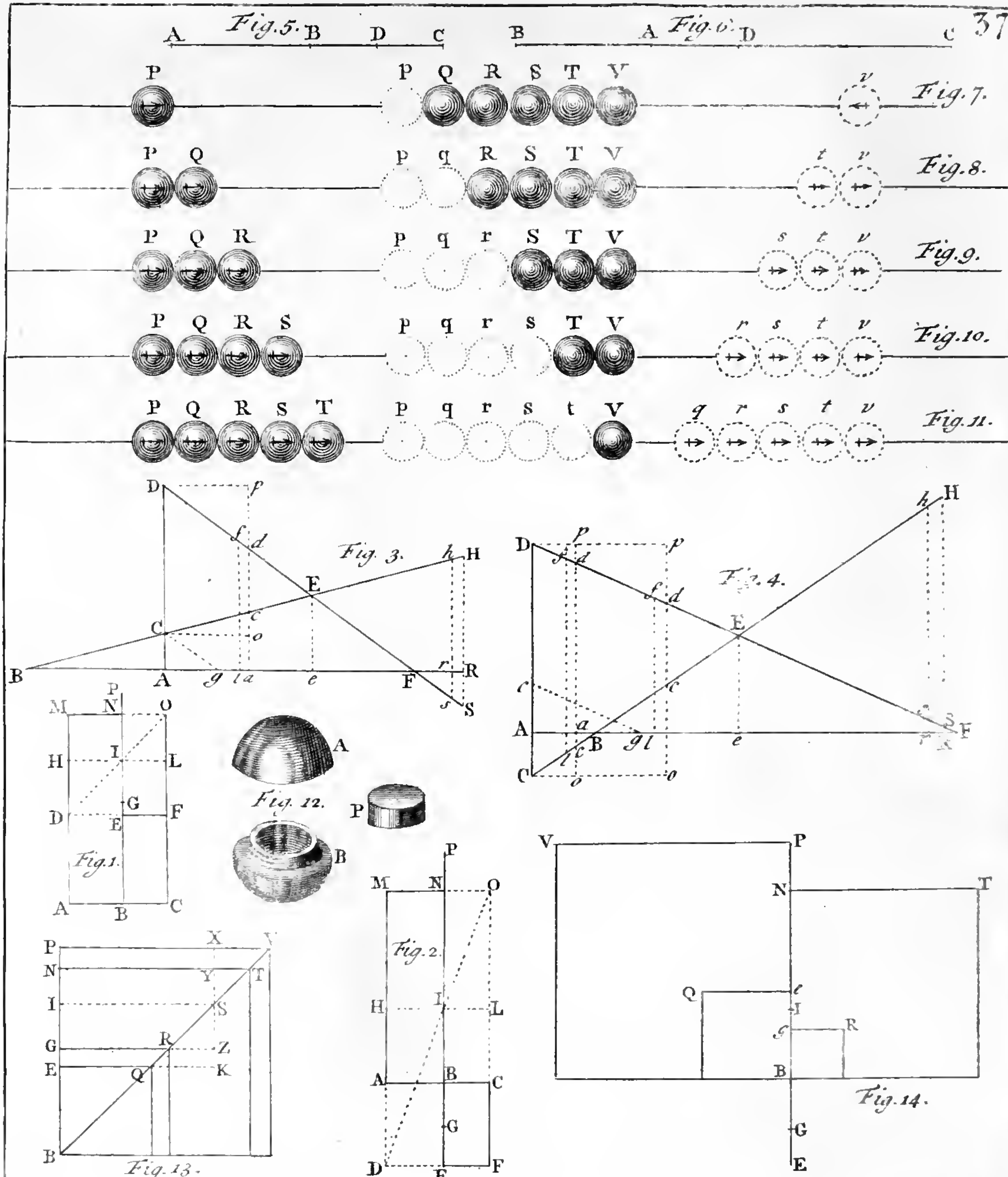
1147. IF *a Body be mov'd, and its Celerity is to be increas'd, or diminish'd, the Direction remaining, it is evident,* that *an Impression is requir'd, which is proportional to the Difference of the Squares of the Velocity, which the Body had before the Action, and of that which it has after the Action*; for the Force communicated, or taken away, is proportional to this Difference *. * 753.

1148. *Let us suppose two Actions, at the same time, to act upon the Body, in the same Direction.* Whilst the Velocity is increas'd, the Force impress'd on the Body increases in the duplicate Ratio of this *; that is, the Increase of the Force follows the Proportion of the Increase of the Triangle, which, whilst it is increas'd, keeps the same Angles, and one of whose Sides represents the Velocity †; the Force whilst the Velocity is A g, is to the Force, when the Velocity is A l, as the Area A g r is to A l s. * 753. Pl XXXVIII. Fig. 1. † 19. El. 6.

Let

Let us conceive the Actions to act upon the Body alternately, during equal Intervals of Time; that by the first Action the Force A *d o* is communicated, by the second the Force *d o p e*; again, that by the first Action the Force *p e f q*, and by the second *f q r g* is communicated, and so on: the Sum of the white Areas represents the whole Force, communicated by the first Action; and the Sum of the black ones denotes the whole Force, impress'd on the Body by the second Action. As the Actions acted during equal Times, these Forces, namely, the Sums of the Areas, are as the Actions themselves; in which Ratio also is any white Area whatsoever to the black one next to it. If the Moments of the Time were infinitely small, as they are, when the Actions act together, these Areas may be look'd upon as Parallelograms, and the neighbouring Parallelograms will have the same Height; and will therefore be
* 1. El. 6. to one another as the Bases *: therefore the Base of a white one is to the Base of a neighbouring black one, *as the first Action is to the second*; and in the same Ratio is the Sum of the Bases of the white Parallelograms to the Sum of the Bases of the black; that is, *so is the Velocity, which the first Action communicated, to the Velocity, arising from the second.* Which same Demonstration takes place in any Acceleration of a Body whatsoever, when many Actions drive it together.

1149. If there be an Action given upon a Body in motion, in a Direction different from that of the first Motion, we have seen above
* 357. that there is a Change given in the Direction *; and we have ex-
† 360. amin'd what relates to the Velocities in this Case †; we must now speak of the Forces. Let the Body be mov'd along A D, with a
Pl. XXXVIII. Celerity represented by this Line; and let a new Force drive it
Fig. 2. 3. 4. along A E, with a Celerity denoted by this other Line; the Body
1150. carried with two Celerities is mov'd along A B *. Yet *an equal*
* 360. *lateral Velocity is not communicated in all Cases, by an equal Impression.* We put A B, and A E, in these three Figures, respectively equal: in *Fig.* 3. the second Motion, in part conspires with the first Motion; so that in this Motion the Acceleration of the Motion along A D is contain'd. In the same manner the Retardation of the Velocity along A D is contain'd in the Motion along A E in *Fig.* 4. Therefore the Impressions, by which the Bodies are driven along A E, that they may communicate the Velocity denoted by
* 714. this Line to each of the Bodies, are not equal to one another *, nor to the Impression, whereby this Velocity could be communicated
† 707. to a quiescent Body †.


37
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 3.
Fig. 4.
Fig. 1.
Fig. 12.
Fig. 2.
Fig. 13.
Fig. 14.

In the Case of *Fig.* 2. only, where the Angle E A D is a right one, the lateral Motion neither conspires with, nor acts contrary to, the Motion along A D; and the Impression, whereby the Body is mov'd, acts upon the Body, as if it were at rest: therefore, in this Case, the Force communicated to the Body is proportional to the Square of its Velocity *; and as the Impression, whereby the Motion is chang'd in this Case, has nothing common with the first Motion, it cannot diminish the first Force, which acts in the Direction A D: therefore the whole Force, which the Body now has, is proportional to the two Squares of the Lines A D and A E, which agrees with what is demonstrated; for the Body is carried with the Celerity A B *, whose Square is equal to the two Squares mention'd †. 1151.

* 753.

* 360.

† 47 El. 1.

From this the Measure of the Forces, if it were unknown, might be discover'd. To a Body, which has a Force, answering to the Celerity A D, a Force is communicated, which answers to the Velocity A E; which being communicated to the Body, as if it were at rest, cannot alter the first Force; therefore the whole Force of the Body is equal to the Sum of these Forces, whilst its Velocity is A B; therefore the Force, which answers to this Velocity, is equal to the Sum mention'd. Which can't be in every Case, unless the Forces are proportional to the Squares of the Velocities *. 1152.

* 47. El. 5.

From hence we deduce, that it matters not, either with respect to the Impressions, whereby the Body is mov'd, or with respect to the Forces, or Velocities, whether the Body be carried along A B with the Celerity A B, or along A D and A E with Celerities proportional to these Lines, which contain between them a right Angle. Wherefore *the Motion along* A B, *according to a Direction as* A D, *contains nothing but the Motion with the Velocity* A D. 1153. 1154.

Plate XXXVIII. Fig. 2.

We also infer, that *the Motion of a Body may be resolv'd into two others,* many ways; which will be done, *if a Line, which is plac'd in the Direction of the Motion given, and denotes the Celerity by its Length, be the Hypotenuse of a right-angled Triangle*; for *the two other Sides of it will, by their Situation, give the Directions of the Motions sought, and by their Lengths respectively express the Velocities of them: and the Forces in these Directions will be proportional to the Squares of the Velocities.* 1155.

Now to determine, with what Force a Body must be mov'd along A E, that the Celerity A E may be communicated to it, in the Case in which this Motion conspires in part with the first Motion; I re- 1156.

Plate XXXVIII. Fig. 3

ſolve the Motion along A E into two Motions along Af and A g, containing a right Angle, and Eg is drawn parallel to Af. Along Af, ſuch a Force muſt be communicated to the Body, whereby the Body, if it had been at reſt, might have been carried with this
* 1151. 753 Celerity, and which is proportional to the Square of Af*; but along Ag a Force muſt be communicated, whereby the Celerity A D may be increas'd by the Quantity Ag; that is, may become Ah; which Force is proportional to the Difference of the Squares
* 1147. of Ah, A D*. Theſe Forces will be to be communicated together along A E, that the Body may be carried with this Celerity; and the whole Force of the Body will be proportional to the Square of the Line A D, the Difference of the Squares of the Lines Ah and A D, and the Square of Af; the two firſt of theſe three Quantities being collected into one Sum, we have the Square of the Line A B; to which that the innate Force of the Body is proportional,
* 753. follows from what has been demonſtrated before*; ſince it is ma-
† 360. nifeſt, that the Body is carried with the Celerity A B †.

1157. Plate XXXVIII. Fig. 4. * 360. If we reſolve the Motion along A E, after the ſame manner, into the two Motions along Af and A g*; the Motion along A D is retarded by this ſecond Motion; whence it follows, that, to carry the Body along A E, with a Celerity repreſented by this Line, a Force muſt be communicated to it, which is proportional to the Square of Af; and the Impreſſion, with which it is mov'd, ſhould moreover be able to diminiſh the Velocity A D by the Quantity Ag: In this Caſe, the Body will only have a Force remaining
* 1147. along the Direction A D proportional to the Square of A h*; to which, if there be added a Force proportional to the Square of
† 1151. Af †, we have a Force proportional to the Square of A B; which
‖ 360. 753. again agrees with what has been demonſtrated before ‖.

1158. It plainly follows from what has been demonſtrated before*, that
* 1147. 1148. this Propoſition, that the Force follows the Proportion of the Square of the Velocity, can't be referr'd to that, with which another acts in the ſame Line; for this Reaſon, when we reſolve the Force into two, theſe Forces will not be proportional to the Squares of the Velocities, unleſs the Directions of both contain a right Angle; for otherwiſe they may conſpire in part, or act contrary to
* 1151. each other*.

1159. Plate XXXVIII. Fig. 2. From whence we infer, that *the Force reſolv'd can't be reſolv'd again in ſuch manner, that each may be proportional to the Squares of the Velocities.* The Motion along A B is reſolv'd into two Motions of the ſame Body along A D and A E, and the Forces are proportional

portional to the Squares of the Velocities; but if the Motion along A E be divided again into two, along A F and A G, containing a right Angle, these last Forces will not be proportional to the Squares of the Velocities; and we can't apply here what is said in N. 1155, in which we treat of Forces, which do not only not conspire together, nor act contrary to one another, but have nothing common with a third. But here the Motion of the Body along A B is resolved into three Motions along A D, A F, and A G; in which A F and A D conspire in part, A D and A G in part act contrary to one another; and it is manifest from what has been before demonstrated *, that the Resolution, which may * 707. 714.
be applied to the Velocities, as the Demonstration of N. 360, is the same, whether the Motions conspire in the Resolution, or act contrary to one another, can't be referred to the Forces.

In N. 1156, 1157. the Motion along A B is compounded 1160.
of two, one of which we resolve into others, but in such Pl.XXXVIII
manner, that all the Motions after the Resolution might be Fig. 3. 4.
given in two Lines, containing a right Angle: wherefore the Motions in each of the Lines, might be considered separately; which never can be done, when various Motions are given in more than two Lines; for then some Motions of necessity conspire in part, or act contrary to one another: we have demonstrated nothing concerning these, yet they may be deduced from the same Theory of Forces. But this belongs not to this Place; for there is not given a Solution of one Force into three others, except there are given three Actions, which can't be determined separately, but should always be considered together; but we shall see in the following Chapter, that there is often given one Action only, when the Force is resolved into two; which therefore may be considered separately, and determined.

 CHAP.

CHAP. VIII.

Of Oblique Percussion.

DEFINITION I.

1161. *THAT is called the Angle of Incidence, which the Direction of the Motion of a Body, moving towards another, makes with a Perpendicular drawn to the Surface of it in the Point, in which it is struck.*

DEFINITION 2.

1162. *The Angle of Reflection is that, which the Direction of the Motion of a Body after the Percussion, makes with the same Perpendicular.*

1163. *If an elastick Body* P, *runs obliquely against a fixed, elastick Ob-*
Pl.XXXVIII *stacle* F G, in the Direction P *a*, *it will return along* *a p* *in such*
Fig. 5. *manner, that the Angle of Incidence* P *a* B *will be equal to the Angle of Reflection* B *a p*. The Motion along P *a*, which we suppose to represent the Celerity of the Body by its Length, may be resolved into two, the Direction of one of which is parallel to the Line B *a*, of the other perpendicular to it; and the Body will run against an Obstacle at *a*, as if it came to it with the Celerities C *a*,
* 1155. B *a*, and according to these Directions *. The Motion along C *a* is not changed by the Stroke, and the Body continues its Motion with the Celerity *a* E, C *a* and *a* E being put equal; with the Motion along B *a* it runs directly against the Obstacle, and re-
† 1086. turns along the same Line †, that is along *a* B, with the same Celerity with which it went; but the Body being carried with these two Motions returns along *a p*, the Diagonal of the Rectangle,
‖ 360. formed by the Lines *a* E, *a* B ‖ : but it is plain that the Triangles B P *a*, B *a p* are equal; whence appears what was proposed. By a like Method we discover the Motions of Bodies striking against each other obliquely.

1164. The Body Q is at rest; the Body P strikes against it, in the
Pl.XXXVIII Direction and with the Celerity P A. Through the Centers of
Fig. 6. 7. both Bodies, when P comes to A, draw the Line D B, and P B perpendicular to it, and compleat the Parallelogram A B P C; the Motion along P A is resolved into two others, along P B and PC, or

or B A, C A *. The Body P does not act upon the Body Q by * 1155.
its Motion along C A; therefore the Action arises from the Motion along B A only, that is, *the Body* P, *acts upon the Body* Q, 1165.
by an oblique Stroke along P A, *with the Celerity* P A, *in the same manner as if it should run against it directly along* B A, *with the Celerity* B A. Wherefore the Motion of the Body Q from that Action, whether the Bodies are elastick or not, is determined from what has been said of direct striking.

The Motion of the Body P after the Stroke is deduced from the 1166.
same Principles. The Motion along C A is not changed; therefore the Body P is carried in the Direction A E, with that Motion, with equal Celerity; therefore let A E be equal to C A.

The Change of the Motion B A is determined in respect of the Body P, after the same manner as the Motion of the Body Q is, from what has been explained concerning direct Collision. Let the Celerity after the Stroke be A D, in Fig. 6. when the Body goes forwards, and in Fig. 7. when it goes backwards. From this Motion, and the Motion along A E, arises a compound Motion along the Diagonal A p, which by its Situation, and Length, represents the Direction, and Celerity of the Body P after the Stroke *. * 360.

When the Bodies are equal, and elastick, the whole Motion along 1167.
B A is destroy'd by the Percussion *, and the Motion along C A * 1125.
only remains; in which Direction *the* Body P is then also carried. In this Case always, both *Bodies*, after the Stroke, *are separated in Directions, containing a right Angle, in what Manner soever the Body* P *comes to the other.*

A Machine,

Whereby some Experiments concerning oblique, and compound Collision, are made. 1168. Pl. XXXIX.

Two wooden Planes CDE, CDE, whose Sides CD, CD, are Fig. 1.
about three Feet and a half long, and the Sides D E, D E, a Foot and a half long, being put into a vertical Situation, and fastened together by the Hinges A and B, are disposed in such Manner, that they may make any Angle whatsoever.

The Experiments are made with this Machine, with ivory Balls of an Inch and an half Diameter.

These Planes are so joined together, that if other Planes be supposed parallel to these, at a Distance a little greater than the Semidiameter of the Balls, their Intersection may be the Axis itself

about

about which they turn: which is performed if Hinges as G be made ufe of, (Fig. 2.) whofe Parts *b*, *b*, are faftened in the Wood, for the greater Firmnefs.

In the Center of the upper Hinge A, there is joined to it a fmall Cylinder *a*, (Fig. 2.) in whofe Bafe there is a Hole, which is joined to another fide one fo, that the Thread *i h* may pafs through both, whereby the Ball P is fufpended, and which is faftened by a Pin.

By help of the Screws F, F, F, F, F, the Machine is put into a vertical Situation fo, that the Thread *h i* may coincide with the Axis of the Machine.

At *m*, *m*, two Pins are inferted in the Planes mentioned; from thefe the Balls Q, Q, are fufpended, at fuch a diftance from the Planes, that each may almoft touch that, to which it is applied; that is, that a Line, which is conceived to pafs thro' the Centers of the Balls P and Q, may be parallel to this Plane. It is moreover required, that thefe Balls, being placed at the fame Height, may touch one another.

The Threads, whereby the Balls Q and Q are fufpended, go thro' Holes in the Pins before-mentioned, and are faftened to the Pins *l l*, that they may be raifed, and let down conveniently, and that the Centers of all the Balls may be fituated in the fame Plane, parallel to the Horizon. A Brafs Ruler R, bent fo, that the Ball P in its Motion may afcend along it, is turned about one of the Ends, and the Center of Motion coincides with the Axis of the Machine. It is of ufe to mark out the Way of the Ball P, and the Height to which it afcends.

Each of the Balls Q is let down along the Plane, to which it is applied; and the Height, from which it is let down, is obferved by means of the Index faftened to the Plane; for which purpofe there are four Holes in each Plane, which point out equal Angles, in refpect of the Motion of the Threads.

When the Ball Q is let down from a certain Height, it ftrikes againft the Ball P; and drives it forward in the fame Direction.

Experiment 1.

1169. Pl. XXXIX. Fig. 3. There is here reprefented an horizontal Section of this Machine, on which are fufpended the Ball P, and one of the Balls Q; the Planes being fet to a right Angle, with whatfoever Direction, and from whatfoever Height falling, the Body P ftrikes againft the Body

Body Q, after the Stroke the Bodies move in the Directions of the Planes.

From the ſame Principles, from which we deduce the Motions of the Bodies, when one is at reſt we alſo determine the Motions of two Bodies after the Percuſſion, when both are moved, howſoever they be carried againſt one another. The chief Caſes are repreſented in Plate XXXVIII, and all are reſolved after the ſame manner. 1170.

Let the Body P be moved in the Direction and with the Celerity 1171.
P A; the Body Q in the Direction and with the Celerity Q *a*; Pl. XXXVIII
draw the Line B *b*, paſſing through the Centers of both Bodies, Fig. 8. 9. 10.
when they touch one another; let C A and *c a* be perpendicular to 11. 12. 13.
this Line, and complete the Parallelograms P B A C and Q *b a c*. The Motion of the Body P is reſolved into two others, the Celerities, and Directions of which, are repreſented by C A, B A. The Motions, into which the Motion of the Body Q is reſolved, are repreſented by *c a*, *b a*. The Bodies don't act upon each other, by the Motions along C A and *c a*; therefore theſe Motions are not changed, and after their Meeting they are repreſented by A E and *a e*, which are equal to C A and *c a*. The Percuſſion from the Motions along the Lines B A, *b a*, is direct, and is determined in the 4th and 6th Chapters. Let the Motion of the Body P be towards D, and its Celerity A D; the Motion of the Body Q towards *d*, and its Celerity *a d*; therefore after their meeting the Motion of the Body P is compounded of the Motions along A E and A D, and it is moved along the Diagonal A *p*. The Motion of the Body Q after the Stroke is compounded of the Motions along *a e* and *a d*, and this Body is carried along the Diagonal *a q*; and the Lengths of thoſe Diagonals repreſent the Celerities of the Bodies after their meeting *. In Fig. 8, 9, and 10, the Bodies are ſup- * 360.
poſed to be non-elaſtick. Fig. 11, 12, and 13, repreſent the ſame Caſes, when elaſtick Bodies are given. In Fig. 8. ſome Letters are wanting; becauſe the Points, denoted by them, coincide with others.

By this Method we reduce oblique Strokes to direct ones *in the* 1172.
Line, in which if the Bodies were moved, they would really *concur directly*; in this Line *the Changes of the Velocities are inverſely as the Maſſes* *, *the lateral Velocities not being changed.* We have now * 987.
demonſtrated this in a peculiar Caſe, in which the Directions of the lateral Motions make a right Angle with the ſaid Line.

And

And this Propoſition is univerſal, howſoever the Reſolution of the Motions be made, if both be but reduced to the ſaid Line.

1173. Plate XL. Fig. 1. Let the Directions of two Bodies meeting at C be A C, B C; F *f* the Line in which the Percuſſion is direct; A L, B M, perpendicular to it; we have now a rectangular Reſolution of the Motions; of the firſt into A L and L C, of the ſecond into B M, and M C, as in N. 1171. With the Motions L C, M C, the Bodies concur directly, and L C, C *l* being put equal, as alſo M C, C *m*, if *m l* be divided at O, ſo, that *m* O may be to O *l*, as the Maſs A is to the Maſs B, C O will be the Velocity common to both Bodies after the Stroke. A Perpendicular to F *f* being erected at O, let O *b* be equal to B M, and O *a* equal to L A; we have C *a*, which repreſents the Direction, and Velocity of the Body A, after the Stroke; and C *b* repreſenting the ſame with
* 1171. reſpect to the Body B *.

1174. *When the Bodies are not elaſtick*, of which we ſpeak here, they are not ſeparated *after the Percuſſion*, their Motions being referred to the Line in which the Percuſſion is direct; that is, *they are continually, whilſt they remain in Motion, in the ſame Perpendicular, to the* very *Line, in which the Percuſſion is direct.*

1175. Let us now return to the firſt Motions along A C, B C, the Velocities of which theſe Lines expreſs; we reſolve theſe at pleaſure, into the Motions along A F, F C, and B G, G C, ſo that, for each Body, one of the Motions may be given in the Line F *f*, in which the Percuſſion is direct. We muſt now demonſtrate this; that the lateral Velocities A F, B G, are kept, the direct ones F C, G C being changed in an inverſe Ratio of the Maſſes of the Bodies A and B. Let the Motion after the Percuſſion be reſolved; the Motion along C *a*, A I being drawn parallel to A F, is reſolved into the Motions along C I and I *a*; and *b* H being drawn parallel to B G, the Motion along C *b* is reſolved into two along C H and H *b*. The Triangles *a* I O and A L F are equiangular, becauſe their Sides are reſpectively parallel; but we made the
† 34. El. I. 26. El. I. Sides *a* O, A L equal †; therefore I *a*, A F are alſo equal. After the ſame manner we demonſtrate that the Velocities are equal, which are repreſented by the Lines B G, H *b*; and it is manifeſt that each Body keeps its lateral Velocity.

Let us ſuppoſe C *f* equal to F C; and C *g* equal to C G: the Velocities in the Line F *f*, before the Percuſſion were C *g*, C *f*; after the Percuſſion they are C H, C I; therefore the Changes are *g* H, I *f*, which we affirm to be inverſely as the Maſſes, that

2 is,

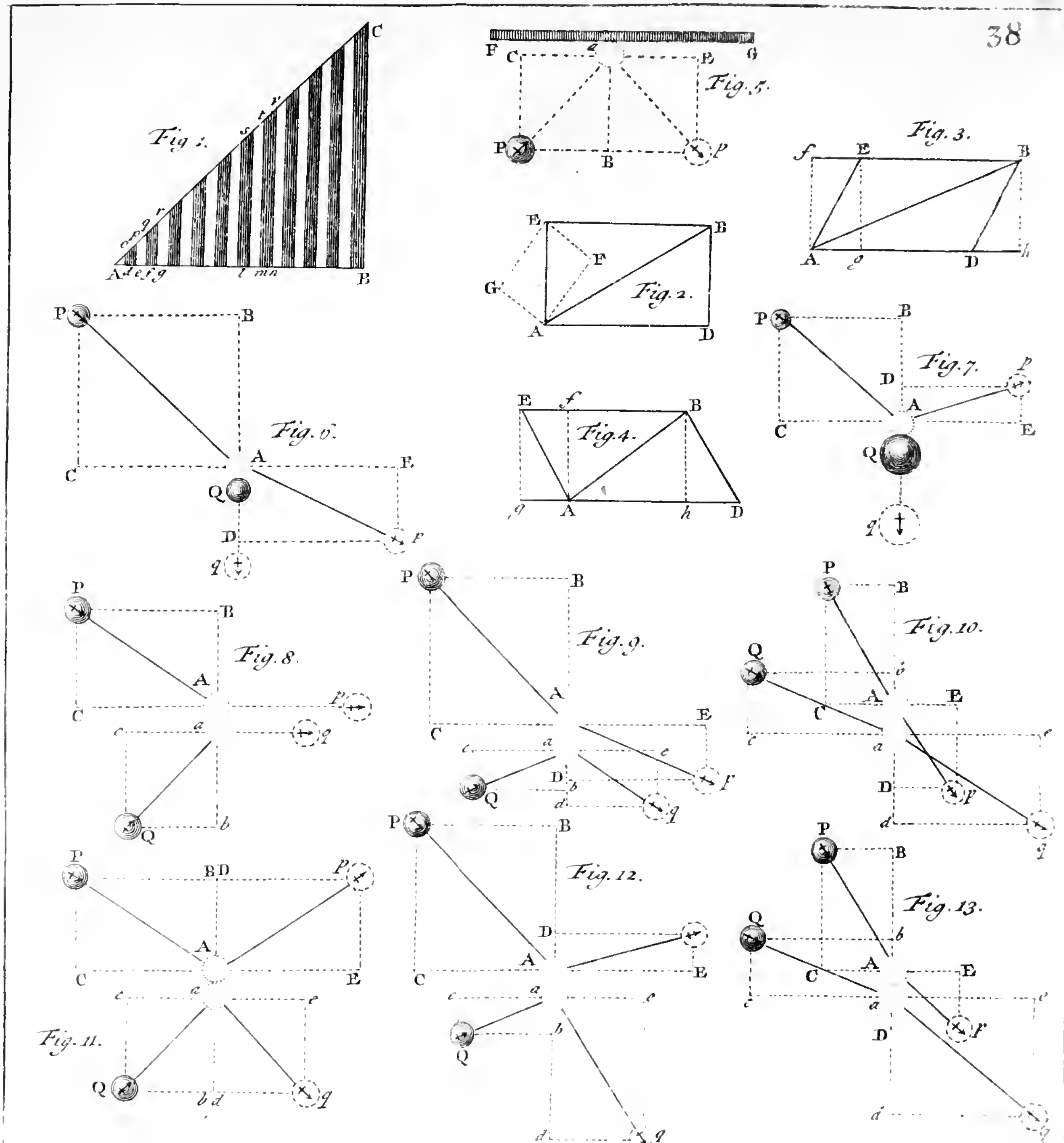


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.

is, directly as A is to B. By reason of the Equality of F C, C *f*, as also of L C, C *l*, F L, *l f* are equal. In the equiangular and equal Triangles A F L, *a* O I, the Sides F L, O I are equal; therefore *l f*, O I are equal; *l* I, which is common, being taken away, (or added if the Figure requires it) there remain O *l*, I *f*, which are equal. After the same manner we demonstrate *m* O, *g* H to be equal. But *m* O is to O *l*, as A is to B; therefore *g* H is also to I *f* in the same Ratio of A to B. Which was to be demonstrated.

BOOK II.

PART III. Of Compound Collision.

CHAP. IX.

Of Double Collision.

1176. *WE call that Compound Collision when many Strokes take place, at the same Time, in the same Body.*

This happens, when more Bodies than two meet; or when one Body runs against many Planes, at the same Time.

In this Chapter I shall examine some Things relating to the striking of a Body against two Planes, and some Things concerning the Collision of three Bodies, in such Cases in which there are given but two Strokes; I shall afterwards speak of three Collisions.

1177. Plate XLI. Fig. 1. 2. The Body P runs against the Angle GCF, with the Velocity AP, in the Direction AP; we must determine with what Action it runs against each of the Planes GC and FC.

We must observe, that the Body loses its whole Force, for we suppose the Obstacle fixed.

Draw AB and AD, which make Right Angles with CG and CF; let PE and PH be respectively parallel to these.

Now if we suppose the Body P, to be carried along AE and AH in the same Time, with Velocities proportional to these Lines, it will really be moved along AP, with the Velocity AP*; therefore we may consider the Body, whilst it is moving to P, to be carried with the Velocities HP and EP, and to run against the Planes CG, CF, directly, and according to these Directions; so that the Question may be reduced to this, With what Force can a Body act along AE and AH, at the same Time?

If the Angle FCG were a Right one, the Angle EAH would be a Right one also; therefore these Motions would neither act contrary to each other, neither would they tend towards the same Part; and their Actions would be proportional to the Squares of the Velocities AE, and AH*.

* 360.

* 1155.

2 But

But when the Angle, which the Planes make, is acute, or obtuse; you must draw E L, H I perpendicular to A P, as in these Figures. In the Motion along A E the Motion along A P is contained, with the Velocity A L; in the Motion along A H is contained the Motion along A P, with the Velocity A I; and there is nothing else of the Motion along A P, contained in these Motions, by reason of the Right Angles A L E, and A I H *; * 1151.
so that it matters not, with respect to the Motion of the Body, whether it be moved along A H and A E at the same Time, with Velocities proportional to these Lines; or in the Line A P, with the Velocities A I and A L. In both Cases the Body is really moved along A P, with the Velocity A P; which is therefore equal to both the Velocities A L, A I, which also appears elsewhere. For A L and I P are equal by reason of the similar and equal Triangles A E L, H I P having their Sides respectively parallel, of which A E and H P are equal *. * 34. El. 1.

Now since the Motion along A L is contained in the Motion along A E, whereby it acts against the Plane G C; and the Motion along A I is contained in the Motion along A H, whereby the Body acts against the other Plane; it follows that the Actions against the Planes are the Forces, whereby the Body is carried, at the same Time, with the Velocities A L and A I: but these Forces are as the Velocities themselves *, and the whole Force of * 1148.
the Body, which is proportional to the Square of the Velocity A P †, † 753.
should be divided into two Parts, which are to one another as A L and A I; these Parts are the Rectangles A P by A L, and A P by A I.

1178. If, the Angle G C F being supposed obtuse, the Direction of the Motion A P makes an obtuse Angle also with one of the Legs, Plate XLI Fig. 3.
as C F, the Body exerts an Action against this last Plane only, proportional to the Square of the Line A D, perpendicular to F C, and does not lose its whole Force by the Stroke; but continues its Motion after the Stroke along C F, with a Velocity proportional to the Line D C.

These Things follow from the Resolution of Motion *; for we * 1155.
demonstrate elsewhere that there is no Action given against the Plane G C.

In the Case of *Fig.* 2. where the Angle, which A B makes with C F, is an acute one, the Action against the Plane G C is diminished, this Angle being encreased; if it be a Right Angle, as in *Fig.* 3. the Direction of the Body being *a* P, the Diagonal A P of

the Parallelogram A E P H coincides with the Side A H, and the Sides A E and A L vanish, and together with these the Action a-
* 1177. gainst the Plane G C is also taken away *; which therefore, the Inclination of the Way of the Body with respect to this Plane being encreased, will be lost.

In determining what relates to the direct Collision of three Bodies, moved in the same Line, we make use of a Method like that, which we used in Chap. 4. where we spoke of the Collision of two Bodies. *Where three Bodies are given, there are two respec-*
1179. *tive Velocities given*, upon *which* depend the Actions of the Bodies
* 948. upon one another *, and the Introcessions of the Parts, which, the Bodies *remaining*, and these two Velocities, *is always the same*;
* 934. 956. and therefore *the Force destroyed by the Stroke* also *.

1180. *When the Bodies are at rest after the Stroke, the Sum of the Forces is the smallest of all, the respective Velocities being given*; for if a less Sum should be given, a less Force would be destroyed
* 960. by the Stroke, which cannot be *.

1181. But we demonstrate in *Scholium* 1. of this Chapter, That *the Force, the respective Velocities being given, is the smallest of all, if two Bodies being moved towards the same Part, another be carried towards the contrary Part in such Manner, that the Product of the Mass of this last Body by its Velocity may be equal to the Sum of the Products of the Masses of the other two, of each multiplied into its Velocity.*

But that in this Case the Bodies are at rest after the Stroke, and that therefore the Sum of the Forces is the smallest of all, we also deduce from what is demonstrated concerning the Collision of two Bodies, to which we refer the Collision of three Bodies.

1182. Let there be three Bodies A, B, C; let the Velocity of the first
Plate XLI. Fig. 4. be fh; of the second gi; of the third li. We suppose the Products of A by fh, and B by g, i, taken together, to be equal to the Product of C by li.

Plate XLI. Fig. 5. Let us suppose the Body C to be resolved into two Parts D and E in such manner, that D being multiplied by li, may be equal to A multiplied by fh, and E being multiplied by li may be equal to B multiplied by gi; that is, let D be to E, as A multiplied by fh is to B multiplied by gi. In this Case A is to D, as li is to
* 16. El. 6. fh *; and these Bodies meeting are at rest after the Stroke †: B
† 962. ‖ 962. and E also are at rest ‖; because B is to E, as li is to gi. But these four Bodies do not differ from the three given, which are moved with the Velocities mentioned.

The

The Force lost in any Stroke whatsoever, the respective Velocities being given, is equal to the Sum of the Forces, in the Case in which the Bodies are at rest *; but this Sum can be expressed on- * 1179.
ly by the respective Velocities given; and as is demonstrated in
Scholium 1. *In every direct Concourse of three Bodies, the Force* 1183.
lost follows the Proportion of the Sum of the three Products, which are made, by multiplying the two Masses into one another, and by the Square of their respective Velocity, this Sum being divided by the Sum of the three Masses.

The Bodies A, B, and C being given; 1. The Mass of the Body A must be multiplied by the Mass of the Body B, and this Product by the Square of the respective Velocity of A and B. 2. The Product of the Mass of A by the Mass of C must be multiplied by the Square of the respective Velocity of these Bodies. 3. Lastly, the Masses of B and C being multiplied into each other, the Product must be multiplied by the Square of the respective Velocity of these Bodies; but the Sum of these three Products must be divided by the Sum of the Masses, and we shall have the Force lost by the Stroke.

If the three Bodies are not elastick, for of such we speak, *after* 1184.
the Stroke they are carried with the same Velocity *, and this is * 939.
the Velocity, which a Ship would have, in which the Bodies should be moved according to the Law mentioned in N. 1181; because after the Stroke the Bodies would be at rest in the Ship, being carried with the same Velocity with the Ship. *We discover the Velocity*, of the Ship in *Scholium* 1. and it is had, *by multiplying the Masses of all the Bodies by their Velocities, and dividing the Sum of the Products by the Sum of the Masses, if the three Bodies tend towards the same Part; if otherwise, the Products of the contrary Motions must be subtracted one from the other.*

We see that what relates to the Collision of three Bodies, agrees 1185.
in many Things with what has been demonstrated concerning two Bodies, which also may be referred to what has been demonstrated concerning the Changes of Velocities in an inverse Ratio of
the Masses *. For as we demonstrate in *Scholium* 1; *The Changes* * 987.
of the Velocities of two Bodies, arising from the mutual Action of these 1186.
Bodies in Collision, are inversely as the Masses of the Bodies, although the Motion of one of them be changed by another Action also, at the same Time.

Now if we suppose the Bodies to be perfectly elastick, these are 1187.
moved, in the Ship mentioned, by the Action of the Spring only,

and they mutually recede from one another with the fame Celerities, and with the Forces, with which they came towards one another; for in this Cafe each of the Springs, which, whilft they are relaxed, generate Forces equal to thofe, with which they were
* 1083. bent *, undergoes a Refiftance required to produce this Effect, namely a Refiftance equal to that, which they underwent in the bending; for a Body refifts in the fame Manner, whilft it lofes a
* 713. certain Force, and whilft it acquires it *.

1188. Whence we deduce this general Conclufion, That *the Change of Velocity, in the ftriking of any elaftick Bodies whatfoever, is, with refpect to all the Bodies, double of that, which would take place in the fame ftriking againft one another, if the Bodies were non-elaftick:* therefore the Rules of N. 1110. 1111. may be applied here alfo.

1189. Plate XLI. Fig. 4. In this Demonftration we fuppofe the elaftick Parts of the Bodies to be preffed inwards, only by the mutual Action of the two Bodies A and B; that is, that that Spring only is bent, which is between thefe Bodies, when they meet at *a*, *b*; and that no part of this Action is transferred towards bending the elaftick Parts between *b* and *c*.

That thefe Things are fo, it feems to follow from the very fudden bending of the elaftick Parts, and the returning of them,
* 1127. 1128. which we have fhewn before *.

But if we fuppofe the Parts to be preffed inwards more flowly, as the Parts of foft Bodies are, elaftick Bodies are not feparated in the fame Manner, as they came towards one another, and it is more difficult to determine their Motion.

1190. *For in the Concourfe of three foft Bodies* A, B, *and* C, *which come together at the fame Time, the Introceffions are equal* between *a* and *b*, and between *b* and *c*; *tho' their Actions are unequal:* for whilft C acts upon B, if this Action exceeds the Action, which A exerts upon B, at the oppofite Part, *c* does not only prefs inwards the Parts between *b* and *c*, but it alfo preffes *b* fo, that the Action between *b* and *a* is increafed; wherefore by the mutual Action of the Bodies *c* and *b*, not only the Parts between thefe Bodies are preffed inwards, but the Introceffion of the Parts between *a* and *c* is alfo encreafed; and this Action is difperfed in fuch manner, that *b*, which is at reft between *a* and *c*, is equally preffed on both fides: wherefore, the Introceffions on both Sides, if the Parts yield inwards with equal Eafe, are equal; but the Sum of both Cavities follows the Proportion of the Force deftroyed in
* 841. 934. making them *.

EXPE-

EXPERIMENT I.

In this Experiment we make use of the Cylindrick Box O, two Inches and a quarter long, which is hollow at each End, to the Depth of more than half an Inch. These Cavities are filled with Clay, as was done in other Experiments *. This Cylinder is suspended by four Hooks, of which *v*, *v* are as far distant from one End, as V, V are from the other; but *v* and its corresponding Hook V are an Inch and an half distant from each other. 1191. Pl. XXVIII. Fig. 11. * 939. 969.

We must now make use of the Machine, which we have often used before *, and suspend the Box mentioned in the Middle of it, the middle Hooks *g f*, and *h*, *i* being fastened in such manner, that the Distance between them may be an Inch and half †. Pl. XXVII. Fig. 1. * 760. † 764. 765.

The Hooks next to these are moved towards them so, that the upper Plates may come together; the rest are moved from these in such manner, that the Distance between the Hooks may be three Inches: which may be easily done, if the Length of the Rulers *b b* and *d d* be so determined, that there may be this Distance between the Hooks, when the last small Tubes, to which the Hooks are joined, are removed, as far as possible, from the Middle of the Ruler; as is represented in the Figure. Plate XL. Fig. 2.

We must now suspend at the Sides of the Box the two Rectangles *, often made use of in the Experiments; and join to them the similar Cones *h*, *h* †, one of which we made use of in the 2. 7. 8. Experiments of Chap. 3. of this Book. But such is the Disposition of the Hooks in the Machine, that, if you reduce the Threads to their due Length, the Bodies, when at rest, are in the same Horizontal Line, and the Vertices of the Cones touch the Clay, in the Centers of the Bases of the Cylinder, which contains it. We measure the Velocities, as we did in other like Experiments. Plate XL. F. 3. * 769. † 738. 771.

Now let the Mass of R be two, the Mass of S nine; both Bodies are let go at the same Time in such manner, that they may together run against the Body O at rest, S having two degrees of Velocity, and R nine; after the Stroke they will be at rest also, as in Experiment 4. of Chap. 4. of this Book *: but the Cavities will be equal to one another, and together equal to the Cavity, which we had in the Experiment mentioned, if we make use of the same Clay; as is discovered by measuring the Cavities, and is easily deduced from the following Experiment. 1192. * 969.

The

* 969. † 979. * 841. The Force deſtroyed, in the Experiment mentioned *, is 198 †; the ſame is deſtroyed here alſo; therefore each of the Cavities is equal to 99 *: which we demonſtrate by the following Experiment alſo.

Experiment 2.

1193. Plate XL. Fig. 4. The Surfaces of the Clay in the Body O being again made plane, let each Maſs R, and S, be four; theſe being let go at the ſame Time, and with equal Velocities, which want little of 5, let them run againſt the Body O, and they will be at reſt; and the Cavities, each of which is made by the Deſtruction of a Force, which wants but little of $4 \times 5 \times 5 = 100$ *, and is therefore equal to 99, are equal to one another, and to thoſe, which we had in the foregoing Experiment. * 757.

1194. Plate XLI. Fig. 4. Now *if we ſuppoſe the elaſtick Parts to be bent, in ſuch manner as the Parts of the ſoft Bodies mentioned yield inwards*, the Inflection between *a* and *b* will be equal to that, which is given between *b* and *c*; and *the Bodies are to be conſidered as if ſeparated by the Action of two Springs, equally powerful, and placed between them.* We ſhall ſee in *Scholium* 2. how the Separation is determined in this Caſe.

1195. In the direct Colliſion of three Bodies, all their Motions being in the ſame Direction, there may be given a Caſe different from that, which we have hitherto examined; for if one Body be perforated, two Bodies together, one of which penetrates the other, may act upon a third, at the ſame Time.

To determine what ſhould happen in this Caſe, I will put four Bodies inſtead of three; but ſo determined and moved, that the Motion may entirely agree with that propoſed; I will alſo chuſe a ſimple Caſe.

1196. Plate XLII. Fig. 1. Let C be the Body upon which the others act; this we ſuppoſe to be at reſt; the Body A runs directly againſt it, with any Velocity whatſoever; the two Bodies B, B, which are equal, and have equal Velocities, different from the firſt, ſo run againſt it, that their Actions together may be looked upon as direct. The Surface of the Body C is plane; the Bodies A, and B, B are terminated cylindrically, and their Ends are Right Cylinders, and the Bodies are moved according to the Direction of their Axes.

Theſe three Bodies come to C at *a, b, b*, at the ſame Time; and their Actions begin at the ſame Time. But in the Caſe of this Figure, the Velocity of the Bodies B, B, which is diminiſhed, whilſt

whilst the Velocity of the Body C is continually increased by all the Actions, is reduced to an Equality with this last Velocity, at the Time, when the Velocity of the Body A exceeds this Velocity, which is common to the Bodies C, and B, B. At this Moment the Action of the Bodies B, B ceases, and their Velocity is not diminished any more; but A continues its Action upon C, whereby the Velocity of this last is still encreased; wherefore C is separated from B and B; and all Action ceases, when the Velocities of the Bodies C, and A, have come to an Equality, which continue their Motion with this same Velocity.

The Computation of the Velocities, in like Cases, is somewhat more intricate; the Geometrical Construction, whereby we determine the same, is more plane; I will give it here, the Demonstration will be found in the 3d *Scholium* following.

The Lines D X, D E being drawn, which make a Right Angle; 1197. Plate XLII. Fig. 2.
the Point E being determined at pleasure, I suppose the Line D E to represent the Velocity of the Body A, and D F is determined, that it may represent the Velocity of the Bodies B, B.

At a Distance taken at pleasure, G O is drawn parallel to F D; and the Points O and M are determined, by Lines drawn thro' E and F, parallel to D X.

G H being taken at pleasure, we determine the other Points in the Line G O by the following Proportions.

G H is to H I, as the Base of the Cylinder *d*, by which the Body A is terminated, is to the Sum of the Bases of the Cylinders *e*, *e* (*Fig.* 1.)

G H is to O N, as the Mass of the Body A is to the Mass of the Body C.

H I is to M L, as the Sum of the Masses of the Bodies B, B, is to the Mass of the Body C.

Now the Lines D H, D I, F L T, E N X being drawn, the two middle ones mutually intersect one another at Q; and Q P, which is perpendicular to D X, represents the common Velocity of the Bodies B, B, and of the Body C, in the Moment when they are separated.

If P Q be continued upwards so as to cut E X in R, the Line P R will represent the Velocity of the Body A, at that same Time.

Then Q S is drawn parallel to D H, and cutting E X and S; and S V, parallel to E D, or Q P; and S V represents the common Velocities of the Bodies A and C, after all Action is ceased.

1198. Sometimes, the Line E X cuts D I between D and Q, and then E X determines the Point Q by its Interſection with D I.

Plate XLII. Fig. 3. G *i* being taken equal to H I, D *i* is drawn, to which Q S is drawn parallel; and the Point S is determined in the Line F T. In this Caſe Q P repreſents the Velocity of the Body A after the Stroke, and S V the Velocity, with which the Bodies B, B, and C, continue their Motion together.

We ſhall alſo explain, how two Bodies moved in different Directions, and directly running againſt a third Body at the ſame Time, move this laſt Body.

1199. Plate XLI. Fig. 6. 7. Let there be given the Bodies A, B, at the ſame Time, and with any Velocities whatſoever, running directly againſt the Body C at reſt, in the Directions A X, B K. Theſe Directions being produced, let K D be the Velocity of the Body A, and K E the Celerity of the Body B; raiſe D F perpendicular to K D, and E G at Right Angles to K E; divide K D in H ſo, that K H may be to H D, as the Maſs of the Body A is to the Maſs of C; after the ſame Manner K E muſt be divided in L, that K L may be to L E, as the Maſs of B is to the Maſs of C. Now drawing F H, G L, mutually interſecting one another at N, the Line K N will ſhew, by its Situation, the Direction, and, by its Length, the Velocity of the Body C after the Stroke.

1200. But the Bodies A, and B, continue their Motion in the Lines K D, K E, as their Direction can be changed by no Action. But the Velocities are determined by N I and N M being drawn from N perpendicular to K D and K E; and K I is the Velocity of the Body A, and K M of the Body B after the Stroke.

1201. There is no Action given, whereby the Bodies A and C, in a direct Stroke, can be ſeparated, as they are not elaſtick *; and tho' the Body C be moved by the Action of the Body B, the Action of the Body A is indeed by that diminiſhed, but C is not ſeparated from A, in the Direction K D; for then C would take from the Action of A; therefore after the Stroke A and C are moved with the ſame Velocity in the Direction K D. Therefore if C runs thro' A N with the Velocity, which we expreſs by this Line, A will be moved with the Velocity K I; for the Motion along K N, in the Direction K D, contains nothing beſides the Velocity K I *; and the Body A will loſe the Velocity D I.

* 931. * 1154.

We ſhould moreover attend to this, that, the Lines N O, N P, being drawn parallel to K E and K D, the Body C is driven on by

by the Stroke of the Bodies A and B, in the Directions K O and K P at the ſame Time, and indeed with Velocities proportional to theſe Lines, if it be mov'd along K N, with the Velocity K N *: * 360.
but the Change of the Velocity of the quieſcent Body C, ariſing from the Action of the Body A, will be K O; therefore K O is to I D, if N be well determined, as the Maſs of A is to the Maſs of C *; that is, as K H is to H D; which only obtains, if the Point * 1186.
N be given in F H: for P N being produced till it cuts F D in Q, we have P N to N Q, as K H is to K D; but P N is equal to K O, and N Q to I D *. After the ſame manner we demonſtrate the * 34. El. 1.
Point ſought N to be given in the Line G L; and therefore in the Interſection of this Line with the Line F H, which was to be demonſtrated.

1202. If the Bodies are elaſtick, the Changes of the Velocity are double *: therefore if K N be produced, and doubled, we ſhall * 1188.
have the Motion of the Body C along K n, with the Velocity K n; and taking I i equal to I D, and M m to the Line M E, we ſhall have K i and K m the Velocities of the Bodies A and B.
1203. *In this Caſe the Sums of the Forces before and after the Stroke are equal* *; * 1085.
which alſo we ſhall demonſtrate in *Scholium* 4. to follow from this Determination of the Velocities.

1204. When the Angle E K G is obtuſe, and the Motions of the Bodies A and B are in part contrary to one another, what was obſerved in N. 1189. ought to be applied here alſo.

1205. In theſe, we have ſuppoſed the Changes of the Velocities in both Colliſions to be made in the ſame Time; that is, that both Actions ceaſe at the ſame Time: this is the Caſe if the Surface of the Body C be plane in the Places, in which the Stroke is made, and the Bodies A and B be terminated by parabolical Figures, of which we ſpoke above *, the Parameters of the Parabolas being * 852.
put directly as A to B, and inverſely as the Sum of A and C to the Sum of B and C, as we demonſtrate in *Scholium* 5.

1206. If the Times of the Actions are unequal, we muſt firſt determine the Velocities of all the Bodies, at the Time when one of the Actions ceaſes, as we have ſeen in N. 1196, the Direction of the Body C alſo being diſcovered, at this Time. We muſt then enquire into the ſecond Change, ariſing, from that Action only which remains. But the Determination of the Moment, when one of the Actions ceaſes, ſhould be ranked among the moſt difficult Problems, if we except ſome particular Caſes.

1207. It would be very difficult to confirm by Experiments what is demonſtrated concerning double Colliſion. Experiments cannot be made on ſoft Bodies, becauſe all ſuch Bodies which we can make uſe of in theſe Experiments, if they have no Spring at all, which is neceſſary in making the Experiments, ſtick together after the Stroke; and beſides, as in elaſtick Bodies, we can never diſcover, whether the two Bodies come to the third at the ſame Time exactly, except from the way of the Body C; which way therefore can't be determined by Experiment. In that Caſe only, in which the Bodies A and B are equal, and moved with equal Velocities, it appears at firſt Sight, that theſe Bodies did ſtrike againſt C at the ſame Time, if the way of this laſt divides the Angle D K E into two equal Parts. The following Experiment relating to this Caſe may be made.

EXPERIMENT 3.

1208. An horizontal Section of the Machine, deſcribed in N. 1168. is
Pl. XXXIX. repreſented here. The three Ivory Balls, mentioned in that De-
Fig. 4. 5. ſcription, being applied to this Machine, if the Bodies Q, Q, be let go from equal Heights at the ſame Time, and the Parallelogram *a b c f* be made, whoſe Sides *a b*, *a c* are the Directions of the Bodies Q, Q, contiued, and equal to the Subtenſes of the Arcs, along which the Bodies Q, Q, deſcend; the Body P, if the Angle Q P Q be an acute one, aſcends with a Velocity, leſs than that with which it could aſcending run thro' an Arc, whoſe Subtenſe ſhould be the Diagonal of the Parallelogram mentioned.

1209. But the Angle being put obtuſe, it aſcends to a greater Height, than that which is determined by the Diagonal of the Parallelogram.

1210. Which agrees with what is explained in N. 1199, 1202.
Pl. XLI. We ſuppoſed both the Bodies, which ſtrike againſt the quieſcent
Fig. 6. 7. Body, to ſtrike againſt it directly; but if the ſtriking were oblique,
†1165.1171. it might be reduced to a direct ſtriking, as is demonſtrated of two Bodies †; and ſetting aſide the lateral Motions, the Changes would be deduced from the direct ſtriking only, as if the Bodies were moved with this only; afterwards the lateral Velocities would be
1211. to be conſidered again, as in the ſtriking of two Bodies.

If the Body C ſhould be moved alſo, the Problem would be ſolved after the ſame manner, by conſidering a Ship, in which this Body would be at reſt, and determining the Motions in the Ship; which being given, the Motions without the Ship are eaſily had.

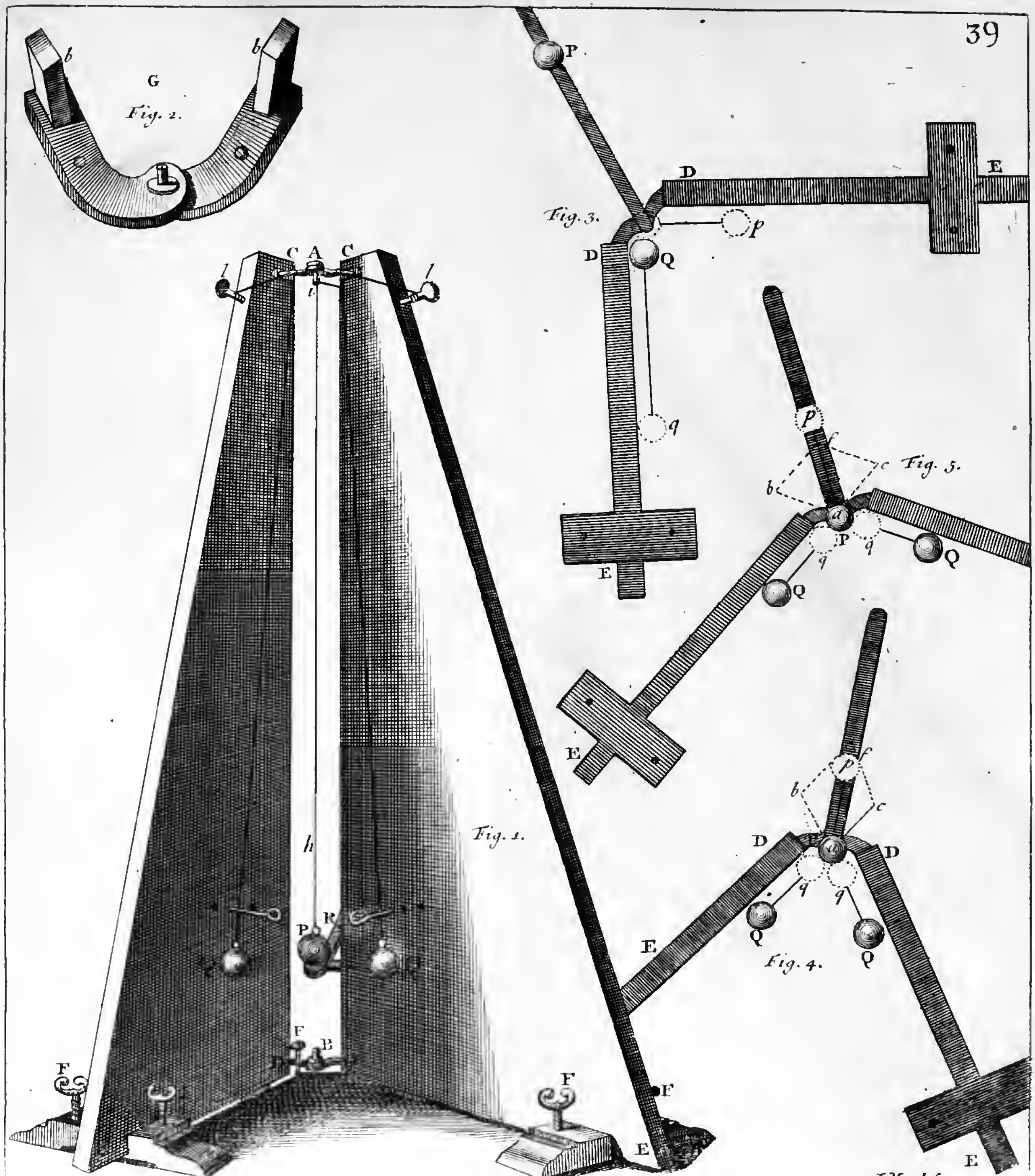


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
J. Mynde fc.

It would not be more difficult, to apply a direct Solution to these; but it would be of no great Service, to multiply the Methods.

SCHOLIUM I.

The Demonstrations of N. 1181. 1183. 1184. 1186.

LET there be given three Bodies A, B, C, running against one another directly; the Velocity of the first being put a, of the second b, of the third c: the Sum of the Forces is $Aaa + Bbb + Ccc$ *. If A and B tend towards the same Part, and C to a contrary Part, we said in N. 1181. that this Sum would be, the respective Velocities being given, the smallest of all, if $Aa + Bb = Cc$; which indeed follows from the rest of the Bodies after the Stroke, demonstrated in N. 1182; but it is also prov'd directly, if we conceive any Velocity whatsoever increased, or diminished, by any Quantity whatsoever as x, and a Computation be made of the Sum of the Forces. 1212. * 757.

For Example, let the Velocity of the Body A be $a + x$, that the respective Velocities may be kept, B is moved with the Velocity $b + x$; and the Velocity of the Body C will be $c - x$. The Sum of the Forces is $Aaa + 2Aax + Axx + Bbb + Bbx + Bxx + Ccc - 2Ccx + Cxx$; which exceeds the first by the Quantity $Axx + Bxx + Cxx$, taking away $2Aax + 2Bbx - 2Ccx$, which mutually destroy one another; but as there is an Excess given, howsoever we conceive the Velocities to be changed, keeping the respective Velocities, it follows that the Sum in the Case mentioned was the smallest.

The same Things being laid down, the respective Velocities of the Bodies A and B is $a - b$ *; of the Bodies A and C is $a + c$ †; and lastly the respective Velocity of the Bodies B and C is equal to $b + c$ ‖. The Force lost, these respective Velocities being given, is in every Case equal to the Sum of the Forces in this peculiar Case, in which this Sum is the smallest *, and in which $Aa + Bb = Cc$. We said that this Force lost was equal to $\frac{AB\times\overline{a-b}^2 + AC\times\overline{a+c}^2 + BC\times\overline{b+c}^2}{A+B+C}$ †. Which to demonstrate, we must prove this Quantity to be equal to $Aaa + Bbb + Ccc$; 1213. * 918. † 919. ‖ 919. * 1179. † 1183.

or $AB\overline{a-b}^2 + AC\times\overline{a+c}^2 + BC\times\overline{b+c}^2 = Aaa + Bbb + Ccc\times\overline{A+B+C}$.

Because $Aa + Bb = Cc$, also $AAaa + 2ABab + BBbb = CCcc = ACac + BCbc$; whence we deduce $AAaa + BBbb + CCcc = 2ACac + 2BCbc - 2ABab$. But, multiplying

Aaa

$Aaa + Bbb + Ccc$ by $A + B + C$, we have $AAaa + BAaa + CAaa + ABbb + BBbb + CBbb + ACcc + BCcc + CCcc$, and by substituting instead of $AAaa + BBbb + CCcc$, the Value discovered, we have $\overline{Aaa + Bbb + Ccc} \times \overline{A + B + C} = ABaa - 2ABab + ABbb + ACaa + 2ACac + ACcc + BCbb + BCcc = AB \times \overline{a-b}^2 + AC \times \overline{a+c}^2 + BC\,\overline{b+c}^2$; which was to be demonstrated.

1214. Let there be again three Bodies A, B, C; the Velocity of the first m; of the second n; and of the third p. To demonstrate the Rule N. 1184. we call the Velocity of the Ship there mentioned x; and the Velocities of the Bodies A and B in the Ship, if we suppose them to be moved faster than the Ship, will be $m - x$ and $n - x$; but C, if it be moved slower than the Ship, is carried in it towards the contrary Part with the Velocity $x - p$. When we consider the Case, in which the Bodies are at rest after

* 1180. 1181. 1212. the Stroke, we have $Am - Ax + Bn - Bx = Cx - Cp$ *.

Whence we deduce $x = \dfrac{Am + Bn + Cp}{A + B + C}$. Which was to be demonstrated.

1215. If all the Bodies do not tend towards the same Part, the Velocities of those, which are carried towards the contrary Part, are negative, and their Products in the Numerator are negative.

1216. Let us put the Bodies A, B, and C, as in the foregoing Demonstrations; and let us suppose these to be carried towards the same Part so, as in the Ship, which is moved with that Velocity, with which the Bodies are moved after the Stroke, let the Velocities of the Bodies A and B, from the Stern to the Prow, be fh, gi, the Velocity of the Body C from the Prow to the Stern, li. In this Case only the Body C is moved slower than the Ship, and is accelerated by the Action of both the others. As the Bodies in the Ship are at rest after the Stroke, the Sum of the Products of A by fh, and B by gi, is equal to that of C by li *.

Plate XLI. Fig. 4.

* 1180. 1181. 1212.

Plate XLI. Fig. 5. † 1182. ‖ 1182.

C being divided into two Parts D and E, as before †, which are to one another, as A multiplied by fh is to B multiplied by gi; if A acts upon D, and B upon E, the Bodies in the Ship are also at rest ‖; that is, considering the absolute Motions, without attending to the Ship, D and E being moved, before the Stroke, with equal Velocities, these are also equally accelerated by the Actions of the Bodies A and B, namely by the Quantity il; and Forces are communicated to them, which are to one another, as the Masses of D and E †; that is, as the Products of A by fh, and B by gi. Whence it follows that, in the Collision of these three Bodies, the Actions of the Bodies A and B upon C, whilst they together accelerate the Motion of this Body, are to one another, as A multiplied by fb and B by gi; in which Ratio also are the Velocities, which are communicated to the Body C by these Actions *.

† 748. Plate XLI. Fig. 4.

* 1489.

The

The whole Velocity communicated $i\,l$ being divided into two Parts $i\,m$, $m\,l$, which are to one another, as A multiplied by $f\,b$ is to B multiplied by $g\,i$, $i\,m$ will be the Velocity communicated by the Action of the Body A.

By multiplying $i\,m$ and $l\,m$ by C, whereby the Ratio is not altered, we have $i\,m$ multiplied by C, is to $m\,l$ multiplied by C, as A multiplied by $f\,b$ is to B multiplied by $g\,i$; whence we deduce $i\,m$ multiplied by C plus $m\,l$ multiplied by C, that is, C multiplied by $i\,l$, is to $i\,m$ multiplied by C, as A multiplied by $f\,b$ plus B multiplied by $g\,i$ is to A multiplied by $f\,b$: but the Antecedents are equal, and therefore the Consequents.

Therefore A is to C, as $i\,m$, the Change of the Velocity of the Body C arising from the Action of the Body A, is to $f\,b$, the Change of the Velocity of the Body A. That is, the Changes in the Velocities of these Bodies, arising from the mutual Action in the Collision, are inversely as the Masses, as we have observed in N. 1186.

SCHOLIUM II.

The Investigation of the Motion mentioned in N. 1194.

LET us suppose three Bodies A, B, C, perfectly elastick; let the Velocity of the first be m; of the second n, of the third p; let them tend towards the same Part; after the Stroke, before the Figure is re- 1217.

stored, the Velocity is $\frac{Am + Bn + Cp}{A + B + C}$ *; let this be called v. * 1184.1212.

The Force destroy'd by the Stroke is

$$\frac{AB \times \overline{m-n}^2 + AC \times \overline{m-p}^2 + BC \times \overline{n-p}^2}{A + B + C}\ *.$$ * 1183.1213.

Let this be equal to $2\,Aff + 2\,Bff + 2\,Cff$.

If all the Bodies should not tend towards the same Part, the Velocity after the Stroke, and the Force destroy'd, might be determined by the same Rules.

Setting aside the Springs, the Bodies in the Ship after the Stroke, moved with the Velocity v, would be at rest; therefore after the Stroke they are moved in it by the Springs only, and they are moved in the Ship with the same Velocities, with which the Bodies, if they were really at rest, would be moved by the same Springs; therefore the Motions in this last Case being determined, we shall have the Motions in the Ship, whence the absolute Motions are easily deduced.

1218. Therefore we put the quiescent Bodies A, B, C, and between them Springs bent, with the Forces, with which the Parts were compressed in the Stroke, which are equal to $2\,Aff + 2\,Bff + 2\,Cff$. When we consider the Case, in which the Parts between A and B, and between B and C, are equally bent in, the Force, with which each Spring is compressed,

is

is $Aff + Bff + Cff$; and the Spring, whilst it expands itself, communicates such a Force to the Bodies *.
* 1087.

The Spring, expanding itself between A and B, communicates to the Body A the Force $Bff + Cff$, and exerts an Action upon the Body B,
* 1089. equal to the Force Aff *. After the same manner, the other Spring communicates to the Body C the Force $Aff + Bff$, and exerts an Action
† 1089. upon B equal to the Force Cff †.

Therefore the Body B is pressed by two Actions towards opposite Parts; if A exceeds C, the Body B is pressed more towards this Body, by an Action which is equal to the Difference of the Actions Aff and Cff; as to the rest, the Actions upon both Parts are equal to one another, and to Cff.

Whilst the Springs press upon one another with equal Actions, each acts as if it had stood against an immoveable Obstacle; and exerts its whole Force at the opposite Part *; that is, the Springs act upon the
* 1089. Bodies A and C in such manner, as to communicate to each the Force Cff, besides the Forces mentioned, wherefore the Force communicated to the Body A is equal to $Bff + 2\,Cff$, and C is moved with the Force $Aff + Bff + Cff$, whilst B is driven towards C with an Action equal to $Aff - Cff$.

But B cannot be moved, except the Spring between B and C be driven with the same Velocity; and the Body C receives the Force just mentioned, from the Spring so moved; in the same manner as a Spring in a Ship, which should stand against an Obstacle, which could not yield, would be moved by the Action of a Spring; that is, the Velocity, with which the Body C recedes from B, or with which it is moved faster than B, is that, which the Impression just mentioned has; which Velocity is f
* 757. $\sqrt{\frac{A+B+C}{C}}$ *. If the Velocity of the Body B be called x, $x + f\sqrt{\frac{A+B+C}{C}}$ will be the Velocity of the Body C. The Sum of the Forces of the Bodies A and B, is $Aff + Bff + Cff$, and besides $Aff - Cff$, that is, this Sum is equal to $2\,Aff + Bff$; whence we deduce $Bxx + Cxx + 2fx\sqrt{AC+BC+CC} + Aff + Bff + Cff = 2\,Aff + Bff$; or $xx + \frac{2fx\sqrt{AC+BC+CC}}{B+C} = \frac{Aff - Cff}{B+C}$;

and $x = \frac{f\sqrt{AB+2\,AC} - f\sqrt{AC+BC+CC}}{B+C}$: adding the Velocity $f\sqrt{\frac{A+B+C}{C}}$, with which C recedes from B, we have the Velocity of C $\frac{fC\sqrt{AB+2\,AC} + fB\sqrt{AC+BC+CC}}{BC+CC}$.

But

But the Velocity of the Body A is discovered, from its Force before determined; and their Velocity is $\frac{f\sqrt{AB + 2AC}}{A}$.

These Velocities are to be subtracted from the Velocity v, or added to it, 1219.
as they conspire with the Motion of the Ship, or act contrary to it.

If in the first Motion A be carried faster than B, that is, if m exceeds n, the Velocity of the Body A, after the Stroke, will be $v - \frac{f\sqrt{AB + 2AC}}{A}$; the other Velocities discovered of the Bodies B and C must be added to v.

In the second Scholium of the following Chapter, we will demonstrate the Sum of the Forces after the Stroke to be equal to $Amm + Bnn + Cpp$; which agrees with what hath been demonstrated before *. * 1085.

SCHOLIUM III.

The Demonstration of N. 1197.

LAYING down those Things, which were explained in N. 1196. 1220.
1197. when we consider Bodies cylindrically terminated, it is plain, Plate XLII.
that each of the Bodies, which together act upon C, changes its Velocity, Fig. 2.
as if it acted alone; for the Change is the same, howsoever C be moved *. * 1034.
The Changes of the Velocities of the Body C, arising from the Actions of the Body A, and B, B, are to one another as GH, is to HI †: so † 1036.
that GI expresses the whole Velocity, which C acquired in a certain Time. The Change of the Velocity of the Body arising from the Action of the Body A, is to the Change of the Velocity of the Body A, at the same Time, as GH is to OM ‖; and the Change of the Velocity of the ‖ 967. 1186. 1216.
Body C, arising from the Actions of the Bodies B B, is to the Change of the Velocities of these Bodies, at the same Time, as HI is to ML. Therefore at the Time when the Velocity of the Body C is GI, the Diminutions of the Velocities of the rest of the Bodies are ON, ML; and the Velocity of the Body A is GN; the Velocities of the Bodies B, B, is GL. These Changes of the Velocities keep a constant Ratio to one another; therefore the Lines DI, EN, FL, by cutting any Line whatsoever parallel to ED, will determine the Velocities of each of the Bodies at the same Time. Whence it appears that the Velocities of the Bodies C, and B, B, are reduced to an Equality, when C has acquired the Velocity PQ; and that then all Action between these Bodies ceases. But the Velocity of the Body A, at this Time, is PR, and its Action upon C is continued, whose Velocity now is only encreased by the Action of A; for this Reason QS, parallel to DH, denotes this Encrease of Velocity, and when this Velocity is VS, A is moved with the same Velocity also; there is no further Action of the Bodies given; and C and A, being separated from B, B, continue to move with a common Motion.

SCHOLIUM IV.

The Demonstration of N. 1203.

1221. Plate XLI. Fig. 6. WE said that the Sum of the Forces after the Stroke, was equal to the Sum of the Forces, before the Stroke, in the Collision explained in N. 1203: therefore laying down the Determination of the Velocities there delivered, we must demonstrate that the Body C acquires so much Force, as A and B lose.

The Square of the Line KN is equal to the Squares of the Lines KO and ON, or KP, and twice the Rectangle under IOK *; the same Square is also equal to the Squares of KO, and KP, and twice the Rectangle under MPK †: whence it follows that these Rectangles are equal; and that the Square of KN is equal to the Square of KO, and the Rectangle under IOK, as also the Square of KP with the Rectangle under MPK; therefore the Square of KN is equal to the Rectangles under IKO and MKP; and the Square of K*n*, which is the double of that of KN, which is quadruple of the Square of KN, will be equal to four times the Sum of the Rectangles under IKO and MKP. These being multiplied by C, we have the Force of the Body C, acquired by the Stroke, equal to $4\,C \times KO \times KI + 4\,C \times KP \times KM$ ‖.

* 12. El. 2.
† 12. El. 2.
‖ 757.

The Force, which the Body A lost by the Stroke, is had by multiplying A by the Difference of the Squares of KD, K*i*, the Velocities before and after the Stroke *: but this Difference, by reason of the Equality of DI, I*i*, is equal to four times the Rectangle under KID †; and the Force lost is $4\,A \times ID \times KI$: but we have seen in N. 1201. A : C :: KO : ID; therefore $A \times ID = C \times KO$, and the Force which A loses is $4\,C \times KO \times KI$.

* 1147.
† 8. El. 2.

After the same manner we demonstrate the Force, which B loses, to be equal to $4\,C \times KP \times KM$; and that therefore the Sum of the Forces lost is equal to the Force, which C acquired. Which was to be demonstrated.

There is little Difference in the Demonstration, when we consider the Case of Fig. 7.

SCHOLIUM V.

The Demonstration of N. 1205.

1222. IN N. 1205. we took notice of a Case, in which, a double Collision being given, both are of equal Continuance.

We spoke of Bodies, which are terminated by Figures, made by the Revolution of Parabolas, in which the Time of the Action does not depend upon the Velocity *; wherefore we may consider each of the Collisions

* 862. 915. 1027.

sions separately; as the Change of Velocity arising from one Action cannot alter the Duration of the other, which does not depend upon the Velocity.

There is a Collision given between the Bodies A and C; as also between B and C. Let a be the Parameter of the Parabolas, which determined the Figure of the Body A; b the Parameter of the Figure of the Body B. If the Bodies should run against plane, and fixed Obstacles, the Times would be $\frac{A}{a}$, and $\frac{B}{b}$ *, but, by reason of the Collision, they now * 915.
are $\frac{AC}{aA + aC}$, and $\frac{BC}{bB + bC}$ †. In the Case in which these Times are † 1207.
equal, $a : b :: \frac{A}{A + C} : \frac{B}{B + C}$. Which was to be demonstrated.

CHAP. X.

Of the Motion of the Center of Gravity.

LET A, and B, be the Centers of Gravity of two Bodies; Plate XLI.
If the two Bodies come to C, *the common Center of Gravity,* Fig. 8.
with Velocities, which are to one another as their Distances from 1223.
this Center, namely as A C is to B C, that is, *inversely as the Masses of the Bodies themselves**, *the Center of Gravity, in this* * 192. 199.
Motion of the Bodies, is at rest; for whilst, in the same Time, they run thro' A a B b, which are as A C, B C, there remain a C, b C, in the same inverse Ratio of the Masses; wherefore, in this Situation also, C is the common Center of Gravity †, which was † 192
not moved in this Motion of the Bodies.

The same Demonstration may be applied to the Motion *of Bodies receding from their common Center of Gravity, with Velocities* 1224.
which are inversely as their Masses; in which case therefore *the Center of Gravity* is also *at rest*.

If the Bodies be moved in different Directions, not passing 1225.
through the Center of Gravity; they may be carried with such Velocities, that the Encrease, or Diminutions, of the Distances from the Center of Gravity may be in an inverse Ratio of the Masses, in which case also the Center of Gravity will be at rest.

If there be given *many Bodies*, as A, B, D, and these, being 1226.
moved in the same Line, come all to the common Center of Gravity Plate XLI.
C, *or recede from it*, with Velocities, which in all the Bodies Fig. 9.

 are

are as their Diſtances from *this Center*, this Center *is* alſo *at reſt*. For, as in the Situation A, B, D, the Sum of the Products of the Maſſes by their Diſtances from C, on one Side of this Point, is equal
* 199. 202. to the like Sum on the other Side *, this will alſo take place *if* all the Diſtances are changed in the ſame Ratio, as is done here;
† 199. wherefore C remains the common Center of Gravity †, which therefore is at reſt.

In this Caſe, *all the Maſſes being multiplied by their Velocities, the Sum of the Products, on one ſide the Center of Gravity is equal to the like Sum on the other ſide*; for we put the Velocities as the
1227. Diſtances from this Center. *Which Equality of the Products, as alſo the ſaid Ratio between the Velocities, is deduced* after the ſame Manner, *from the reſt of the Center of Gravity*, if we ſuppoſe this to be given.

Hence it follows, that in a Ship carried uniformly with a rectilinear Motion, with any Velocity whatſoever, two Bodies may be ſo moved uniformly, along any right Lines whatſoever, that their
1228. common Center of Gravity will be at reſt in the Ship: and in this Caſe, if the Velocity of one of the Bodies be changed, its Direction being kept, *the common Center of Gravity of the two Bodies* will not *be at reſt in the Ship:* but if this be at reſt *for a Moment*, *the Velocities and Directions of the Bodies remaining*, *the Center of Gravity will continue at reſt*; becauſe the Bodies recede from this Center, or come to it uniformly, in Lines, which keep their Situation with reſpect to the ſame Center.

1229. But then we have two Bodies without the Ship, mov'd uniformly in Right Lines, whoſe common Center of Gravity does alſo move uniformly in a right Line.

1230. Now leaving theſe, let us ſuppoſe any two other Bodies, mov'd uniformly any how, in right Lines, diſpoſed at pleaſure, it appears that their common Center of Gravity alſo moves uniformly, if it be mov'd. For let us ſuppoſe a Ship, which is mov'd together with this Center, for a Moment of Time, how ſmall ſoever, the Center will be at reſt in it, during this Time, and will continue at reſt, if the Ship continues to move uniformly, and keeps its Di-
* 1228 rection *; that is, the Center will move with the Ship.

1231. If we ſuppoſe a third Body, which is alſo moved uniformly, in any Direction whatſoever, the common Center of Gravity of the three Bodies will be moved, as if the two firſt were united in their
* 212. common Center of Gravity *, and mov'd with this Center; ſo that the Center of Gravity of the three Bodies will be moved after the

2 ſame

ſame manner, as that of two would be, that is, uniformly *: but as * 1230.
this Demonſtration may be applied to four, and more Bodies, it follows that *the Center of Gravity of any Bodies whatſoever, moved any* 1232.
how uniformly in right Lines, will either be at reſt, or move uniformly in a right Line.

We have already obſerved, that, in the Colliſion of Bodies, the 1233.
reſpective Motions are diſtinguiſhed from the abſolute Motions, in many Caſes; to this we muſt add moreover, that the abſolute Motions of the Bodies, muſt not be confounded with the abſolute Motion of all the Bodies, conſidered together.

DEFINITION.

We call the abſolute Motion of any Bodies whatſoever, conſidered 1234.
together, the Motion of the common Center of Gravity.

In all the Bodies we determine this Motion from the Motion of the Center of Gravity, and it is manifeſt that this may be applied to many conſidered together.

Concerning this Motion of the Center of Gravity we obſerve, what we demonſtrated in the firſt *Scholium* following, that *the Sum* 1235.
of the Forces, of any Bodies whatſoever, is equal to the Sum of the Force, which all the Bodies would have, moved together with that Velocity, with which the common Center of Gravity is carried, and of all the Forces, with which the Bodies are moved in reſpect of this Center. That is, if the Sum of the Maſſes be multiplied by the Square of the Velocity of the Center of Gravity, and all the Maſſes be multiplied by the Squares of the Velocities, with which they are carried in reſpect of the Center of Gravity, or with which they would be moved *in a Ship, in which the Center of Gravity ſhould* 1236.
be at reſt, the Sum of all the Products will be equal to the Sum of the Products of all the Maſſes multiplied by the Squares of their Velocities. Therefore *if, the Motions being changed, the Sum of the Forces be not changed in this Ship, neither will the Sum of the abſolute Forces be changed.*

We ſhall demonſtrate ſome other Things alſo relating to this Motion of the common Center of Gravity.

Let there be given any two Bodies in Motion, whoſe Center of 1237.
Gravity is either at reſt, or moves uniformly; it is manifeſt that, in every Moment, the Line, which paſſes through the Centers of Gravity of each Body, does alſo paſs thro' the common Center of Gravity; and that the Diſtances of the ſaid Centers, from this laſt Center, are in an inverſe Ratio of the Maſſes of the Bodies *. * 192. 202.

In

1238. In this Case also, *if the Motions of the Bodies be changed, and the Changes be in the same Direction, but opposite, and the Changes of the Velocities in this Direction be in an inverse Ratio of the Masses, the Motion of the Center of Gravity will not thereby be changed.*
Plate XL. Fig. 5. Let the Bodies be at H and I; let the common Center of Gravity be G; let these be moved, the first thro' H D, the second, in the same Time, thro' I E; the Way of the common Center of Gravity will be G F. Let us now suppose the Motions to be changed in the Points D, and E; and the Changes to be made along D *d*, E *e*, which we suppose parallel, and in an inverse Ratio of the Masses; and that they represent the Spaces, which might be run thro' by the Bodies, with these new Impressions, in the Time, in which they would have run thro' D B, E A, with the first Motions. The Motions of the Bodies now are along D *b*
* 360. and E *a* *; but the Way of the Center of Gravity is the same. For A B being drawn, this passes thro' the Center of Gravity C,
† 1232. supposing the Motion of this not to be changed †; but it will appear that this Motion was not changed, when it shall be demonstrated that this same Point C, is the common Center of Gravity of the Bodies, when they are at *b* and *a*.

Drawing C *a*, C *b*, in the Triangles A C *a* and B C *b*, we have
* 29. El. 1. the equal Angles C A *a*, C B *b* *; and the Sides proportional A C : B C : : A *a* : B *b*; because each Ratio is inverse of the Masses. By
‡ 6 El. 6. Alternation A C : A *a* : : B C : B *b*; and the Triangles are similar ‡; therefore the Angles A C *a*, and B C *b* are equal; and *a* C, C *b*,
† 15. El. 1. make one Right Line †. And A C, *a* C, are to one another, as
|| 4. El. 6. B C, *b* C ||; and therefore, as A C, C B, are to one another, so are *a* C, C *b*; which therefore are in an inverse Ratio of the Masses; and C is also the Center of Gravity of the Bodies, when they have got to *a* and *b*. Which remained to be demonstrated.

1239. If more Bodies be given, the common Center of Gravity of them all is not changed, tho' two Bodies change their Situation, if
* 212. the common Center of Gravity of both remains *.

Therefore the Motion of the Center of Gravity of many Bodies, moved uniformly, is not disturbed, the Motions of any two of them being changed, according to the Conditions above-men-
* 1238. tioned *; nor if such Changes are repeated at pleasure. But all the mutual Actions of the Bodies, are in the same Line, and op-
† 361. posite †, and the Changes of the Velocities, arising thence, are in an
|| 967. 1186. inverse Ratio of the Masses ||; therefore *the Motion of the common*
1240. *Center*

Center of Gravity can never be disturbed by the mutual Actions of the Bodies.

Therefore it appears that *the absolute Motion of many Bodies, considered together, in any Collision whatsoever, is not changed*; and that therefore *the common Center of Gravity of many Bodies is moved in the same Line, and with the same Velocity, before and after the Stroke.* Which may be also deduced, in each of the Collisions explained before, from what we have delivered concerning them. 1241. 1242.

I will now demonstrate this, that what belongs to this Matter, may be illustrated the more.

*The Center of Gravity of two, or three Bodies, running against one another directly in such manner, as to be at rest after the Stroke, if they are not elastick, is at rest before the Stroke *. In this same Concourse, if the Bodies are elastick, this Center is at rest after the Stroke also †.* 1243. 1244.

* 962. 1223. 1180. 1181. 1226.
† 1100. 1188. 1224. 1226.

If two or three Bodies, run against one another directly, and a Ship be supposed so moved, that the Bodies, not being elastick, they may be at rest in it after the Stroke, the Center of Gravity is at rest in it before the Stroke *: and in such a Conflict the same Center is at rest after the Stroke also, if the Bodies are elastick †. Whence it follows that this Ship is moved, with that Velocity, with which *the* common *Center of Gravity* of the Bodies is carried, before, and after the Stroke, *the Motion of* which Center therefore *is not changed.* 1245.

* 1243.
† 1244.

After the same Manner, in the Case explained in N. 1196. and 1197. we should suppose a Ship, carried with the same Velocity with the common Center of Gravity of all the Bodies. In this Ship all the Bodies tend towards this Center of Gravity with Velocities, which are as the Distances from this Center †; and the Product of the Body C, by its Velocity in the Ship, is equal to the Products of the other Bodies, multiplied by the Velocities also, which they have in the Ship*. In this Ship, whilst the Percussion continues, the Velocities of all the Bodies are diminished, whilst C loses G I, A loses O N, and the Velocity of the Bodies B, B, is diminished by the Quantity M L. But the Product of C by G I is equal to the Products of A by G N, and B, B, by M L; therefore the Diminutions are as the first Velocities, and the remaining Velocities are in the same Ratio *; and the Center of Gravity remains at rest, as long as the mutual Actions of the three Bodies continue †. One of these ceasing, those two Bodies only 1246. Plate XLII. Fig. 1. 2.

† 1227.
* 1227.
* 19. El. 5.
† 1226.

act

act upon one another, the Motion of whose Center of Gravity is
|| 1245. not changed ||; wherefore the rest of the Center of Gravity of the three Bodies in the Ship is not disturbed.

1247. We shall demonstrate in the 2d *Scholium* following, that the Proposition, of which we are speaking, takes place also in the Motions mentioned in N. 1194. 1199. 1202.

1248. *In the oblique Concourse of two Bodies* we considered two Motions, one by which they run against one another directly, another la-
* 1171. teral *, which is not changed in the Stroke; wherefore neither is the lateral Motion of the Center of Gravity changed; but neither can the Motion of the Center of Gravity be changed according to
* 1245. another Direction, because it is not changed by a direct Stroke *. Therefore this Motion varies in no respect, and *the common Center of Gravity of the Bodies keeps its Velocity and Direction.*

SCHOLIUM I.

The Demonstration of N. 1235.

1249. As long as the Bodies are moved in the same Line, the Proposition, mentioned in N. 1235, appears by a simple Algebraical Computation.

Let there be the Bodies A, B, C; let the Velocity of the first be m; of the second n; of the third p; the Velocity of the Center of Gravity d. Let the Bodies tend towards the same part; and let m and n be greater than d; and p less: therefore the Velocities, with which the Bodies tend to-
* 1227. wards the Center of Gravity, are $m - d$, $n - d$, $d - p$; and $A \times \overline{m - d} + B \times \overline{n - d} = C \times \overline{d - p}$ *; wherefore $2Amd - 2Add + 2Bnd - 2Bdd = 2Cdd - 2Cdp$, by multiplying the whole Equation by $2d$. But we must demonstrate $Amm + Bnn + Cpp = \overline{A + B + C} \times dd + A \times \overline{m - d}^2 + B \times \overline{n - d}^2 + C \times \overline{d - p}^2$. This last Quantity may be thus expressed $Amm - 2Amd + 2Add + Bnn - 2Bnd + 2Bdd + Cpp - 2Cpd + 2Cdd$. But $-2Amd + 2Add - 2Bnd + 2Bdd$ and $-2Cpd + 2Cdd$ mutually destroy one another, and this Quantity is only equal to $Amm + Bnn + Cpp$. Which was to be demonstrated.

1250. Again, let there be three Bodies A, B, C, whose Centers of Gravity only
Pl. XLI. we consider; let the common Center of Gravity be D; let us suppose the
Fig. 10. Bodies to be so moved along AE, BE, CE, with Velocities proportional to these Lines, as to meet in one Point. The Direction and Celerity of the Center of Gravity D is DE. The Velocities with which the Bodies tend towards the common Center of Gravity, are AD, BD, CD; for these would have been the Velocities of the Bodies in a Ship, in which the Center

Center of Gravity ſhould be at reſt. Therefore we muſt demonſtrate
$A \times AE^q + B \times BE^q + C \times CE^q = \overline{A+B+C} \times DE^q + A \times AD^q + B \times BD^q + C \times CD^q$.

Draw A F, B G, C H, L D L, perpendicular to D E. The Diſtances of the Bodies A, B, C from the Line L D L are F D, G H, H D; therefore, becauſe D is the common Center of Gravity, $A \times FD + B \times GD = C \times HD$ *; whence it appears that D is the commom Center of * 199. 217.
Gravity of thoſe Bodies, theſe being ſuppoſed at F, G, and H †. If in † 199.
this Situation we ſuppoſe the Bodies to be moved, A with the Velocity F E, B with the Velocity G E, and laſtly C with the Velocity H E; therefore $A \times FE^q + B \times GE^q + C \times HE^q = \overline{A+B+C} \times DE^q + A \times FD^q + B \times GD^q + C \times HD^q$ *; by adding on either ſide $A \times$ * 1249.
$AF + B \times BG^q + C \times CH^q$, and by ſubſtituting the Squares of the Hypoteneuſes of the right-angled Triangles A F D, B G D, C H D, A F E, B G E, C H E, inſtead of the Squares of the Sides *, we ſhall have what * 47. El. 1.
was propoſed.

It would be a like Demonſtration, if the Bodies ſhould proceed from one Point.

Laſtly, let us ſuppoſe the Bodies to be moved any how; A along A *a*, 1251.
B along B *b*, D along D *d*; and that they paſs thro' theſe Lines, in the Pl. XL.
ſame Time, as the Center of Gravity runs thro' C *c*: we will demon- Fig. 6.
ſtrate the Propoſition to take place in this Caſe alſo.

From C, the Center of Gravity of the Bodies, placed at A, B, and D, I ſuppoſe Lines drawn, C E parallel and equal to B *b*; C F parallel and equal to A *a*; laſtly C G parallel and equal to D *d*. If the Bodies ſhould be moved, A along C F, B along C E, and D along C G, they would recede from, or approach to the Lines H I and I L, drawn at pleaſure, in the ſame manner, as in the Motions along A *a*, B *b*, and D *d*: therefore, in both Caſes, the Diſtance of the common Center of Gravity from theſe Lines is changed after the ſame manner *. Therefore, ſince in the firſt * 217.
Caſe this Center is carried from C to *c*, it will be carried after the ſame manner in the ſecond Caſe alſo; and *c* is likewiſe the common Center of Gravity of the Bodies placed at F, E, G.

The Propoſition, of which we are ſpeaking, may be applied to theſe Bodies, if their Motions be given along C F, C E, C G *; therefore it * 1250.
may alſo be applied to them whilſt they are moving along A *a*, B *b*, D *d*: for in both Caſes the Motions of the Bodies are the ſame, the Motion of the Center of Gravity the ſame, and the Motions in reſpect of the Center of Gravity the ſame; by reaſon of the parallel and equal Ways of the Bodies in both Caſes.

SCHOLIUM II.

The Demonstrations of N. 1247.

1252. * 1247. WE said * in the Case N. 1194, which we explained particularly in N. 1217, that the Motion of the Center of Gravity also is not changed; which to demonstrate, we must prove that the Bodies are so separated from one another, that, considering those Motions only, by which they are separated, their Center of Gravity is at Rest; for then if we suppose the Bodies to be separated in the Ship, moved with that Velocity, with which the Bodies are moved jointly before their Separation, the common Center of Gravity will continue in Motion, with that Velocity, with which the Ship is carried.

1253. Those Things being laid down, which were explained in N. 1218. we must demonstrate that A multiplied by the Velocity there determined, the Product of which is $f\sqrt{AB+2AC}$, is equal to the Sum of the Products of the Bodies B and C, each being multiplied by the Velocities there discovered. These Products are

$$\frac{fB\sqrt{AB+2AC}-fB\sqrt{AC+BC+CC}}{B+C} \text{ and}$$

$$\frac{fC\sqrt{AB+2AC}+fB\sqrt{AC+BC+CC}}{B+C} \text{ whose Sum is}$$

$$\frac{fB\sqrt{AB+2AC}+fC\sqrt{AB+2AC}}{B+C}, \text{ that is, } f\sqrt{AB+2AC},$$

which was to be demonstrated.

1254. From these Demonstrations easily appears, what we observed at the End of N. 1219, that the Sum of the Forces, before and after the Stroke, in the Motion mentioned in N. 1217. is the same. The Forces, with which we supposed the elastick Parts to be bent, are the Forces, with which the Bodies * 1213. 1226. came to the common Center of Gravity *; they were separated from one another, the same Sum of the Forces being kept, as follows from the Computation itself: that is, that was the Sum of the Forces with which they receded from the Center of Gravity, when the Velocity of this Center * 1252. 1253. was not changed by the Stroke *; whence it follows, that the Sum of the absolute Forces was the same before and after the Stroke also †.

† 1286. 1255. In N. 1247. we said, that also the common Center of Gravity of the Bodies in the compound Collisions mentioned in N. 1199. 1202. continued its Motion after the Concourse of the Bodies, in the same Direction, and with the same Velocity.

Pl. XLI. Fig. 6. 7. If we suppose the Bodies A and B, to continue their Motion beyond K, with the same Velocity, with which they were moved before the Stroke, the Body C being at rest after the same manner, neither the Direction nor * 1245. the Velocity of the common Center of Gravity will be changed *. Therefore

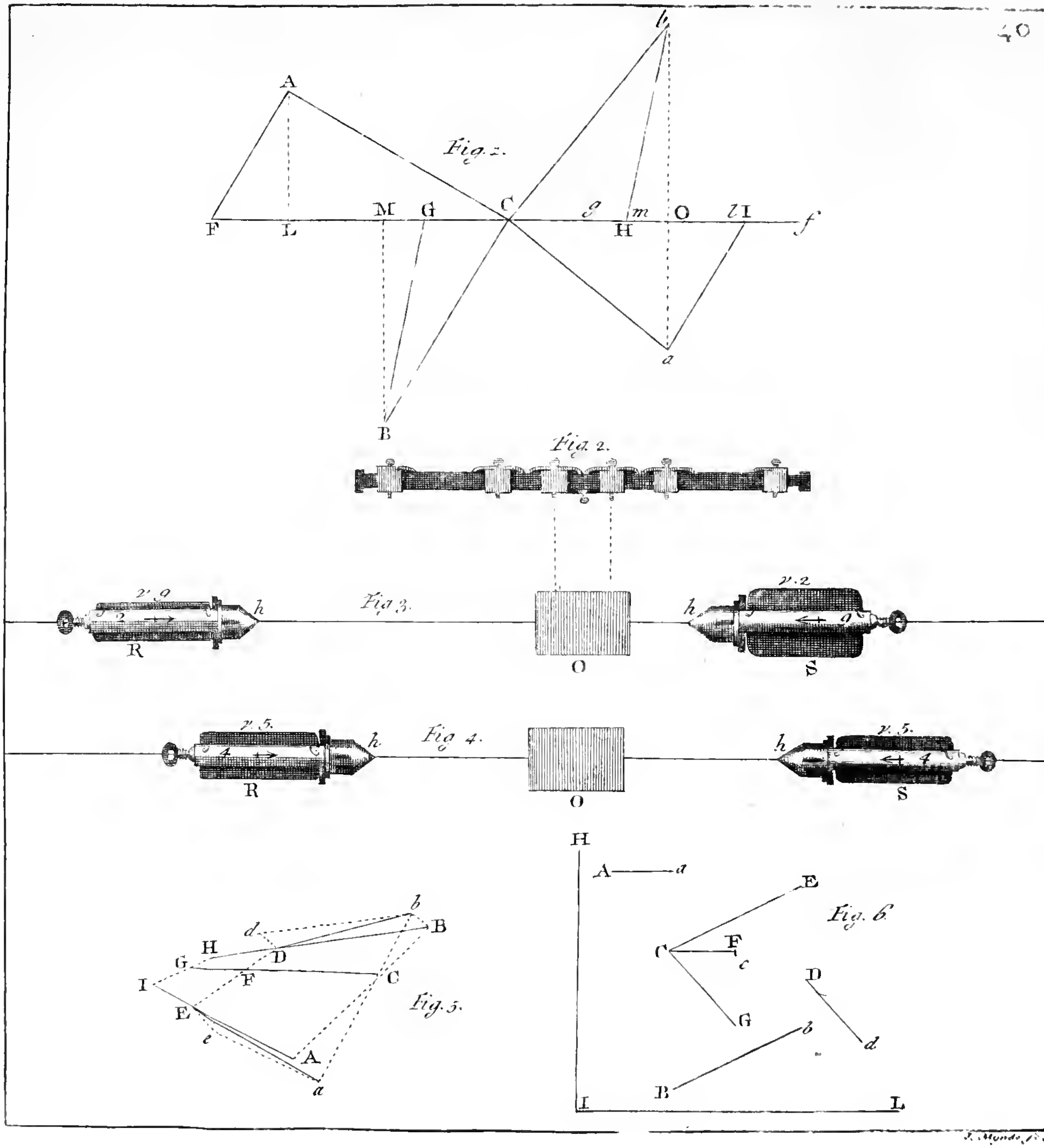

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Fig. 1.
A
b
F
L
M
G
C
g
H
m
O
h I
f
a
B
Fig. 2.
Fig. 3.
h
R
O
S
Fig. 4.
Fig. 5.
Fig. 6.
H
A
a
E
C
F
c
D
G
b
d
B
I
L
J. Mynde fc.

fore the Proposition will appear, if we demonstrate, the Center of Gravity to be in the same Point, supposing the Bodies C at K, A at D, and B at E; or supposing C at N, A at I, and B at M; or lastly, supposing C at *n*, A at *i*, and B at *m*. But it will appear, in these three Cases, that the Center of Gravity is the same, if we demonstrate the Distances of it from the Lines K F and K G not to be changed.

The Demonstration is the same in respect of both Lines, wherefore I shall speak of K F only.

The Distance of the Point N from this Line is N M; that of the Point *n* is 2 N M; the Distances of the Points D, I, and *i* from the same K F are discovered by these Proportions

$$PN : NM :: \begin{cases} KD : \dfrac{NM \times KD}{PN}. \\ KI : \dfrac{NM \times KI}{PN}. \\ Ki : \dfrac{NM \times Ki}{PN}. \end{cases}$$

Which being discovered, the Distances of the common Center of Gravity of the Bodies, from the Line mentioned K F, in the three Dispositions mentioned of the Bodies are discovered to be $\dfrac{NM \times KD \times A}{PN \times \overline{A+B+C}}$, $\dfrac{NM \times C}{A+B+C} + \dfrac{NM \times KI \times A}{PN \times \overline{A+B+C}}$, and $\dfrac{2\,NM \times C}{A+B+C} + \dfrac{NM \times Ki \times A}{PN \times \overline{A+B+C}}$ * * 217.

which we demonstrate to be equal.

From the Construction it follows $PN : NQ :: A : C$; therefore $PN \times C = NQ \times A$. But N Q is equal to I D, and is equal to $KD - KI$; therefore $PN \times C = KD \times A - KIA$; and $PN \times C + KI \times A = KD \times A$.

After the same manner 2 N Q is equal to 2 I D, that is, *i* D, and is equal to $KD - Ki$; whence we deduce $2\,PN \times C + Ki \times A = KD \times A$.

By multiplying these three equal Quantities $KD \times A$, $PN \times C + KI \times A$, and $2\,PN \times C + Ki \times A$, by N M, and the Products being divided by $PN \times \overline{A+B+C}$, we shall have equal Quotients, not different from the Distances discovered, which was to be demonstrated.

SCHOLIUM III.

The Investigation of the Motions after the Concourse mentioned in N. 1004.

IF we apply the Proposition demonstrated in this Chapter *, that the Center of Gravity is carried with the same Velocity, before, and after the mutual Action of the Bodies, to the Action mentioned in N. 1004. we 1256. * 1240.

 shall

ſhall be able to determine the Velocities of the Bodies after their Concourſe.

* 1245. Three Bodies, after the Stroke, are carried according to the Direction of the firſt Motion, with the Velocity, with which the Center of Gravity is carried before the Stroke *; for there is no Action given, by which they can be ſeparated directly; this Velocity is diſcovered by the Rule delivered in N. 992. So that they are moved as ſoft Bodies after a direct Stroke; but, the Bodies ſtruck keep the Force, which is loſt in this ſtriking of ſoft Bodies, in the Caſe which we examine; and therefore they are carried la-
* 1151. terally by this Force *: this Force is given †; wherefore the lateral Velo-
† 985. 1010. city, namely that which makes a right Angle with the firſt Direction, may be diſcovered: and therefore we eaſily determine the Directions and abſolute Velocities, with which the Bodies ſtruck are moved after the Stroke.

Pl. XXXV. Fig. 1. Let the Maſs of the quieſcent Body be called Q; let the Maſſes of the other be P, P; and their Velocity v.

* 992. After the Stroke the Body Q is moved with the Velocity $\frac{2\,P\,v}{2\,P+Q}$ *; the Bodies P, P, are carried with the ſame Velocity, in the ſame Direc-
* 985. 1010. tion; but theſe are moreover carried with Forces equal to $\frac{2\,P\,Q\,v\,v}{2\,P+Q}$ *;
† 757. wherefore the lateral Velocity of both is $\frac{v\sqrt{Q}}{\sqrt{2\,P+Q}}$ †, and the abſolute
‖ 1151. Velocity $\frac{v\sqrt{4\,PP+2\,PQ+QQ}}{2\,P+Q}$ ‖. *Example.* Let Q be $=6$; $2\,P=10$; $v=8$; then the Velocity of Q after the Stroke will be 5; that of the Bodies P 7.

CHAP. XI.

Of the triple Colliſion of three Bodies.

1257. * 1170. IT is eaſily deduced from what has been explained above concerning the oblique Colliſion of two Bodies *, that *Bodies may concur in oblique Motions, without any mutual Action.* We always light upon this Caſe, *when, both Motions being reduced,* by a Me-
† 1171. thod there explained †, *to the ſame Line, paſſing thro' the Centers of Gravity of both in the Situation of their Concourſe, the Conſequent Motion does not exceed the Antecedent.*

1258. We ſpeak now of three Strikings; if therefore there be given three Bodies meeting, ſo as to make three Concourſes, we muſt firſt

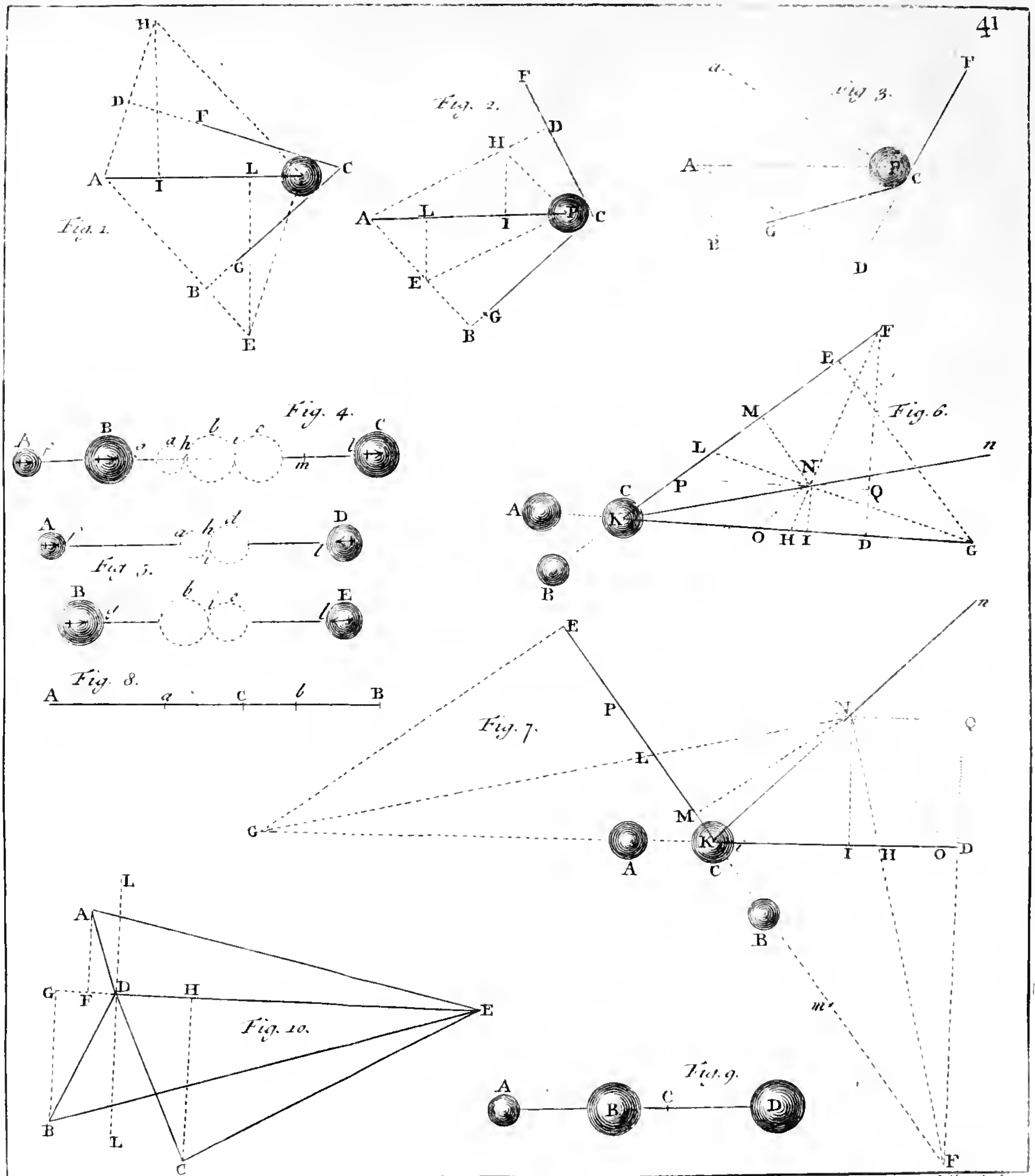


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.

first examine them separately, according to this Rule *, that it * 1257.
may appear the striking is given every where. For if three should not be given, the Collision would be to be referr'd to one or other of those before explained †. † 1210.

1211.

Let there be three Bodies A, B, D; let their common Center of Gravity be C; let them be moved along A a, B b, D d, with Velocities, proportional to these Lines, so that they may meet at a, b, d. If we examine separately the Motions of the Bodies A and B only; then the Motions of the Bodies B and C only; as also the Motions of the Bodies A and C only; we discover that there are three Collisions given, which we suppose to be of equal Continuance. 1259. Plate XLII. Fig. 4.

Now to determine the Motions after the Stroke, we seek for the common Center of Gravity *, when the Bodies touch one another; let this be c. Thro' C and c a Line is drawn, which is continued to *c* so, that C c, c *c* may be equal. 1260. * 222.

We must draw Lines, in which the Strikings are direct; and the Motion of each Body must be resolved into two in the Lines, in which this Body runs directly against the other two. The Motion along A a is resolved into two along A I and A L *, or L a and I a; the Motion along B b is resolved into two along E b, F b; lastly the Motion along D d is resolved into two along G d, H d; but the Velocities of all these Motions are proportional to these Lines. 1261. * 360.

Thro' *c* Lines are drawn perpendicular to the said Lines, in which the Strikings are given, continued if it be required. These are perpendicular *e i* to E I; *h l* to H L; and lastly *f g* to F G. 1262.

To one of these, as *e i*, there is drawn, at a Distance determined at pleasure, the parallel *m o*, which cuts the other two in *x* and *z*.

In this parallel we determine the Points *o* and *m* so, that *o x*, *x z*, *z m*, may be to one another as the Masses of B, D, A. In which Determination we observe this Law; the Line *o m* is parallel to *i e*, which is perpendicular to b a, and has a particular relation to the Bodies A and B; and *z x*, which is now determined, is put proportional to the third Body D. The Line *l h*, which is perpendicular to the Line D d a, and which therefore has a peculiar relation to the Bodies D and A, determines the Point *x*; and we seek for the Line *x o* proportional to the third Body B: that is, as D is to B, so *z x* is to *x o*. After the same Manner *z m* is found proportional to the other Body A. 1263.

The

1264. The Mafs of B determined the Point *o*, thro' this we draw *op*, parallel to *da*, which paffes thro' the Centers of the other Bodies. And thro' *m*, which was determined by Help of the Mafs A, I draw *mp* parallel to *d b*, paffing thro' the Centers of the other Bodies in the fame Manner; and thefe new Lines mutually cut one another at *p*: and we draw the Lines *c o*, *c p*, *c m*, which we produce indeterminately.

1265. Thro' the Center of Gravity c we draw c V, parallel to E a, to which *o m* is perpendicular; and in this Line we mark the Points Q and V in fuch manner, that c Q may be equal to I a; and c V may be equal to E b. Thro' this Point V we draw V T parallel to b d; and thro' Q we draw Q T parallel to a d; thefe determine the Point T by a mutual Interfection.

1266. The Part Q V, of the Line c V, we divide at R fo, that Q R may be to R V, as the Mafs of the Body B is to the Mafs of the Body A; from T thro' R is drawn a Line, which, being continued cuts the Line *c p* continued in P. Thro' this Point is drawn P O, parallel to *p o* and a d, which cuts the Line *c o* continued in O; after the fame Manner P M, which is parallel to *p m* and b d, determines the Point M, in the Line *c m* continued. The Points O and M being joined, this Line will be parallel to the Lines *o m*, *i e*; thro' O and M alfo are drawn the Lines O N, M N, refpectively parallel to the Lines *l h*, *f g*. Thefe muft be drawn in fuch manner, that the Triangle O N M may include the Point *c*; for this reafon, in this Figure O N is drawn parallel to *l h*, and not to *f g*.

1267. Now drawing c O, c M, c N, thefe Lines fhew the Motions of the three Bodies after the Percuffion, fetting afide the Magnitudes of the Bodies; but the true Motions muft be demonftrated.

1268. The Point O is common to the Lines O M, O N, which are perpendicular to b a, d a; the Point a is common to thefe laft Lines, and c O fhews the Motion of the Body A; and drawing a *A*, parallel, and equal to c O, we have the true Way of the Body A, whofe Velocity is expreffed by this Line.

M relates to the Body B in the fame Manner; and b B being drawn, parallel, and equal to c M, we have the Way, and Velocity of the Body B, after the Percuffion.

So alfo d D, which is parallel and equal to c N, denotes the Way and Velocity of the Body D after the Stroke.

1269. Thefe Things are fo, when the Bodies are not elaftick. When they are elaftick, the Ways of the Bodies before the Concourfe

 are

are to be continued, and the Continuations are to be put equal to he Ways themselves. A a is continued that a α may be equal to A a; and drawing α *A*, this is produced to *a*, that α *A* and *A a* may be equal: the Motion of the Body A, if the Elasticity be perfect, will be along a *a*, with a Velocity proportional to this Line.

After the same Manner we discover the Motions of the rest of the Bodies along b *b*, and d *d*.

SCHOLIUM.

The Demonstration of the foregoing Construction.

WE suppose the Bodies first to be soft; therefore they are not separated by their mutual Action; and when we consider the Case, in 1270.
which all the Actions are of equal Continuance, neither are they separated by other Actions. Therefore, in these three Collisions, the concurring Bodies are carried, after the Stroke, with the same Velocity in the Lines, in which they concur directly: that is, setting aside the Magnitudes of the 1271.
Bodies, they both continually remain in the same Perpendicular to the said Plate XLII.
Line, in which they concur *. Therefore the Bodies A and B, which Fig. 4. * 1174.
meet in the Line E I, are not separated, in respect of this Line, and of *c* V, although they recede laterally from them.

The last of these *Bodies, supposing the before-mentioned Resolutions of* 1272.
their Motions *, *meet with* the *Velocities*, I a, E b, *which are changed by* * 1261.
the Stroke; *but a like Resolution being made after the Stroke, the Changes are in an inverse Ratio of the Masses* *; *tho' there be given another Action* * 1172.
upon the Body also, at the same Time †. † 1186.

All these Things take place in three Collisions; but if there should not be three given, the Reasonings, which depend upon this Hypothesis, that there are three Collisions given, would be false; for this reason, we said that we must first examine, whether there be really three *. * 1258.

What we said of the Changes of the Velocities in an inverse Ratio of 1273.
the Masses *, takes place, whatsoever the Solution of the Motions be †; * 1272.
but this may be conceived to be such, that a Striking may be impossible; † 1172.
as in this Case of ours in respect of the Motions along F b, G d; but, if we only consider the true Striking, the Demonstration N. 1175. is universal, whatsoever the Resolutions of the Motions be.

The two Conditions, which we have shewn *, are sufficient for the So- * 1271.
lution of the Problem, of which we are speaking; for if the Motions after 1272.
the Concourse be so determined, that they may agree to the three Collisions, we have what we seek; for there is but one such Solution.

But as the Construction after this Method is very difficult, it is better to 1274.
add a third Condition also; namely that the Center of Gravity be moved with the same Velocity, before, and after their meeting *. * 1240

The

1275. The Motion of this Center before the Percussion is along C c, which Line also expresses its Velocity. Therefore the Motion after the Stroke is along c *c*; for this Line is equal to C c.

Let there be drawn thro' *o* a parallel to O N, and thro' *m* a parallel to M N; these meet at the Point *n*, which is given in the Line which is to be drawn from *c* to N; as easily appears from the similar Triangles, which occur in this Figure.

Let us now suppose the Bodies to be placed, A at *o*, B at *m*, D at *n*; these Bodies will have *c* for their common Center of Gravity: for first, they are in Æquilibrio about the Line *l h*; for the Distance of the Bodies A and D from this Line is to the Distance of the Body B from it, as *x o* is to *x m*; but *x o* is to *x m*, as the Mass of B is to the Sum of the Masses
* 1263. of A and D *; whence the said Equilibrium is deduced †. 2dly, The Di-
† 197. 217. stance of the Bodies B and D, placed at *m* and *n*, from the Line *f g*, is to the Distance of the Body A, placed at *o*, from the same Line, as *m z* is to *z o*; that is, as the Mass of A is to the Sum of the Masses of B and
* 1263. D *; therefore the Bodies are in Equilibrio about the Line *f g* also †.
† 197. 217. Whence it follows, that the common Center of Gravity of the three Bodies is given in the Intersection of the Lines *f g*, *l h*, that is, at *c*.

The Triangle O M N is similar to the Triangle *o m n*, and the Point *c* has the same Relation to both; therefore this also is the common Center of Gravity of the three Bodies, if they be placed A at O, B at M, D at N.

1276. Now if we suppose, setting aside their Magnitudes, the Bodies to be moved after the Stroke, A along c O, B along c M, D along c N, as was
* 1267. said in the Solution *, this will be sufficient for the third Condition †; as
† 1274. also for the first Condition ‖. Now if we demonstrate, the second Condi-
‖ 1271. tion ‖‖ to take place in one Collision, the Solution will be determined;
‖‖ 1272. that is, all these Things cannot concur in another Solution; whence it will appear that the Solution is true, and that the second Condition takes place in other Strikings also.

1277. We consider the Bodies A and B; after the Resolutions of their Mo-
* 1261. tions *, the Velocities are I a, E b, in the Line in which the Stroke was made; the first is equal to c Q, and is the Velocity of the Body A; the second, which is the Velocity of the Body B, is equal to c V.

The Motions are to be solved after the Stroke, as before it; that is, thro' O there must be drawn a Parallel to the Line A I, or a d, that is,
* 1266. O P must be continued to S *; and the Velocity in the Line c V, parallel to the Line b a, in which the Stroke is made, will be c S; and the Change of Velocity of the Body A in this Line is Q S. The Motion of the Body
† 1266. B is resolved by drawing a Parallel to the Lines B E, b d; this is M P †, which cuts c V in X; and the Change of Motion in the Line c V is X V. The Triangles P R S, Q R T are similar, by reason of the Parallels P S,
* 1265. 1266. Q T *. After the same Manner, the Triangles P X R, T V R are simi-
† 1265. 1266. lar, by reason of the Parallels P X, T V †. Whence we deduce,

R S

$$RS : QR :: RP : RT :: XR : RV.$$

By Compos. $RS + QR = QS : QR :: XR + RV = XV : RV.$

By Alternat. $QS : XV :: QR : RV :: B : A$ *; * 1266.

that is, the Changes of the Velocities QS and XV, the first of the Body A, the second of the Body B, are inversely as these Bodies: which remained to be demonstrated.

But it appears from this, that there is but this one Solution of the Problem, because this last Demonstration does not proceed, unless the Lines OS, MX, mutually cut one another in the common Intersection of the Lines *cp*, TR, that is, in P.

BOOK II.

PART IV. Of the Laws of Elasticity.

CHAP. XII.

Of Elastick Fibres.

1278. WE have already seen, what Elasticity is, and whence it
* 96. arises *; and what follows from it in the Congress of Bodies, striking against one another, whether directly, or obliquely; it remains that we examine the Laws of Elasticity, and that from Phænomena.

All Bodies, in which we observe Elasticity, consist of small Threads or Filaments, or at least may be conceived as consisting of such Threads; for a Body may be conceived divided into Threads; and that those Threads, laterally joined, make up the Body. That therefore we may examine Elasticity in the Case which is the least complex, we must consider Strings of musical Instruments, and such as are of Metal; for Cat-gut Strings form a Spiral, which surrounds a strait Thread in the Middle, with which the outward Part being twisted, coheres in a certain manner; wherefore these Strings cannot be considered in the same manner as those Fibres, of which Bodies are formed.

1279. *The Elasticity of Fibres consists in this, that they can be extended, and taking away the Force by which they are lengthened, they will return to the Length which they had at first.*

1280. *Fibres have no Elasticity, unless they are extended with a certain Force*; as appears in Strings that have their Ends fixed without being stretched: for if you remove them a little from their Position, they do not return to it. But what the Degree of Tension is, which gives Beginning to Elasticity, is not yet determined by Experiments.

1281. *When a Fibre is extended with too much Force, it loses its Elasticity*; and this Degree of Tension is also unknown. This we know, that the degree of Tension in Fibres, which constitutes Elasticity, is confined to certain Limits.

 Hence

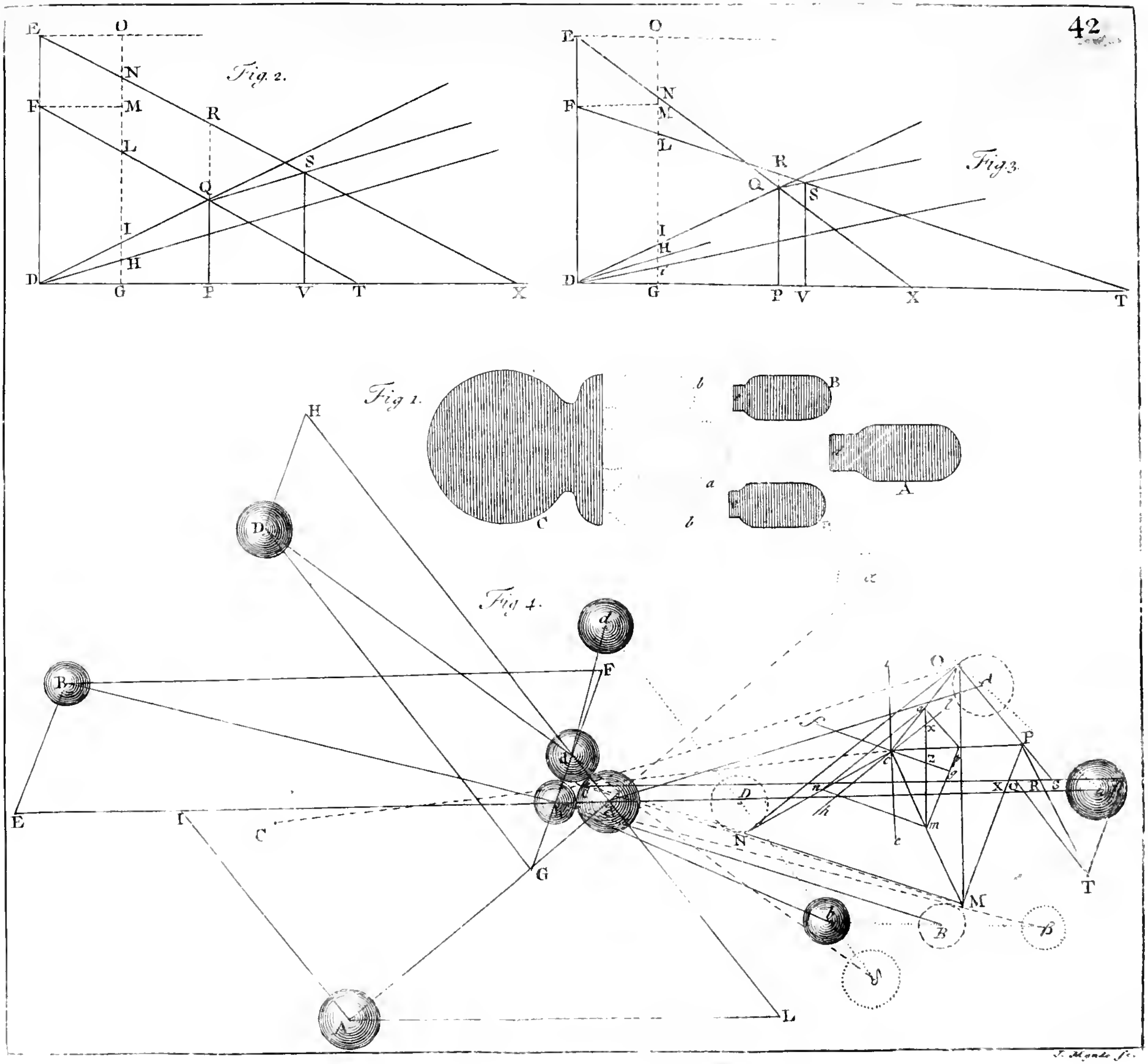


Fig. 2.
Fig. 3.
Fig. 1.
Fig. 4.
J. Mynde sc.

Hence appears the Difference of Bodies that are elaftick, and fuch as are not fo; why a Body lofes its Elafticity, and how a Body deftitute of Elafticity acquires that Property. A Plate of Metal, by repeated Blows of a Hammer, whereby the Fibres are ftretched, becomes elaftick; and being heated lofes that Virtue, whilft the Situation of the Parts is difturbed by the Action of the Fire. 1282.

Between the Limits of Tenfion, that terminate Elafticity, there is a different Force required for different Degrees of Tenfion to ftretch Chords to certain Lengths: what this Proportion is, muft be determined by Experiments, which muft be made with Chords of Metal, as was faid before. But as thefe Chords are fcarce fenfibly lengthened, the Proportions of the Lengthening cannot be directly meafured; therefore they muft be meafured by fome other Method.

Let A B be an horizontal Chord ftretched with a certain Force, whofe Ends are fixed at A and B: Let it be bent by a Weight hanging in the Middle of it, fo that it may come to the Pofition A C B. Plate XLIII. Fig. 1.

Definition I.

The Line C *c drawn from the middle Point of a String or Chord after its Inflexion, to the middle Point of the fame when it was in its natural State, is called the Sagitta (Arrow) of the Chord.* 1283.

Let *c e* be an Arch of a Circle defcribed about the Center B, with the Radius B *c*. Half the Chord by the Inflexion was ftretched the Length C *e*, which Quantity has a certain Relation to the Sagitta C *c*, which fhall be fhewn in its place. 1284.

The Weight alfo, by which the Chord is bent, has a certain relation to the Force with which the Fibre is ftretched, that is, is drawn along B C: and fo in feveral Experiments by comparing the Sagitta C *c*, and the Weights with which the Chords are inflected, the Proportions of the Lengthenings are determined, as will be fhewn in what follows. 1285.

But before we determine thefe Proportions more diftinctly, we muft explain fome general Phænomena, depending upon Elafticity; and which take place in the fame Manner, whatfoever the Laws of Elafticity be. Wherefore they are obferved in the fame manner, whether the Chords are made of Metal, or Catgut.

1286. Let the Chord A B be ſtretched: and at various Times bent, to A *c* B, A *c* B, A C B; but always ſo, that the Sagitta may be very ſmall. Let us ſuppoſe c B to repreſent the Tenſion; as we here ſpeak of the ſmalleſt Sagitta, the Lines *c* B, *c* B, C B, hardly exceed c B, and their Lengthenings are inſenſible; wherefore any Law whatſoever of Elaſticity being given, very ſmall Forces will give theſe Lengthenings: therefore the Error will not be ſenſible, if we affirm *c* B, *c* B, C B, to repreſent reſpectively the Tenſions of the Fibre, in each of theſe Inflections. But the Tenſion of the Fibre, in the Situation A C B, is the Force which draws the Point C thro' C B and C A; and the Sagitta being doubled repreſents the Force, which draws the ſame Point downward *, which is *the bending Force*. This therefore, *as long as the Sagitta is ſmall, will have that Ratio, to the ſtretching Force, before the Inflection, which is given between the Sagitta doubled, and the Chord halved.* From which Propoſition we deduce the following Concluſions.

Plate XLIII. Fig. 2.

* 332.

1287.

1288. *Let the Chord be ſtretched by a Weight at pleaſure, and bent by any leſs Weight; theſe Weights being changed any how, if they are changed in the ſame Ratio, the Sagitta is not varied*; namely if this be ſmall, for we ſpeak in all that has been ſaid, of this Caſe.

1289. Moreover, *the Tenſion remaining, the ſmall Sagitta are to one another, as the bending Forces.*

1290. Beſides *Chords, which differ any how, if they are of the ſame Length, and equally ſtretched, are bent with equal Forces.*

1291. *If many Chords have different Lengths, but are equally ſtretched, and bent with equal Weights, the Sagitta* doubled *will be* in each as the Chords halved *; and therefore the Sagittæ themſelves, *as the Lengths of the Chords.*

* 1287.

1292. *If the Chord* A B, *ſtretched any how, be* ſo *inflected* as to acquire the Figure A C B, *then left to itſelf*, by its Elaſticity it will return to its firſt Figure, and in that Caſe the Motion of the Point C is accelerated: for when the Chord is let go from the Poſition A C B, this Point is driven with the Force, that is able to retain it in that Poſition; this Motion is not deſtroyed, but there is ſuperadded to it, in all the Points of the Sagitta, a Velocity communicated by the Force, by which the Point C might be retained in them. The Celerity is the greateſt of all at c, and by that Celerity the Point C is carried further, and then returning, *it will perform ſeveral Vibrations*; in which the Point C runs out but ſhort Spaces: for which Cauſe the Force, by which the Point C is acted

Plate XLIII Fig. 2.

acted upon in all Distances from c, is as the Distance in each Point *. *And* because *the Elasticity of the Chord* is the moving * 1289.
Cause, this Cause is transferred together with the Fibre itself so, as to press it, tho' in Motion, as if it were at rest; wherefore this Force *is of the same kind with Gravity* *. Therefore this Motion * 371.
agrees with the Motion of a Body vibrating in a Cycloid †, *and* † 414.
how unequal soever the Vibrations are, they are performed in the same Time.

If there be two equal and similar Chords, but unequally stretched; 1293.
unequal Forces are required to inflect them equally; therefore the Vibrations are performed in unequal Times.

We may compare the Motions of these Chords with the Motions of the Pendulums which vibrate in Cycloids *, and describe simi- * 1292.
lar Cycloids by different Forces; which Forces are inversely as the Squares of the Times of the Vibrations †; in Chords therefore † 432.
likewise *the Squares of the Times of the Vibrations are to one another inversely*, as the Forces by which they are equally inflected; which are *as the Forces by which the Chords are stretched* *. * 1288.

When the Chords are similar, equally stretched, but of different 1294.
Lengths, their Motion is likewise compared with the Motion of Pendulums. When we speak of the different Gravities in the Motion of Pendulums, we consider the Velocities communicated to Bodies in like Circumstances; and because these Velocities are as the Forces themselves, therefore we mention the Proportion of the Forces; which also may be referred to the Demonstration of the foregoing Number.

But in the present Case we ought to consider the Velocities, generated in like Circumstances, and compare the Ratio of the Velocities with the Ratio of the different Gravities.

The Chords, A C B, *a d b*, which are inflected by equal Weights, Plate XLIII. Fig. 1. 3.
are moved as Bodies upon which Gravities should act, which are to one another as *a b*, is to A B; for in this Ratio are the infinitely small Velocities, which are communicated to these Bodies, by equal Forces * : these Chords are also moved as Pendulums, whose * 138.
Lengths are as *c* B is to D *b*, or A B to *a b* †; therefore the Squares † 1291.
of the Durations of the Vibrations, which are inversely as the Forces, and directly as the Lengths of the Pendulums *, are in a * 433.
Ratio compounded of the inverse Ratio of *a b* to A B, that is, of A B to *a b*, and the direct Ratio of A B to *a b*; which compound Ratio is the Ratio of the Squares of the Lengths. Therefore *the Lengths of the Chords are, as the Times of the Vibrations.*

By

1295. By a like way of reaſoning we may compare *the Times of the Vibrations of Chords of different Thickneſs, ſuppoſing the Chords equal, and ſtretched by equal Forces*; theſe are equally inflected by equal
* 1290. Forces *; and are therefore moved as equal Pendulums, upon which Gravities act, which *are* inverſely, *as* the Quantities of
† 138. Matter in the Chords †: that is, inverſely as the Squares of the Diameters; which Ratio muſt be inverted again, that we may have
‖ 432. the Proportion of the Squares of the Durations of the Vibrations ‖, therefore *the Diameters* themſelves are as the Durations.

1296. *Any Chords of the ſame kind being given, the Durations of the Vibrations* may be compared together; for they *are in a Ratio compounded of the inverſe Ratio of the ſquare Roots of the Forces, by which the Chords are ſtretched* *, *and the Ratio of the Lengths of*
* 1293. † 1294. *the Chords* †, *and the Ratio of the Diameters* ‖. If you multiply
‖ 1295. the Diameter by the Length, and divide the Product by the ſquare Root of the Force, that ſtretches the Chord, and go thro' the ſame Operation for the ſeveral Chords, the Quotients of the Diviſions will be to one another as the Times of the Vibrations.

I will now treat of the Laws of Elaſticity.

A Machine,

Whereby the Laws of Elaſticity are explored by Experiments.

1297. Plate XLIII. Fig. 4. The principal Part of this Machine is the Board A B, about three Foot long; and one Foot broad, and about an Inch thick. It is ſupported by Feet.

To this Board is joined a thicker Piece of Wood E F, into which is put an Iron Bar, three quarters of an Inch broad, and thick; its Ends are bent ſo, as to be at right Angles to the Bar itſelf, and make ſmall Arms two Inches long. The Length of the Bar, excluſive of the Parts bent, is three Foot. This is hid in the Wood E F; the ſmall Arms D, D, only appear, and they don't appear entirely. The Piece of Wood EF is joined to the Board by Screws, whoſe Heads appear at *c c*.

1298. Upon each of the Arms is put a ſmall Plate, which is repreſented by itſelf at G G; this has a Tail at K; this Tail is put into the Arm, and faſtened by the Screw M, which is turned by the Key S, ſo as to be immoveable.

The Handle H is faſtened to this Plate, thro' which the Screw I paſſes, by help of which a ſecond ſmall Plate L, which is leſs than the other, is joined to G G. The

The ſmall Plate L, has a ſmall Hollow in the midſt of its upper Surface, that the Screw I, which goes into this Cavity, may always preſs upon the ſame Place. Both the Plates are of Steel, and have their contiguous Surfaces rough.

The Fibre, or thin Plate, whoſe Elaſticity is try'd, is ſtretched, 1299. and its Ends, on either Side, are faſtened between the thin Plates mentioned, by preſſing them together by the Screws *i*, *i*.

The Fibre ſo ſtretched is parallel to the Surface of E F, and paſſes through the copper Plate *n*, which is repreſented by itſelf at N; and is ſuſpended upon the Middle of the Fibre.

A greater Copper Plate P is joined to the Machine ſo, as to be 1300. parallel to the Plane of E F, and to have its anterior Surface at a ſomewhat leſs Diſtance from this Plane than the Fibre itſelf.

This Plate has an Index Q R, which turns about freely; its Mo- 1301. tion is meaſured by Diviſions of a Portion of a Circle, of one Foot diameter. In its Circumference ſixteen Inches are divided into an hundred Parts; which Parts may be ſubdivided.

The Axis of the Index is retained at the back Part of the Plate P, 1302. that the Motion of the Index may not be hindered; for this Index muſt have a Tail, that it may be in Æquilibrio in every Situation.

Between the Surface of the Plate P and the Index, there is put upon the ſame Axis a ſmall Pulley, which is moved together with the Index: but if it be retained, the Situation of this laſt may be altered; as the Situation of the Index of a Clock may be changed, without moving the Wheels.

The Pulley has two narrow Grooves going round it; the neareſt to the Surface, receives a Chain, like that of a Watch; the Diameter, meaſuring from the Bottom of the Groove, and adding the Breadth of the Chain, is three Quarters of an Inch. The other Groove receives a ſilken Thread, and the Diameter of the Pulley, meaſuring from the Bottom of it, and adding the Diameter of the Thread, is alſo three quarters of an Inch.

One End of the Chain is faſtened in the Groove, and goes about 1303. Part of the Pulley; the remaining Part of the Chain being ſtretched vertically downwards, anſwers to the Plate N, which is hung on the Middle of the Fibre. There is a Prominence *o* in the upper Part of this Plate N, which has a Slit; thro' this there paſſes a thin Piece, which is taken hold of by a Hook, joined to the End of the Chain. To the Thread, which goes about the Pulley in a contrary Direction, there is faſtened a Copper Cylinder

T,

T, exactly of the ſame Weight with the Plate N and its Hook V. This Cylinder has two Uſes; it always keeps the Chain ſtretched, and ſuſtains the Plate N with its Hook V; which therefore does not preſs upon the Fibre with its Weight.

1304. It follows from what has been before explained *; that the
* 1284.1285. Laws of Elaſticity may be explored by making uſe of this Machine.

If a Fibre, or metallic Chord be ſtretched, by applying ſeveral Weights ſucceſſively to the Middle of it, the Sagitta may be obſerved. The Plate N draws the Chain; this goes about the Pulley, whoſe Diameter is equal to a ſixteenth Part of the Diameter of the divided Circle, which the End of the Index runs along *;
* 1301.1302. but each Diviſion of this Circle, anſwers to a hundredth Part of an Inch in the Sagitta.

But the Sagittæ being given, their Lengthenings may be compared together; and the Encreaſes of the Tenſions, cauſing them, may be determined; but this Method is liable to great Inconveniences. By this, as the Weights are changed in an arithmetical Progreſſion, I could not come to a regular Series of Lengthenings, and Tenſions, without applying Corrections in the ſmaller Sagittæ, ſcarce exceeding the thouſandth Part of an Inch. But as, theſe Corrections being neglected, which are diſcovered with Difficulty, and yet muſt be ſought for, becauſe theſe very ſmall Errors can ſcarce be avoided, we can diſcover nothing regular, I looked for another Method; and this I thought the more neceſſary, becauſe, when the Sagittæ are greater, that is, of one Inch, or more, another Irregularity is diſcovered, which I believe is to be attributed to a too great Inflection in the Middle.

For theſe Reaſons I ſelected a certain mean Sagitta, according
1305. to which I bent the Fibre in all the Experiments; and it will appear in the Experiments themſelves, that no other Inflections are required.

But it follows from what is ſaid before *, that the ſmaller Sa-
1306. gittæ are of Uſe, when we treat of determining the Tenſion itſelf,
* 1287. before the Inflection.

The mean Sagitta which I make uſe of, is 0,4 of an Inch.
1307. The Lengthening C *e*, of the Part C B, is diſcovered in this Situ-
Plate XLIII. Fig. 1. ation, by dividing the Square of the Sagitta C *c* by the Diameter
* 36. El. 3 of the Circle, of which *c e* is an Arc, that is, by A B *; which Length in my Machine is 34,5 Inches. Therefore C *e* is equal to

O,

O, 0046 of an Inch: that is, it is not a two hundredth Part of an Inch, and the Lengthening of the whole Fibre wants somewhat of an hundredth Part of an Inch; it is equal to O,0092 of an Inch.

But we discover this general Law of Elasticity, That *the lengthening of the Fibre*, cæteris paribus, *follows the Proportion of the lengthening Force.* 1308.

A Fibre has no Elasticity, as we have seen, except it be stretch'd by a certain Force *; whilst it is so stretched, it is lengthened; but we are not speaking of this lengthening. * 1280.

We suppose the Fibre so stretched as that it may be made elastic; any Force whatsoever being superadded, we affirm that the Lengthening arising from it, follows the Proportion of this very Force.

Experiment.

We make use of a Brass Chord, of the Sort that is used in some musical Instruments. In the Experiment, which I am now about to explain, I made use of such an one, as weighed 24 Grains. This was the Weight of the Part stretched between the Plates which retain it; and the Length of this Part is, as we said, thirty four Inches and an half *. 1309. Plate XLIII. Fig. 4. * 1307.

The Chord is applied to the Machine †; first, one End is fasten'd, the other End is drawn by Pincers, which passes between the Plates G G, and L, and the Chord is stretched, the other End of which is then also fastened, by turning the Screw *i*. 1310. † 1209.

The Chord passes thro' the Hole O, as was said; in the upper Part of which there is a small Incision, that the Chord may always answer to the same Point of the Plate.

To the Hook V of the Plate N is fastened a Weight equal to a Drachm, or rather greater, the Division to which the Index answers is observed; and the Plate has moreover two, three, or four Drachms hung upon it, and the Space passed thro' by the Index is observed. By this we determine the Encrease of the smaller Sagitta, from the Addition of one Drachm; and by comparing several Trials with one another, all Doubt is removed. As long as the Sagitta does not exceed ten Divisions, that is, is less than the tenth Part of an Inch, it is encreased and diminished according to the Ratio of the bending Weight; whence it follows that the Tension is not changed in these Inflections *. 1311. * 1289.

The Encreaſe found of the Sagitta is equal to the Sagitta itſelf, when the whole bending Weight is equal to one Drachm.

1312. This being diſcovered, we muſt determine the Encreaſe of the Sagitta from the Weight of the Chord itſelf. This, being left to itſelf, is bent by its own Weight; but, as its Weight acts in the middle of it, it may be looked upon as conſiſting of two right Lines; each of which having one End ſuſtained, acts in the Middle with half its Weight; without conſidering its ſmall Inclination to the Horizon.

1313. By the Trials taken notice of I diſcovered, that three Drachms give a Sagitta, equal to four; I hang on this Weight of three Drachms, and have the inflecting Weight of three Drachms and twelve Grains, or a fifth Part of a Drachm, by reaſon of the Weight of the Chord. The Sagitta anſwering, to this is $4\frac{1}{4}$, if the Index anſwers to this Diviſion, the Diviſions
* 1302. ſhew the true Magnitudes of the Sagittæ. If the Index anſwers to another Diviſion, its Situation may be changed *; but, as it is difficult to do this in ſuch manner, as to make it anſwer exactly to the Point required, it is ſufficient to obſerve, in this Experiment, that the Beginning of the Diviſions is removed from the Point, to which the Index now anſwers, four Diviſions and a Quarter; and we ſhall have the Situation of the Index diſcovered, when the Sagitta is equal to forty, or four tenths of an Inch. We had this Situation, when the inflecting Weight was equal to four Ounces and a half, that is, thirty-ſix Drachms.

1314. The Experiment was repeated after the ſame manner, encreaſing the Tenſion of the Chord ſo, as to have the Sagitta five, when a Weight of one Ounce was hung on; it was forty, when eight Ounces and ſix Drachms were hung on.

1315. The Tenſion was encreaſed again ſo, as to make the Sagitta four, when one Ounce was hung on; it was forty, ten Ounces and ſix Drachms being hung on.

1316. Making the Computation with the ſmaller Sagitta *, we diſcover
* 1287. † 1313. the Tenſions before any Inflection, to be in theſe Proportions. In the firſt Caſe †,

O,04 : 8,625 : : 3 Dr. : 6486 Dr. = 5 Pounds and 7 Drachms nearly. In the ſecond Caſe,

O,04 : 8,625 : : 8 Dr. : 1380 Dr. = 11 Pounds, taking away half an Ounce. In the third Caſe,

O,05. : 8,625 : : 8. Dr. : 1725 Dr. = 13 ½ Pounds, wanting three Drachms. If in theſe three Caſes, the Tenſion ſhould not be encreaſed, when the Inflection is encreaſed, the Sagitta would encreaſe as the inflecting Weight *. Then the Sagitta would * 1237. be equal to O,40, in the firſt Caſe, a Weight of thirty Drachms being hung on. In the ſecond Caſe, when a Weight of half a Pound was hung on. In the third, when a Weight of ten Ounces was hung on. But in theſe three Caſes, theſe Weights were to be encreaſed equally, namely by ſix Drachms. But we diſcover the Encreaſe of the Tenſion from theſe ſix Drachms, by this Proportion *, * 1287.

O,40 : 8,625 : : 6 Dr. : 129,4 Dr. = 1 Pound and $1\frac{1}{2}$ Dr. nearly.

The Experiment proceeds in the ſame manner, any other Tenſion whatſoever being given; whence it follows, that the Tenſion 1317. being encreaſed by a Quantity, equal to one Pound and a Drachm and an half, the lengthening of the Fibre is always equal to O,0092 of an Inch; if an equal Weight be again ſuperadded, the new Lengthening will be equal to the firſt alſo, and a double Weight gives a double Lengthening, and the Experiment evinces, that the Lengthening follows the Proportion of the Weight; as was ſaid before *. * 1308.

In Chords of the ſame Kind, Thickneſs, and which are equally 1318. *ſtretched, but of different Lengths; the Lengthenings which are produced by ſuperadding equal Weights, are to one another as the Length of the Chords.* For the Chord is equally ſtretched in all its Parts; therefore the Lengthening of a whole Chord is double the Lengthening of half of it, or of a Chord of half the Length.

The Lengthenings of Fibres of the ſame Kind, and Thickneſs, are 1319. therefore *in a Ratio compounded of the Lengths, and Weights by which they are lengthened* †. † 1308.

If the Fibres are of different Thickneſſes, tho' of the ſame Matter, *the Forces, which lengthen Fibres of equal Length equally,* 1320. *are not to one another, as the Quantities of Matter in the Fibres;* and I diſcovered in the Experiments, which I made with different Chords, that this Ratio is ſometimes greater and ſometimes leſs than that. Whence it follows that *a greater or leſs Elaſticity, in* 1321. *Bodies of the ſame kind,* cæteris paribus, *depends upon a certain peculiar Diſpoſition of the Parts.* Which is alſo deduced from what has been obſerved concerning Elaſticity, in the Beginning of this Chapter †. * 1282.

CHAP. XIII.

Of the Elaſticity of Plates.

1322. WE deduce from what has been ſaid of the Elaſticity of Fibres, what relates to Plates. For a Plate may be conſidered as a Congeries, or Bundle of Fibres, tho' the Elaſticity of
* 1321. all of them may be changed from their being joined together *; if we conſider a thin Plate, not very broad, we may make Experiments concerning its Lengthenings, by hanging on Weights, after the ſame manner as we did with the Fibres. Theſe Experiments will ſhew that the ſame Law of Elaſticity, delivered be-
1323. fore †, obtains here alſo; namely, That *the Lengthening of the*
† 1308. *Plate follows the Ratio of the Force by which it is lengthened.*

Experiment I.

1324. This Experiment is made in the ſame manner, as that which
Pl. XLIII. was explained in the foregoing Chapter *; a Plate is uſed inſtead
Fig. 4. of an elaſtic Chord.
* 1309.

I made uſe of two Watch-Springs; cutting off the Ends having the Holes, one of theſe was divided into two equal Parts; which were well faſtened to the Ends of the other.

Theſe Springs, being joined, made a long Plate of thirty-eight Inches and a Quarter, weighing eighty-four Grains; but the Plate is doubled in the Places where it is joined; wherefore this is the Weight of a Plate thirty-nine Inches long. The Part, which is lengthened by the Inflection, is thirty-four Inches and an half long; adding the Parts doubled, it is thirty-five Inches and a Quarter long, and weighs ſeventy-ſeven Grains; and any inflecting
† 1312. Weight whatſoever is encreaſed by thirty-three Grains †: that is, this Weight was to be added to the Weight applied, that the inflecting Weight might be determined. We have neglected three Grains.

1325. This Plate is applied to the Machine, ſtretched, and faſtened,
* 1310. as was ſaid of the Chord *; but Pincers having a Screw, are to be made uſe of. The Plate is put thro' the Hole O; the Breadth of the Plate filling up the Breadth of the Hole, the Plate moving freely in it.

Now

Now these are the different Trials.

Two Ounces and an half gave the Sagitta ten; and eighteen the Sagitta forty. 1326.

Four Ounces gave the Sagitta ten; twenty-four the Sagitta forty. 1327.

These we had in the smaller Tensions; it is needless to mention the mean Sagittæ, I pass to the greater.

Eight Ounces gave the Sagitta seven, fifty-three Ounces and three Quarters, the Sagitta forty. 1328.

Eight Ounces gave the Sagitta seven; sixty-one and an half, the Sagitta forty. 1329.

The Computations are made, as was said of the Chords †. 1330.
But the inflecting Weight in this Experiment, the Sagitta forty being given, always exceeded by eight Ounces the Weight, whereby this Sagitta should be had, setting aside the Encrease of the Tension. † 1316.

But the Tensions themselves were as follows:

In the first Trial *, it was nearly thirteen and an half Pounds. * 1326.

In the second †, it somewhat exceeded twenty-one Pounds, eight Ounces and an half. † 1327.

In the third ‡, it was sixty-one, nine Ounces and an half. ‡ 1328.

And in the last ||, it wanted two Ounces of seventy-two Pounds. || 1329.

And in each of these Tensions, which differ very much from one another, we had the same Lengthening, namely 0,0092 of an Inch, the Tension being encreased by ten Pounds twelve Ounces and an half.

We had the same in all other Tensions that we examined.

Therefore the Conclusions, which we deduced from the Experiment of the foregoing Chapter *, follow from this also. 1331. * 1317. 1318. 1319.

Many elastic Plates are able to support themselves; that is, if you fix one End, the Plate is not bent by its own Weight; if it be bent by any outward Force, it will, when left to itself, move and make many Vibrations, as was demonstrated before of the stretched Fibre †. 1332. † 1292.

In such an Inflection, the Plate is lengthened by a different Force in different Points; that is, if we conceive a Plate, divided into innumerable infinitely small and equal parts, these parts are lengthened unequally in the Inflection.

If the lengthening of one Particle only should be given, as this would follow the Ratio of the lengthening Force *, *it would,* 1333. * 1308.

2 *when*

when left to itſelf, always return to its former Situation, in an equal Time; for we may apply here the Demonſtration of the Motion of Fibres *.

* 1292.

1334. But different Particles, lengthened ſeparately, would not return in equal Times; becauſe they would carry with them different Quantities of Matter: for we do not ſpeak of the Parts taken ſeparately.

But when a Spring is bent, and left to itſelf, all the Parts perform their Vibration in the ſame Time; the Motion of ſome of them being accelerated, whilſt that of the others is retarded; as in a componnd Pendulum †.

† 424.

But in different Inflections of the whole Plate theſe Accelerations and Retardations, depend upon the ſame Cauſe; namely, upon the Matter to be moved by the Actions of the different Particles of the Plate.

From this we deduce, as theſe Actions acting ſeparately, would always perform their Motions in equal Time in any Inflections whatſoever †; that they now alſo, when they are changed always by the ſame Cauſes, perform their Motion in equal Time; and that
1335. therefore *all the Vibrations of the ſame Plate, how unequal ſoever, are performed in the ſame Time*; *and* that *the Plate is moved according to the Laws of a Pendulum vibrating in a Cycloid*: for we have demonſtrated in N. 414. that this Pendulum is thus moved.

† 1333.

Theſe Laws are, that the Action upon the Body, in each point of the Way to be paſſed thro', is as the Diſtance of the Point from the Place in which the Body can be at reſt *; and that the Force is ſuch, as acts upon the Body in Motion as upon a Body at reſt †, ſuch as we have ſeen the Force of Elaſticity to be in the foregoing Chapter ‡.

* 414. † 371. ‡ 1292.

1336. Pl. XLIII. Fig. 5. Whence we infer, that *the various Inflections of the ſame Plate are proportional to the Forces by which the Plate is retained in theſe Inflections*. Let A B be a Plate, whoſe End A is fixed, let it be retained by two Forces, in the Situation A *b* and A b; if one be double of the other, b *b* and *b* B will be equal.

Experiment 2.

1337. Pl. XLIV. Fig 8. The Plate A conſiſts of many elaſtic Plates joined, it is put into the Box B; and is there moved on both ſides between the Rulers *c d*, *c d*; two Threads are faſtened to the upper Part of the Plate, and are put thro' Holes in the Bottom of the Box, and joined to the

the Brass Plate E; to the middle of which is fastened another Thread, on which is hung the half-pound Weight P. The Weight descends half an Inch; an equal Weight being superadded, it descends half an Inch more; and so on, till the Plate can be no longer pressed together.

Every smaller Plate is inflected in proportion to the Weight *, 1338.
and the Motion of the Weight from all the Inflections joined, * 1336.
follows the same proportion. The Experiment is made with many Plates joined, because the Direction of the Action of the Weight upon the Plates is not sensibly changed in the various Inflections.

What is said of the Inflection of the Plates, may be applied to 1339.
the curve Plate A C B; if this be loaded with two Weights so as Pl. XLIII.
to acquire the Situations *a c b*, a c b, and the Weights be to one Fig. 6.
another, as one to two, the Distances c *c*, and *c* C will be equal *; * 1336.
therefore the Introcessions of the Point C are, as the Weights whereby the Plate is loaded: which also may be referred to the Introcessions of many Plates joined.

We shall determine the whole Action, whereby a Spring is 1340.
bent, by collecting into one Sum all the small Actions, whereby the Inflection is encreased successively.

Let A B be the Space passed thro' in the Inflection; and B C the Pl. XLIII.
Force, which retains the Plate bent in that Situation; drawing A C, Fig. 7.
and D E parallel to B C, D E will be proportional to the Action, which would retain the Plate in the Inflection A D *. Whilst it is * 1336.
bent, it passes thro' all the intermediate Inflections, between the smallest Inflection and the greatest A B; and the whole Action, whereby the Plate was so bent, is equal to all the Actions together, by which it might have been retain'd in all the smaller Inflections, thro' which it passed. This Sum is represented by the Surface of the Triangle A B C; as appears, if we refer hither what is contained in N. 373. and N. 750. *and the following* ones.

Therefore, the whole Force in the Inflection A D is to the whole Force in the Inflection A B, as the Triangle A D E is to the Triangle A B C; whence it appears that *the whole bending Forces are* 1341.
in a duplicate Ratio of the Inflections themselves *. * 19 El. 6.

Let us suppose an Inflection to be made, by hanging on, or 1342.
applying a Weight; and let B C be the Weight, by which the Plate is retained in the Inflection A B. The whole Action of the Weight, whilst it descends along A B, which follows the Ratio of the Weight itself, and the Ratio of the Space passed thro' in the Descent, is proportional to the whole Surface A B C G *, which * 23. El. 6.
is twice as great as the Triangle A B C †; and the Product of the † 41. El. 1.

Weight

Weight B C, which keeps the Plate bent, by the Inflection itself A B, which is equal to the Space run thro' by the Weight in its Descent, is equal to the double of the whole bending Force; that
1343. is, *the whole Action, by which the Spring was bent, is equal to the Force, which the Weight mentioned acquires, in falling from an Height equal to half the Inflection.*

* 1341. 1083. The Spring, whilst it is relaxing, produces an Action equal to that, by which it was bent, if the Elasticity be perfect *; therefore
1344. *the Forces, communicated by the Relaxations of a Spring, are as the*
† 1341. *Squares of the Inflections* †; *and the Velocities are as the Inflec-*
1345. *tions* ‖; *which are as the Forces, by which the Spring is kept in its*
‖ 753. ‡ 1336. *Situation* ‡.

EXPERIMENT 3.

1346. Almost all, that was delivered in Exper. 2. Chap. xi. of this
* 778. Book *, is of use here also; therefore laying down what is had in N. 778; we must observe that the Catches *p q*, *p q*, (Pl. XXVI. Fig. 3.) must be removed from the Plate, which they are applied to, before it is fixed in its Place.

The Axis *t s* also (Pl. XXVI. Fig. 2.) which is joined to the Hammer *m*, is removed. All these are easily taken away, by loosening the smaller Screws. The Pulley R is put upon the Supports *p*, *p*, and turns about freely.

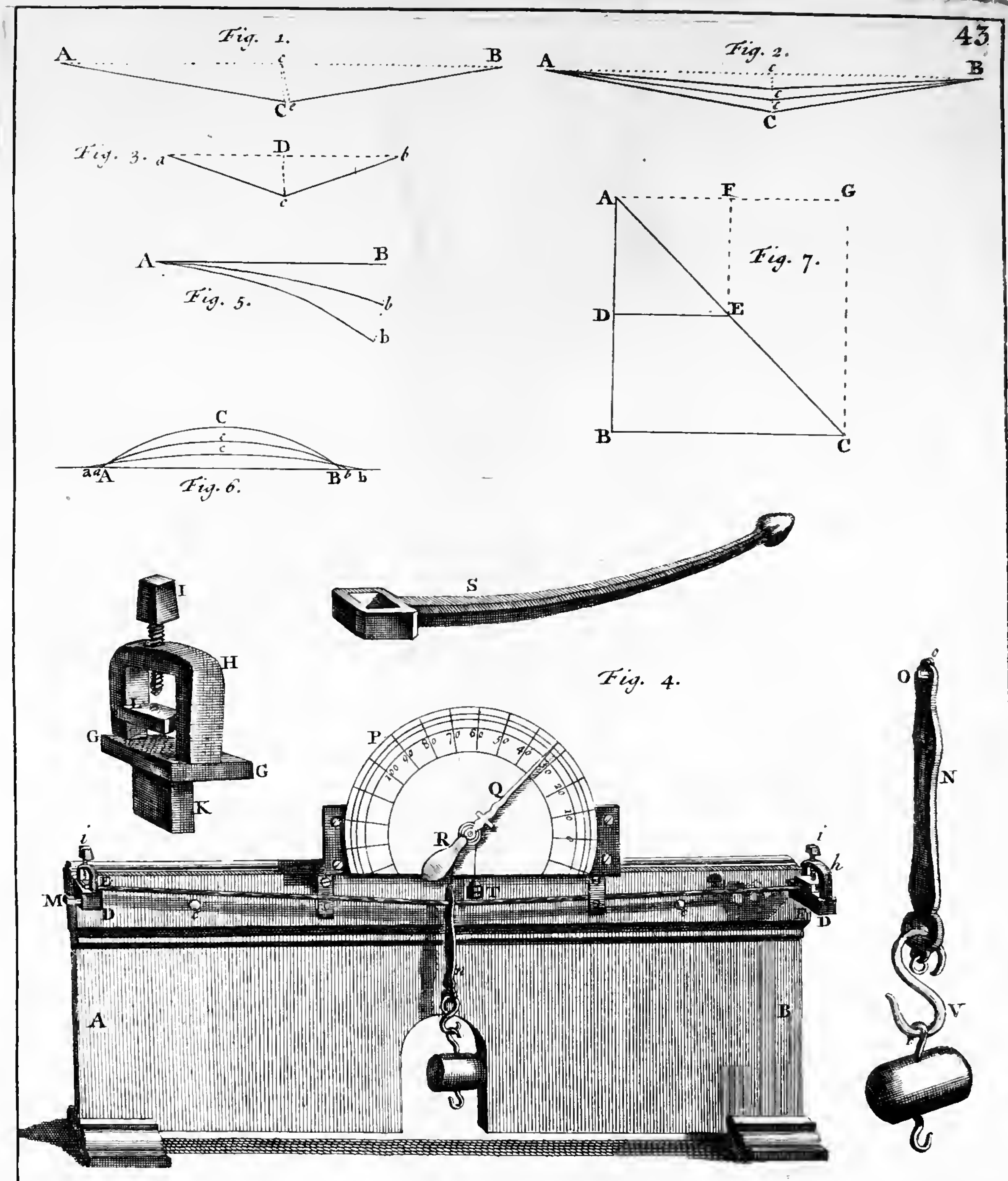
Pl. XLIV. Fig. 1. The Rectangle A being suspended, as was said, a Cylinder is put into it, which makes the Mass 4; as in the second Case of the second Experiment, mentioned in the second Chapter of this Book. The small Tongue of the Spring answers to the Hole in the Plate *g f*, from which the Catches were removed. A small Thread
* 739. is put thro' the Hole in the anteriour part of the small Tongue *; and being doubled, passes thro' the Plate *f g*, and the next to it *b c* also, and is put about the Pulley *r*.

1347. The Weight P of three Pounds is hung on; the Ends of the Thread being first tied together. The Thread is cut suddenly with a pair of Scissars between the Pulley *r* and the Plate *b c*; but the Thread is doubled, but only one of them is cut; and the Body being freed, ascends to the Division 5,2.

1348. If we hang on five Pounds instead of three, the Body ascends beyond the Division 8, 6.

Therefore the Velocities are, as 52 to 86 and an half, or as 3 to 5; that is, as the Weights hung on.

By



J. Mynde sc.

By this Experiment we can determine the Defect of Elasticity in the Spring; by comparing the Ascent of the Weight projected, with the Descent of the inflecting Weight, as is shewn in the following *Scholium*; in which also we shall see, how the Time itself, in which the Spring was relaxed, is determined. 1349.

In the Spring, which we made use of in this and other Experiments, the Elasticity is to perfect Elasticity, as 11. to 12. nearly, taking into consideration the Forces communicated; but the Velocity communicated by this Spring, is to the Velocity, that would arise from perfect Elasticity, as 22. is to 23. nearly; and in this Experiment of ours, supposing the Elasticity to have been perfect, the Velocities would have been 5,43, and 9,03. 1350.

The Time, in which our Spring is relaxed, which is the same whether the Spring be less or more bent *, scarce exceeds two hundred twenty ninth parts of one Second, when it drives forwards the Body, which we made use of in this Experiment. If the moved Body is different, the Time is inversely as the Velocity communicated, or directly in a subduplicate Ratio of the Mass. 1351. * 1335. 1352.

In the last Experiment the Thread passes through the two Plates *f g*, *b c*; because I made use of the same Machine, which I used in the second Experiment of the eleventh Chapter of this Book *, but it may be demonstrated as exactly by a more simple Machine; but it is needless to make use of two Machines, when one is sufficient. Yet I do not think it improper to explain this more simple one also. 1353. * 778.

The Copper-Plate D B C is so bent, as to have its two Parts D B, B C, at right Angles to one another. To the Part D B is joined, at right Angles to it, the Plate E, having a Screw *h*. This Part D B is applied to a Board *, as is seen in Fig. 4. and 5; the Plate E goes thro' an horizontal Slit in the Board, and may be moved a little in this Slit, which is required in the Experiments †; but it is fastened by the Screw *h g*, the Copper-Plate *l* being put between, that the Wood may not be damaged. 1354. Plate XLIV. Fig. 2. 3. * 760. † 778. 743.

These Things being given, the Plate B C stands at right Angles to the Board *; it is vertical and perforated at L. The Pulley R, that turns freely upon its Axis adheres to the Plate B C, the Ends of the Axis going into the Supports *p*, and *p*, and having Holes there to turn in. * 760.

The Figure itself sufficiently shews that there is no Difficulty in the Experiment. 1355. Pl. XLIV. Fig. 4.

1356. Pl. XLIV. Fig. 5. By making use of this same Plate, we easily make the second Experiment of the eleventh Chapter of this Book; taken notice of before: But in this the small Tongue of the Spring must be perforated, and not have its sides indented *. The Pulley R with its Supports is removed: the Rectangle is suspended, as in the said Experiment; the small Tongue of the Spring is so put into the Hole L, that one or other of the Holes of the Tongue itself may come behind the Plate B C; into this Hole is put the Pin M, made of a Brass Wire: the Spring is now bent, and the Inflection may be varied, by means of the different Holes in the small Tongue; the Spring is let loose, by drawing downwards the Pin M suddenly; and that this may be done more conveniently, the String F is tied to it. The rest does not differ from what is described in the Experiment mentioned †.

* 739.

† 778.

1357. By this Method, tho' it is not very nice, the Experiments may yet be made with such Accuracy as to leave no doubt about the Conclusion; because it plainly appears that there must be some Defect in the Machine.

1358. If in the first Experiment of the sixth Chapter of this Book*, we are also willing to recede from the more perfect Method, delivered there, we may make use of a Method, like that, which we have just now been describing.

* 1095.

Pl. XLIV. Fig 6. Instead of the Machine, joined to the Rectangle, and described in N. 1090. *and the following*, we use a Copper-Plate, bent, as is represented at $g f$; which we also join to the Rectangle †, by help of two Screws, going through Holes, one of which is seen at *o*. The anteriour part of the Plate $f g$ has its middle L perforated.

† 773.

Pl. XLIV. Fig. 7. This Plate joined to its Rectangle is represented at B in Fig. 7. The Rectangle A has a Spring joined to it, as in the foregoing Experiments. These Bodies are suspended, as is said in the Experiment of which we are now speaking ‡; the Tongue of the Spring is put into the Hole of the Plate $f g$, and the Spring is bent, and retained by the Pin M, as is seen above *.

‡ 1095.

* 1356.

To separate the Bodies we must draw the Pin M, by the String T, out of the Hole of the Tongue of the Spring; but the Threads, by which the Rectangles are suspended, are so lengthened, as to make the Motion very irregular, when the Rectangles are separated. To remove this Inconveniency, we put into the Board, along which the Bodies are moved, perpendicular to its Surface, a Pin made of a thick Brass Wire, as is represented at S. Then the Plate $f g$ will not descend, when the String T is pulled, for the Pin is so placed as to make the Motion regular enough. The

The reſt may be ſeen in the Experiment itſelf mentioned *. 1359.
If we make this Experiment after this Method, it will be a leſs Expence; but the Experiments will not agree entirely with what is demonſtrated; but it will appear more plainly that the Errors are to be attributed to the Machine itſelf, if each of the Experiments be repeated with the Maſſes tranſpoſed. If, for Example, the Maſs of B be four, and that of A three; we muſt repeat the Experiment, putting the Maſſes, of A four, and B three. * 1095.

As theſe laſt Methods *, which I formerly uſed myſelf, are very ſimple, I thought proper to deſcribe them here; but I did not make mention of them before; becauſe all the Experiments, of which we ſpeak here, do properly belong to this Chapter: but the Order of the Demonſtrations required, that the Experiments themſelves ſhould be firſt delivered; wherefore I have explained one perfect Method only, for all the Experiments. *1354. 1356. 1358.

SCHOLIUM.

The Explication of the Numbers 1350. 1351. 1352.

WE have ſeen before, that the Body in our Machine is raiſed to the Height of one Inch, when it is moved with the Velocity 14,6 *; but in thoſe Experiments the Length of the Threads, by which the Body was ſuſpended, was different from that, which obtained in the Experiments, of which we are now treating, and in thoſe alſo, which were made before with the Spring *; in theſe the Aſcent is one Inch, when the Velocity is 14,2. 1360. * 883. * 779. 780. 781.

The Heights are as the Squares of the Velocities ‖; therefore ‖ 381. 399.

$$\overline{14,2}^2 : \overline{5,2}^2 :: 1 : 0,1341,$$

And this is the Height, to which the Body arrived in the Experiment †; which if we multiply by the Weight of the Body itſelf, the Product will expreſs the Action of the Spring. † 1347

The Rectangle together with what is added to it ‖ weighs four Ounces and a Quarter; the Weight was quadruple in the Experiment, and was equal to $1\frac{1}{64}$ Pound; therefore the Action of the Spring was equal to 0,1362. 1361. ‖ 771.

The Action, by which the Spring was bent, which is equal to the Action, by which it is relaxed, when the Elaſticity is perfect, is had by multiplying half the Inflection by a Weight of three Pounds *. 1362.

The Inflection of the Spring, in Exper. 2. Chap. xi. † was 0,16 of an Inch; and being relaxed, it communicated to the ſame Body, of which we *1343. 1347. † 778.

 are

|| 780. are now ſpeaking, the Velocity 8,4 ||; but the Velocities, in the ſame
* 1361. Body, are as the Inflections *; therefore

$$84 : 52 :: 0,16 : 0,099. = \text{the Inflection sought.}$$

This Inflection being multiplied by three, the half of the Product 0,297, that is, 0,1485, gives the Force ſought.

1363. Therefore we find that perfect Elaſticity, is to the true Elaſticity, as
† 1361. 1485, is to 1362 †, that is, as 495 is to 454; nearly as 11 to 12.

1364. But as the Forces are as the Squares of the Velocities *, the Velocity,
* 753. communicated by the Spring, is to the Velocity, in perfect Elaſticity, as 22 to 23 nearly.

And in the Experiment, of which we are now ſpeaking, the Velocities,
* 1347. which in the firſt * and ſecond † Trial were 5,2. and 8,65. would have
† 1348. been 5,43. and 9,03. if the Elaſticity had been perfect; as we have obſerved in N. 1350.

1365. The Time, in which the Spring, of which we are ſpeaking, is relaxed, is always the ſame, as long as the ſame Body is moved; we will now exa-
*1335. 1362. mine the Caſe mentioned before in N. 780. The Inflection was 0,16 *, and the Velocity 8,4.

In the Experiments, of which we are ſpeaking, the Length of the Pendulum moved is 50,5 Inches; and whilſt the ſuſpended Body is moved, it deſcribes the Arc of a Circle, whoſe Diameter is 101 Inches, and when the Velocity is 14,2. the Sine of the Arc deſcribed is ten Inches; to which anſwers an Arc of 11. Deg. 25″. The Arc deſcribed in the Experiment is diſcovered by this Proportion,

$$142 : 84 :: 11°, 25' : 6°, 45'.$$

whoſe Length is 5,949 Inches.

The Duration of the Vibration of a Pendulum, whoſe Length is 50,5 Inches, is 1″, 1524; and in half of this Time, that is in 0″,5767, the Body runs thro' an Arc of 5,949 Inches; the Spring, whilſt it is relaxed, moves
* 1335. according to the Law of a Pendulum vibrating in a Cycloid *, and may be conſidered as if it ran thro' an Arc ſimilar to that, which the Body itſelf
† 449. runs thro'; wherefore the Spaces paſſed thro' are as the Times †; therefore

$$5{,}949 : 0{,}16 :: 0'',5767 : 0'',0155 = \text{the Time sought.}$$

This Time ſcarce exceeds $\frac{2}{129}$ of one Second; as was ſaid in N. 1351.

1366. The Computation ſhews that *the Time, other Things remaining the ſame, follows the inverſe Ratio* of the Length of the Arc paſſed thro'; whilſt the
* 442. Arc itſelf follows the Ratio *of the Velocity* *. Which is the Caſe, if the Elaſticity be changed, the Inflection remaining.

1367. We have the ſame Duration by another Method, in which what is delivered in the Beginning of this *Scholium* is of uſe; namely that the In-
|| 1362. flection of the Spring is 0,099 ||, when the vertical Aſcent of the Body
* 1360. projected by the Spring is 0,1341 parts of an Inch *.

1368. Now if it be known elſewhere, that the Body in 1″. falls from an Height
*415. 470. of 187,6644 Inches ||; we diſcover the Time of the Fall from a Height
|| 374. of 0,1341 parts of an Inch; and it is 0″,0267 *.

The

The Inflection of a Spring is made according to the same Laws, to which a moving Body is subject, which loses its Motion in making a parabolic Cavity in a soft Body, of which we treated above ‖; wherefore the Rule of N. 897. takes place here also in the Relaxation of the Spring, if instead of the Depth of the Cavity we put the Inflection of the Spring 0,099; and instead of the Height, there mentioned, that, to which the Body was projected 0,1341 Parts of an Inch. 1369. ‖ 891. 893. 1335.

Supposing the Diameter to be to the Circumference, as 113 to 355; we have this Proportion, 1370.

$$0,1341 \times 113 : 0,099 \times 88,75 :: 0'',0267 : 0'',0155.$$

And we discover again 0'',0155.

This is the Time, in which the Spring is relaxed: if we considered its Inflection, the Time would be less; because when a Spring is to be bent, the Body must be projected with that Velocity, with which it would be repelled if the Elasticity were perfect; and the Time is diminished, when the same Inflection is treated of, in the Ratio in which the Velocity is encreased *. * 1366.

If the Mass be changed, other Things remaining, the Velocity follows the inverse subduplicate Ratio of the Mass †; the Inverse of which this direct Ratio is, which the Time follows ‖; as was said in N. 1352. 1371. † 731. 758. ‖ 1366.

CHAP. XIV.

Of Elastic Solids.

IN the two foregoing Chapters we treated of Fibres, and Plates; first we considered the Length only, then Length and Breadth together: but we must now consider the three Dimensions together. In this last Case, we do not speak of the Inflection, or lengthening of the whole Body, as in the two foregoing Chapters; but the Inflection of the Parts has a kind of Agreement with the Introcession of the Parts in soft Bodies. 1372.

Every Elastic Body may be considered as a Bundle of Plates, and *whilst that is struck*, these yield; and the Inflections of them all taken together are equal to the whole Force, which is destroy'd in the Percussion of the Body *. If the Parts of the Body be pressed inwards by a different Force, the Inflections of each of the Plates are different, and in each the Inflection follows the subduplicate Ratio of the Force acting upon the Plate †; but we speak of the same Plates, and the whole Action, whether it be greater or less, is dispersed in the same manner thro' them, wherefore the Inflection of all of them is made according to the same Ratio; and *the* 1373. Pl XLIV. Fig. 9. * 934. † 1341.

Introcession

Introcession of the external Plate, which we can often measure, *is in a subduplicate Ratio of the whole Force pressing the Parts inwards.*

Any one of these Plates, if separately relaxed, would return to its former Situation in equal Time, whether it were more or less
* 1335. bent *; whence we deduce, that, when these are all joined toge-
1374. ther, *the Relaxation of the Parts bent is* likewise *always made in equal Time, when the same Body is moved by this Relaxation.* This will appear if, making the proper Alterations, we apply to this place, what was said of a Plate bent in N. 1334.

If we would apply this to elastic Spheres, and thence deduce Conclusions, we must premise some things.

1375. Let A C B E be a Sphere; let us suppose the Point C to be
Pl. XLIV. pressed inwards as far as D; that is, the Surface A C B to apply
Fig. 10. itself to the Surface of another Body; then, if it makes a Spot
1376. there, the Diameter of the Spot will be equal to the Arc A C B. But this Arc is always very small, and may be looked upon as the Sum of the Subtenses A C, C B; and also as equal to the Line A B.

1377. Drawing A E, the Triangle C A E is right-angled *; where-
* 31 El. 3. fore the Triangles C A D, C A E are similar †; and D C, C A,
† 8. El. 6. C E are proportional ‡, whence it follows that the Square of the
‡ 4. El. 6. Subtense is equal to the Rectangle made of the Abscis C D and
17 El. 6. the Diameter C E ||. Therefore *the Squares of the Subtenses*
1378. A C, a C, *are to one another as the corresponding Abscissæ* C D,
* 1. El. 6. C d *. These Subtenses are as the Diameters of the Spots, *when*
1379. *the Introcessions are equal to the Abscissæ*. Therefore *the Abscissæ are* also, as the Squares of the Diameters of the Spots, that is,
† 2. El. 12. they are *as the Spots themselves* †; which we have seen are equal
|| 1376. to the Bases of the Segments A B C, a b C ||.

1380. *In this same Case* the Abscissæ measure the Inflections of the outward Plate of the Body; therefore the Squares of the Abscissæ are,
* 1373. as *the Forces by which the Parts were compressed* *; which Forces therefore *are* also *as the Squares of the Spots.*

1381. In the Triangle A C B, A B is to *n n*, as C D is to C d; therefore the said Bases of the Segments are also, as A B is to
† 1379 *n n* †.

1382. Now if we suppose the whole Segment A C B to be divided into innumerable Orbs by Planes as a b, *a b*, parallel to the Base A B, the Orbs will be proportional to their corresponding Lines *n n*, *e e*: but if each of the Lines has a Breadth equal to the

 Thickness

Thickneſs of its correſponding Orb, the Parts of the Triangle A C B, will be proportianal, to the correſponding parts of the Segment itſelf; and the Segments themſelves A C B, a C b, *a* C *b*, will be to one another, as the Triangles A C B, *n* C *n*, *e* C *e*. The Triangles themſelves are, as the Squares of the Baſes A B, *n n*, *e e* *, which Baſes are as the Baſes of the Segments †. Therefore *the Segments* themſelves *are*, as the Squares of their Baſes, that is, as the Squares of the Spots; or *as the Forces by which the Parts are preſſed inwards* ‡. * 19. El. 6. 1383. † 1381. ‡ 1380.

If the ſame elaſtic Sphere, runs againſt a fixed elaſtic Obſtacle, with different Velocities, the Squares of the Spots will be, as the Squares of the Velocities *; that is, *the Spots will be as the Velocities*. 1384. * 1380. 753.

EXPERIMENT I.

We make uſe of a heavy marble Plane, that is blue, moiſtened a little, that the Colour may be more intenſe, here, as we did in 1. Exper. of 3. Chap. of this Book. 1385.

An Ivory Ball is let down, which falling ſtrikes againſt the Plane, and leaves a round Spot on its Surface. Let the Ball fall from a Height of nine Inches, and let E be the Spot; then let it fall from a Height of three Feet, which is quadruple of the firſt, and let F be the Spot; laſtly, let it fall from an Height of ſix Feet and nine Inches, which is nine times the firſt Height, and let G be the Spot. In this Experiment the Velocities of the Body are to one another, as one, two, and three *; in which Ratio alſo are the Spots E, F, and G; for, making the right-angled Triangles D A B, D B C, in which the Sides D A, A B, B C are equal to one another, and to the Diameter of the Spot E, the Line B D, will be exactly equal to the Diameter of the Spot F, and the Line C D to the Diameter of the Spot G. But the Spots are as the Squares of the Diameters *; and the Square of the Line B D is equal to the Squares of the equal Lines A B, A D †; and the Square of the Line C D is equal to the Squares of the Lines B C, B D; or of the three equal Lines D A, A B, B C. If we compare the Diameters of the Spots by the Sector, theſe are to one another, as 72, 102, 125, the Squares of which Numbers are nearly as 1, 2, 3. Pl. XLIV. Fig. 11. * 374. 1386. * 2. El. 12. † 47. El 1.

If we conſider different Bodies, but ſuch as are terminated by Portions of equal Spheres, as in the ſaid firſt Exper. of Chap. third *, *the Squares of the Spots*, or the Biquadrates of the Diameters of the Spots, *are as the Maſſes multiplied by the Squares of the Velocities* *. 1387. * 820. * 1380. 757.

If

If the Spheres are different, or if the Bodies themſelves are
1388. ſpherical, or only terminated by Spheres, *as long as we conſider a Matter equally elaſtic, the Segments preſſed inwards are to one another, as the Forces by which they are preſſed inwards.* For this Rule obtains in both Spheres, wherefore it may be applied to different ones alſo, if it takes place in comparing the Segments of two Spheres in one caſe only. But this Caſe is given, when the Abſciſſes are equal; for the Introceſſions of the correſponding Parts are equal, and the different Reſiſtance can only be attributed to the different Quantity of Matter.

1389. Let there be two Spheres M and N; their Segments A F B,
Pl. XLIV. A C B, having equal Baſes, have unequal Heights F D, D C.
Fig. 12. Let us ſuppoſe theſe Heights to be divided into infinitely ſmall and equal Parts, and that there is the ſame Number of Parts in each Height. Let us farther ſuppoſe the Segments to be cut through each of the Diviſions, by Planes parallel to the Baſe; we ſhall have the Segments divided into very ſmall Orbs ſo, that the Thickneſs of each, in the firſt Segment, will be to the Thickneſs in the other Segment, as F D is to D C; by reaſon of the equal Number of Parts in each Height. The Orbs in each Segment, as we re-
* 1379. cede from the Baſe, are diminiſhed according to the ſame Law *, ſo that the correſponding Orbs are equal. Whence it follows that any Orb whatſoever, in the firſt Segment, is to its correſponding Orb in the other Segment, as Thickneſs is to Thickneſs, that is, as F D is to D C; and the Sum of all the Orbs is to the Sum of all the Orbs, that is, the Segment A F B is to the Segment
† 12. El. 5. A C B alſo, as F D is to D C †.

1390. We ſpeak here of ſmall Segments; therefore F D, D A, F G,
* 1377. are proportional *; as alſo D C, D A, C E. Therefore each
1376. Rectangle, that of F D multiplied by F G, and D C by C E, is
† 17. El. 6. equal to the Square of the Line D A †; and the Rectangles are equal
1391. to one another; whence we deduce F D : D C :: CE : F G*; and *the*
* 16. El. 6. *Segments, whoſe Diameters are equal, are inverſely, as the Diameters of the Spheres; and the Forces, by which they are preſſed inwards, are*
‡ 1388. *in the ſame Ratio alſo* ‡.

1392. From theſe things, compared together, we deduce this univerſal Rule, that *the Forces, by which the Bodies are ſtruck, are directly*
* 1380. *as the Squares of the Spots* *, *and inverſely as the Diameters of the*
† 1391. *Balls* †. Which may be thus expreſſed; the Product of the Maſs
‡ 757. by the Square of the Velocity ‡ is, as the Square of the Spot divided by the Diameter of the Ball; and multiplying theſe two

 Quantities

Quantities by the Diameter of the Ball, we change the laſt Rule into this other Rule.

The Square of the Spot follows the Ratio of the Product of the Maſs by the Diameter of the Ball, and by the Square of the Velocities. 1393.

In Spherical Bodies the Maſs follows the triplicate Ratio of the Diameter *; that is, it is as the Cube of the Diameter; and the 1394.
Rule for theſe Bodies may be thus expreſſed; *the Spot itſelf, or the* * 18. El. 12.
Square of the Diameter of the Spot, follows the Ratio of the Product of the Square of the Diameter of the Ball by the Velocity of the Body.

In this Caſe *if the Velocities are equal, the Diameters of the Balls will be to one another as the Diameters of the Spots.* 1395.

We ſpeak here altogether of the ſame Elaſticity; this we have in Bodies made of the ſame Ivory. But I cannot affirm that all Ivory has equal Elaſticity; though I could diſcover no difference in the few Experiments I made concerning it.

EXPERIMENT 2.

In this Experiment, the Balls are let down upon a Marble Plane, 1396.
in the ſame manner as in the foregoing one *; we make uſe * 1385.
of Ivory Balls whoſe Diameters differ any how. Theſe are let down from equal Heights, and the Spots are exactly meaſured. The Diameters of the Ball, being alſo meaſured by a pair of Callipers, we ſhall have the ſame Proportion between theſe and thoſe; as is eaſily diſovered by the Sector.

SCHOLIUM.

Of the Times in which the Inflections of Elaſtic Bodies are performed.

THE Inflection of the Parts of a Body is made according to the ſame 1397.
Laws, to which an elaſtic Plate that is bent is ſubject; if inſtead of the Inflection of the Spring we put the Height of the Segment preſſed
inwards *. Therefore by comparing together what is ſaid in N. 1369, * 1335. 1374.
and 897, we light upon this Rule.

If a falling Body ſtrikes againſt a fixed Obſtacle; the Time of the Fall 1398.
will be to the Time, in which the Parts yield inwards, in a Ratio compounded of the Ratio of the Height, from which the Body fell, to the Height of the Segments preſſed inwards, and of the Ratio of the Diameter of the Cirle to the Quarter of its Circumference.

If we conſider a Ball, the Square of the Semi-Diameter of the Spot, is equal to the Product of the Height of the Segment by the Diameter of the Ball *. * 1377.

1399. Therefore *in a Ball, the said Times are to one another in a Ratio compounded of the Product of the Height, from which the Body fell, by the Diameter of the Ball, to the Square of the Semi-Diameter of the Spot, and the Ratio of the Diameter of the Circle to the Quarter of its Circumference.*

1400. We let down a Ball, whose Diameter was 1,585 Inch, from an Height of a Foot and an half; and the Diameter of the Spot was 0,15 Inch. We now suppose the Time of the Fall from the Height of a Foot and a half to be given 0″,31; making the Computation, we discover the Time of the Inflection to have been 0″,000048, which is equal to ten fifth Minutes and an half, or $\frac{1}{20832}$ of a Second.

1401. The Time mentioned in N. 1133. was determined in the same manner; the Hemisphere was also let down from the Height of a Foot and an half; the Diameter of the Sphere was 2,17 Inches; and lastly, the Semi-Diameter of the Spot was 0,0825 Inch.

1402. One Experiment is sufficient to determine the Times, as long as we treat of the same Elasticity, and of spherical Figures.

Let a be the Height from which a Body is let down; M the Mass of the Body; D the Diameter of the Sphere; d the Diameter of the Spot; the Ratio of the Diameter of the Circle to its Circumference that of m to n; T the Time of the Fall from the Height a; and lastly t the Time in which the elastic Parts are inflected.

The last Rule gives us this Proportion, $T : t :: Dam : \frac{1}{16} ddn$; whence $t = \frac{Tddn}{16Dam}$ and $tt = \frac{T^2 d^4 n^2}{256 D^2 a^2 m^2}$. As we now treat only about discovering the Proportion, we reject all the constant Quantities, and tt follows the Proportion of $\frac{T^2 d^4}{D^2 a^2}$; instead of T^2 I put a, because these Quantities are proportional *; and as d^4 follows the Proportion of the Square of the Spot, instead of this Quantity I write this other MDa †; putting a for the Square of the Velocity ‖; and I have $\frac{MDaa}{D^2 aa} = \frac{M}{D}$,

* 374.
† 1393.
‖ 374.

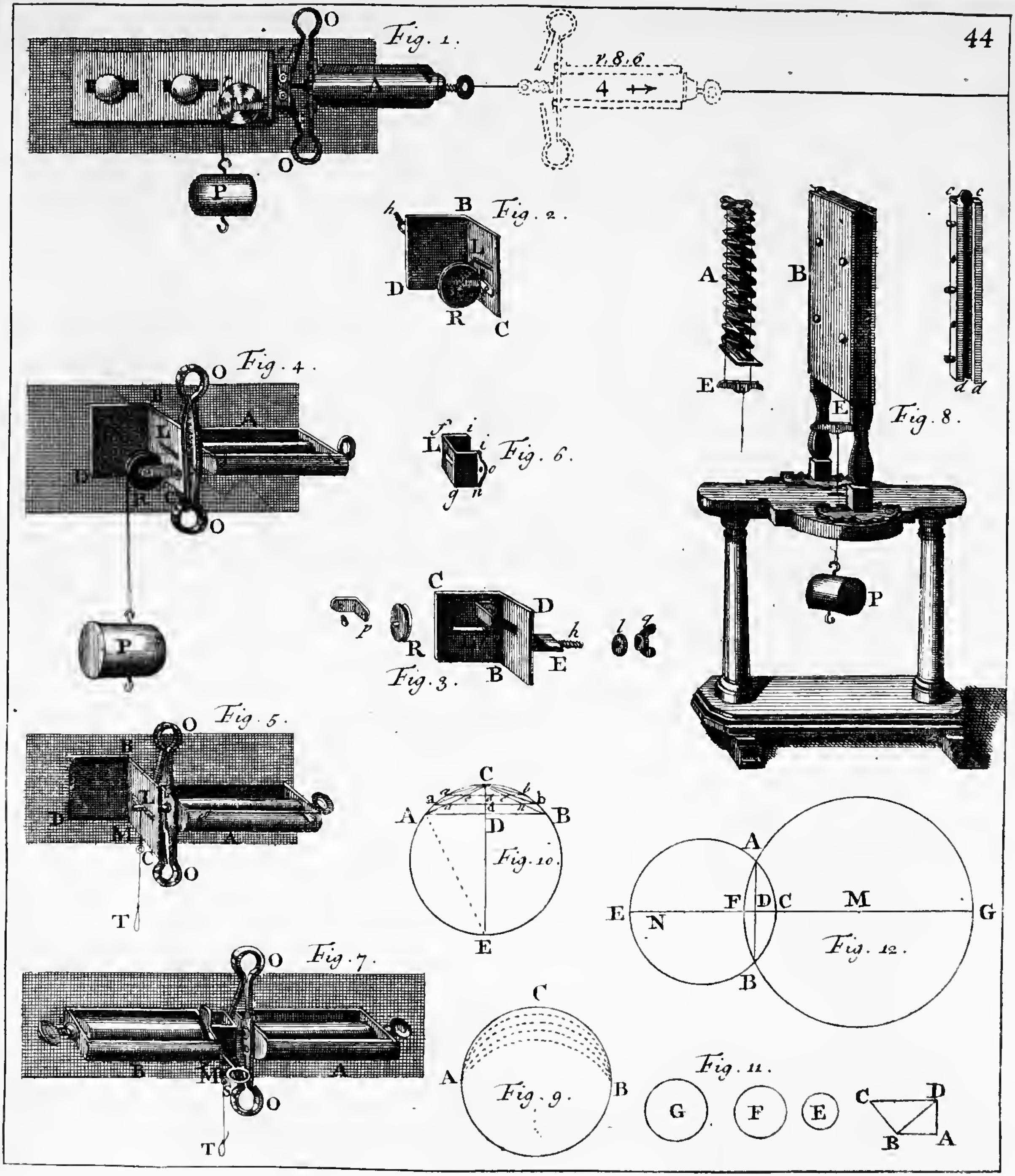
1403. and it appears that *the Square of the Time of the Inflection of the elastic Parts follows the direct Ratio of the Mass, and the inverse Ratio of the Diameter of the Ball, whatsoever the Velocity be.*

1404. *If we consider spherical Bodies*, the Mass is as the Cube of the Diameter *, and *the Time is as the Diameter itself.*

* 18. El. 12.

1405. In what has been explained hitherto, we have only considered the Inflection of one Body; and supposed it to impinge against the plane Surface of an immoveable Body: but in every Impaction both Bodies yield inwards, unless the Cohesion of Parts in one very much exceeds the Cohesion in the other; we supposed the parts of the plane Body to cohere in this manner, and for this Reason we used, in the Experiment, a Stone very hard, if compared with Ivory.

The


44
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.

The Demonstrations would obtain also, if the plane Obstacle should have the same Elasticity as the Ball; for the Sum of the Inflections would be equal to the Segment of a Sphere whose Base the Spot itself should be.

But *if we suppose the fixed Obstacle to be terminated by a spherical Figure also*, the Difficulty will not be greater. The Inflection will be made according to the same Laws; in this Case two Segments are pressed inwards; which during the whole Action have continually equal Diameters; and the Heights in this Encrease keep the same Ratio, which is inverse of the Diameters *: and this Introcession in respect of the Height only of a Segment differs from the Introcession of one Sphere; and the Demonstrations of the Time † may be applied here, if instead of the Height of the Segment we apply the Sum of the Heights, that is, F C. 1406. * 1391. † 1397. Plate XLIV. Fig. 12.

We have seen, by reason of the very small Segments, $\frac{AD^q}{FG} = FD$, and $\frac{AD^q}{CE} = DC$*; therefore $FC^q = \frac{AD^q \times \overline{FG+CE}}{FG \times CE}$; whence it follows, that the Time of the Introcession of the Parts is determined, if in the Rule N. 1398, instead of the Height of the Segment, we put the Value of this; then we have a Rule which differs from N. 1399. in this only, that instead of the Diameter of the Ball we now use the Product of the Diameters divided by their Sum. Whence it follows also, that there is the same Change required in N. 1403. which will give us this Rule: *The Squares of the Times of the Inflections follow the direct Ratio of the Masses and the Sum of the Diameters, and the inverse Ratio of the Product of the Diameters.* * 1390.

We may apply this to the Collision of two Bodies, striking against one another, which are terminated by spherical Figures; only instead of the Mass we must substitute the Product of the Masses divided by their Sum *. 1407. * 1027.

In this Case the Time itself of the Inflection also, the double of which is the whole Time of the Collision, is determined; for the Square of the Time discovered in N. 1400. is to the Square of the Time sought, as the Weight of the Ball there used, divided by the Diameter, is to the Sum of the Diameters of the concurring Bodies, multiplied by the Product of the Masses, and divided by the Product of the Sum of the Masses, multiplied by the Product of the Diameters. 1408.

The End of the Second Book.

Mathematical Elements OF NATURAL PHILOSOPHY CONFIRM'D BY EXPERIMENTS.

BOOK III.

PART I. Of the Gravity and Preſſure of Fluids.

CHAP. I.

Of the Gravity of the Parts of Fluids, and its Effect in the Fluids themſelves.

A Fluid is a Body whoſe Parts yield to any Force impreſſed, and by yielding are very eaſily moved one amongſt ano-
* 68. ther *. Whence it follows, that *Fluidity ariſes from this, that the*
1409. *Parts do not ſtrongly cohere, and that the Motion is not hindered by any Inequality in the Surface of the Parts,* as happens in Powders.
1410. But *the Particles, of which Fluids conſiſt, are of the ſame Nature with the Particles of other Bodies, and have the ſame Properties*; for Fluids are often converted into Solids, when there is a more ſtrong Coheſion of them, as in Ice. On the contrary, melted Metals give us an Inſtance of a Solid changed into a Fluid.

Fluids agree in this with ſolid Bodies, viz. *That they conſiſt of heavy Particles, and have their Gravity proportionable to their Quantity of Matter, in any Poſition of the Parts.* If in the Fluid itſelf that Gravity be not ſenſible, it is owing to this, that the

2 lower

lower Parts ſuſtain the upper, and hinder them from deſcending: But it does not follow from thence, that the Gravity is taken away; becauſe a Fluid contained in a Veſſel will preſs down the End of a Balance, which carries the Veſſel, in proportion to its Quantity. The following Experiment will alſo ſhew, that the Gravity is preſerved in any part of the Fluid.

EXPERIMENT 1.

Immerge in Water the Phial A cloſe ſhut, and hanging by a Horſe-hair, held in the Hand; if the Phial be opened, whilſt it is immerſed, the Water, which enters into it, encreaſes its Weight very much; tho' it has a Communication with the external Water. 1412. Plate XLV. Fig. 1.

From this Gravity it follows, that *the Surface of a Fluid contained in a Veſſel, to keep it from flowing out, if it be not preſſed from above, or if it be equally preſſed, will become plain*, or flat, *and parallel to the Horizon*. For, as the Particles yield to any Force impreſſed, they will be moved by Gravity, 'till at laſt none of them can deſcend any lower. 1413.

The lower Parts ſuſtain the upper, and *are preſſed* by them; *and this Preſſure is in proportion to the incumbent Matter, that is, to the Height of the Fluid above the Particle that is preſſed*; but, as the upper Surface of the Fluid is parallel to the Horizon*, *all the Points of any Surface, which you may conceive within the Fluid parallel to the Horizon, are equally preſſed*. 1414. * 1413. 1415.

If therefore *in a Part of ſuch a Surface there is a leſs Preſſure than in the other Parts, the Fluid*, which yields to any Impreſſion *there*, will be moved; that is, *will aſcend till the Preſſure becomes equal*. 1416.

EXPERIMENT 2.

Take a Glaſs Tube A, open at both Ends, and ſtopping one End with your Finger, immerſe the other in Water; when the Tube is full of Air, the Water will riſe in it but to a very ſmall Height: If you take away your Finger, that the Air that is compreſſed may go out, the imaginary Surface that you conceive in the Water, juſt at the Bottom of the Tube, and parallel to the Horizon, is leſs preſſed juſt againſt the Hole of the Tube, ſo that the Water will riſe up into the Tube, and not be at reſt, 'till it comes up to the ſame Height with the external Water. 1417. Plate XLV. Fig. 2.

The Preſſure upon the lower *Parts*, which ariſes from the Gravity of the ſuperincumbent Fluid, *exerts itſelf every way, and every way equally*. 1418.

This

1419. This follows from the Nature of a Fluid; for its Parts yield to any Impreſſion, and are eaſily moved; therefore no Drop will remain in its Place, if, whilſt it is preſſed by a ſuperincumbent Fluid, it is not equally preſſed on every Side: but it cannot be moved on account of the neighbouring Drops, which are preſſed in the ſame manner, and with the ſame Force, by the ſuperincumbent Fluid; and therefore the firſt or loweſt Drop is at reſt, and preſſed on all Sides, that is, in all Directions. I affirm alſo, that it is equally preſſed; for the lateral Preſſure is to the vertical, as this is to that; therefore, if, to make an Equilibrium, the vertical Preſſure could differ from the lateral; for example, exceed it, by any Cauſe whatſoever, this ſame Cauſe ſhould alſo make this laſt Preſſure exceed the firſt, by reaſon of the reciprocal, and entirely ſimilar Relation: and as this obtains in any Directions whatſoever, it follows that the Preſſures cannot be unequal.

Experiment 3.

1420. Plate XLV. Fig. 2. Let the Glaſs Tubes B, C, D, be immerſed in Water, in the ſame Manner, as was ſaid of the Tube in the laſt Experiment; and, upon taking away the Finger, the Water will riſe in all the Tubes to the ſame Height as in the Tube A; in A the Preſſure is directed upwards, in B downwards, in C ſidewiſe, and in D obliquely; yet the Preſſure is equal in each. If you pour in a greater Quantity of Fluid into the Veſſel, it will alſo riſe equally in each Tube.

1421. Hence it follows, that *all the Particles of Fluids* are preſſed equally on all Sides, and therefore *are at reſt*; and that they do not continually move among themſelves, as ſeveral have ſuppoſed. If on ſome Occaſions there be ſuch a Motion, this is owing to a peculiar Cauſe.

1422. *In Tubes that have a Communication, whether equal or unequal, whether ſtrait or oblique, a Fluid riſes to the ſame Height*; that is, all the upper Surfaces are in the ſame horizontal Plane: which is eaſily deduced from what has been ſaid.

1423. Pl. XLVII. Fig. 1. Let A be a Veſſel, and B a vertical Tube, and D an inclined Tube; they muſt communicate by means of the Tube C E; let there be a Fluid poured into them, and let *f h g* be a Surface parallel to the Horizon; if the Heights *f i* and *h k* be unequal, the Fluid will aſcend where the Height is leaſt *. For the ſame
* 1416. Reaſon, unleſs the Preſſures at *g* and *h* be equal, the Fluid will not

not be at rest; but we demonstrate these to be equal, when *k* and *p* are in the same horizontal Plane.

Let *v p l*, *s o*, *r n*, *q m*, be horizontal, and *p s*, *x o r*, *t n q*, *l m g*, vertical; the Pressure upon each of these horizontal Surfaces is every where equal *. The Point *s* sustains the Pillar *p s* of the Fluid, *o* is equally pressed, and *r* sustains the Pressure *x r*; after the same Manner it appears that the Pressure upon *q* is *q t*, and that the Point *g* is pressed, as if it sustained the Pillar *g l*. Therefore the Pressures are equal, when *k l* and *v* are in the same horizontal Plane. * 1415.

EXPERIMENT 4.

Pour Water into the Machine represented here, and after any Agitation it will not rest, unless all the Surfaces be in the same horizontal Plane. The Glass Vessel A is joined to the Glass Tubes B and D, by help of the Brass Tube C E. 1424. Plate XLVII. Fig. 2.

All Fluids are not equally heavy, that is, have not the same Quantity of Matter in the same Space; but what has been said will agree to every Fluid. 1425.

When Fluids of different Gravities are contained in the same Vessel, the heaviest lies at the lowest Place, and is pressed by the lighter, and that in proportion to the Height of the lighter. 1426.

Take Water tinged with some Colour, for example red, and pour it into the Glass Vessel A to the Height of *b c*; immerge into it the Glass Tube *d e*; the Water will rise in it to the Height *b c* *. Now pour in Oil of Turpentine, which is a Fluid lighter than Water, and immediately the Water will rise in the Tube; and so much the higher as the Oil is poured in, to a greater Height: yet the Water in the Tube does not rise to the same Height as the Oil in the Vessel; because, since Water is heavier, there is not required the same Height of Water as there would be required of Oil to produce the same Pressure. 1427. Plate XLV. Fig. 3. * 1416.

If you have a mind to try this Experiment with Mercury and Water, you will find a greater Difference in their Heights, by reason of their greater Difference of Gravity. 1428.

EXPERIMENT 6.

Let the End of a Tube be immersed in Water, and pour Oil into it. The Water in the Tube is depressed as far as *d*; and the Height of the Oil *d e* will be greater than the Height of the Water in the Vessel. If the Tube be immersed deeper, the Water 1429. Plate XLV. Fig. 4.

will

will run into it in greater Quantity; if you raise it up, the Water will again go out of it, and the Oil will follow it, if it be raised to such a Height, that the Pressure of the Oil overcomes the Pressure of the Water in the lower part of the Tube.

CHAP. II.

Of the Action of Fluids against the Bottoms, Sides, and Tops, of the Vessels, that contain them.

1430. THE *Bottom and Sides of a Vessel, which contains a Fluid, and the Top also,* when the Fluid is raised above it in a Tube, *are pressed by the Parts of the Fluids which immediately touch them; and* because Re-action is equal to Action *, those Parts all sustain an equal Pressure. But as the Pressure of Fluids is equal every way, the Bottom and Sides are pressed as much as the neighbouring Parts of the Fluid; therefore *this Action encreases, in proportion to the Height of the Fluid* *, and is every way equal at the same Depth, depending altogether upon the Height, and not at all upon the Quantity of the Fluid. Therefore when the Height of the Fluid, and Magnitude of the Surface pressed remain the same, the Action upon this Surface will always be equal, howsoever the Figure of the Vessel is changed. Therefore this is a

1431. general Rule, that *the Pressure, upon any Surface whatever, is equal to the Weight of a Pillar of the Fluid, whose Base the Surface is, and Height, in each of the Points, the vertical Distance of the upper Surface of the Fluid from these Points.*

* 361.

* 1418. 1414.

1432. That such is the Pressure upon the Bottom of a vertical, prismatical Vessel, no one will question; for the Bottom sustains the whole Weight of the Fluid, and that only: but when the Height of the Fluid, and Base of the Vessel remain the same, it follows from what has been demonstrated, that the Pressure upon the Bottom is not altered; although the Vessel, by having its Figure changed, contains more, or less of the Fluid. This agrees with the Experiments, and may, in all of them, be deduced from the Nature of Fluidity, as will more plainly appear, after the Experiments are explained.

A Machine,

By which Experiments concerning the Preſſure of Fluids are made.

Take the hollow Cylinder A, open at both Ends, and well poliſhed within; whoſe Diameter and Height alſo ſomewhat exceed three Inches and an half, and in which the Water, when three Inches high, weighs one Pound. 1433. Plate XLV. Fig. 5.

The Ring I is joined to it by a Screw, that it may be ſuſtained by a Trevet. But the Feet are faſtened to the Ring by Screws, that they may be taken away upon Occaſion. One of the Feet is repreſented by itſelf at L; this has its upper Part ſo bent, that it may be removed from the Ring I, when the Part *p o*, of its upper End, is joined to the under Surface of the Ring. This Part is dove-tail'd, that it may ſtick of itſelf, before the Screw *n* is applied, which makes it faſter. 1434.

To the Cylinder A is joined a moveable Braſs Bottom; this conſiſts of the round Plate R, to which is joined the Tail *t s*, which is Cylindrical, and perpendicular to the Center of it. This Cylinder *t s* goes thro' the Plate R, and its lower Part ends in the Screw S. The upper Surface of the Cylinder M, which is half an Inch thick, is applied to the lower Surface of the Plate R, a Leather being put between; Care muſt be taken that the Leather exceeds not the Baſe of it. This Cylinder is hollow, and made thin that it may weigh little. This Cylinder is repreſented open at top in the Figure; then it weighs leſs, and its Edge goes into a Groove, in the Bottom of the Plate R; the Water is hindered from getting in by a Leather; but it will be better to ſolder this Cylinder to the Plate R. The Screw S paſſes freely thro' the Cylinder, to which is applied the Plate O, having in its Center an inſide Screw, anſwering to the Screw S. 1435.

By turning the Screw, the Leather N is faſtened between O and M: this Leather exceeds the Bottom by half an Inch every way, and covers the external Surface of the Cylinder M, when the Bottom is joined to the Cylinder A; and hinders the Water from running out, whilſt the Bottom is moving; which anſwers beſt, when the Leather touches the Surface of the moveable Cylinder.

I make uſe of Calf's-Leather; which muſt be ſoaked in Oil, and after a few Days it muſt be taken out and ſoaked as long in Water; after which Preparation the Leather muſt be well rubbed over with Oil and Water, and moved ſeveral Times up and down the 1436.

the Cylinder, and left in that Condition two or three Days. The Leather being thus prepared, may be ufed in the Experiments many Years; if it be kept in a dry Place. When the Experiments are to be made, the Leather muft be joined to the Bottom, anointed with Oil and Water, and then left in the Cylinder, fome Hours, or rather Days, before the Machine is ufed. And immediately before the Experiments, it muft be anointed again with Oil and Water; then the Bottom is eafily moved, and holds Water well: the Motion is alfo helped if the Infide of the Cylinder A be alfo oiled. The Leather made ufe of, muft be neither too thin, nor too thick; which muft be left to the Judgment of the Workman.

1437. The Tail *t s* directs the Motion of the Bottom; for it goes thro' the Hole *m* in the Plate B, which is put upon the great Cylinder A, and fticks faft in Holes cut in the Edge of it; the Tail muft be oiled. In the upper Part of it there is a Hole at *s*, that the Brafs Chain T may be joined to the Bottom, by means of the Hook *v*, which Chain goes thro' the Tube F, which we fhall fpeak of prefently; that the Bottom may, by help of this Chain, be faftened to the Arm of a Balance.

1438. The Cylinder A is covered by the Top C, which has a Screw; and to hinder the Water from running out, the Leather Ring G is put between, which is preffed tight, by help of the Screw, whereby the Top is joined to the Cylinder; the Cover has a fquare, dove-tail'd Plate *e*, and to the Cylinder A is joined the Handle *h*, that by means of the Key E, and the Pin H, the Cylinder may be opened, and fhut more conveniently. The Top has a Hole in the Middle of it; and the hollow Cylinder D, having a Screw on its Outfide, is joined to it; that the Tube F may be joined to the Machine, and the Water is hindered from running out here alfo, by applying a Leather, which is preffed tight by means of the Key.

1439. In the Experiments, Weights, like P, are put upon the Cylinder A; many are required, two of four Pounds each, two of two Pounds, two of one Pound, as many of half a Pound, and of a quarter alfo. Thefe Weights are cut in fidewife, that the Tube F may go into them.

1440. But to put thefe Weights upon the Top C conveniently, the wooden Ring Q is firft laid upon it; which is reprefented inverted; this alfo has a lateral Incifion at *q*, whereby it receives the

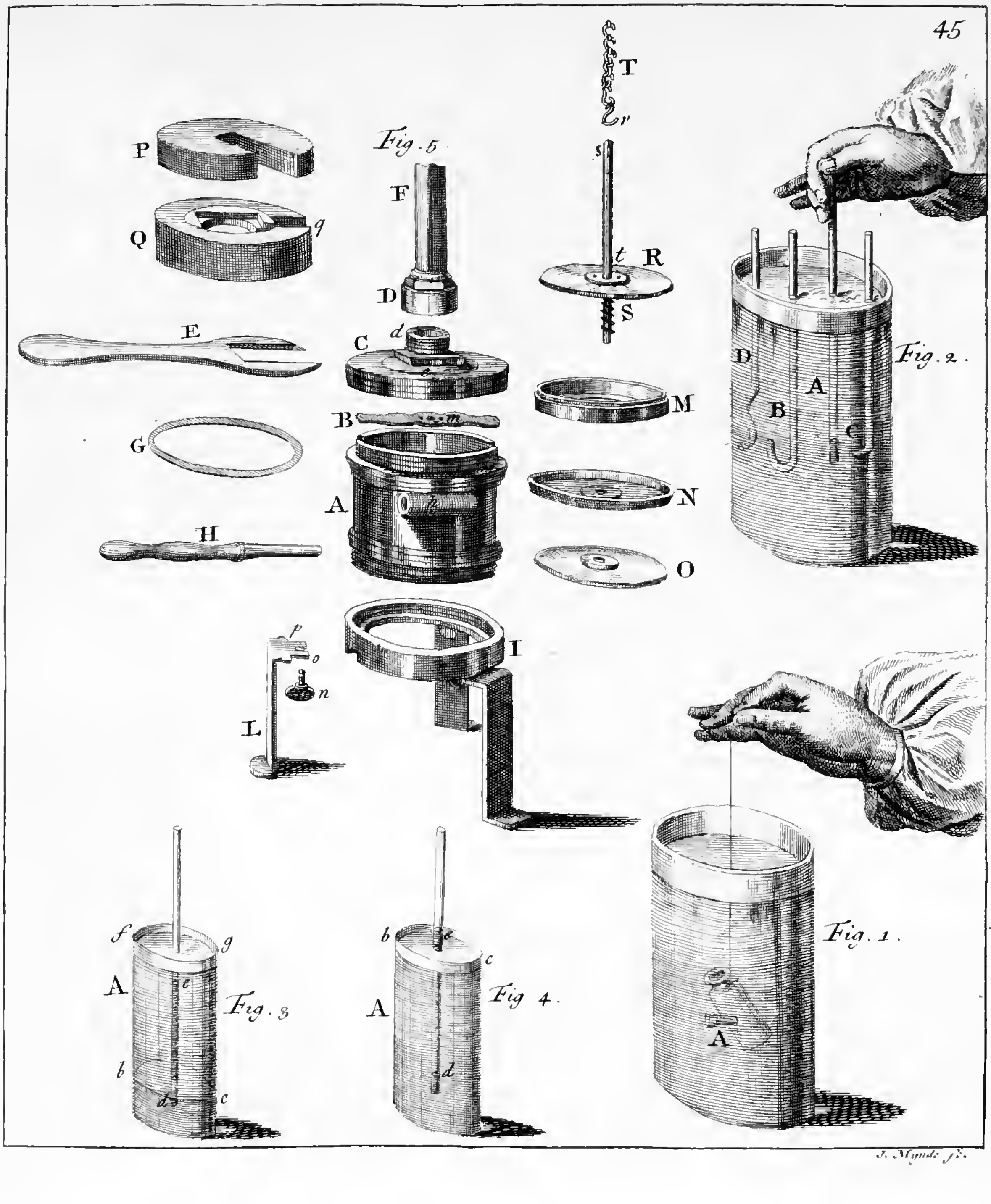


Fig. 5
P
Q
q
F
D
T
v
s
t
R
S
E
C
d
e
B
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A
k
H
M
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A
B
C
Fig. 2.
p
o
n
L
I
f
g
A
e
Fig. 3
b
d
c
b
c
A
Fig 4.
d
Fig. 1.
A

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the Tube F; then it is thruſt down, and is ſo hollowed, that the Cylinder D, and the Plate *e* may go into it.

EXPERIMENT I.

Three Staves, ſpread at Bottom, ſuſtain the wooden Head I. 1441.
Theſe are cut in ſuch manner, that, whilſt they ſupport the Head, Pl. XLVI.
their Ends are applied to it ſidewiſe alſo; the Staves are faſtened Fig 1.
to the Head by Joints, that, when the Machine is to be removed, they may be eaſily brought together. The Head I conſiſts of two Parts, the lower being Hexagonical, and the upper O Spherical. The Iron Screw V C goes thro' this Head, which is bent at the Bottom, into the Hook V, on which a Balance is hung, which is eaſily raiſed to the Height, required in the Experiment, and which is plainly ſhewn in the Figure, by turning the outward Screw D, which is alſo of Iron.

The Parts of the Machine, explained above *, are joined as is 1442.
ſaid. The Chain, which is faſtened to the moveable Bottom †, * 1433.
is hung upon the Arm of a Balance Beam at G; and this Bottom † 1437.
occupies the Middle of the Height of the Cylinder A, when the Beam is horizontal. A Weight is put into the Scale E, to make it be in Equilibrio with the Weight of the Bottom and the Chain; we muſt by Trials ſettle the Weight required for this. To the
upper End of the Tube F *, which is thirty-two Inches long, * 1438.
there is joined by a Screw a Cylindrick Funnel, whoſe Diameter is about nine Inches, and Height four.

The Beam being horizontal, pour Water into the Tube F, ſo that it may riſe into the Funnel; and be three Feet above the Bottom of the Cylinder; which Height is to be obſerved by a coloured Circle, within the Funnel. We muſt add a Weight of twelve Pounds to the Weight, which was put into the Scale E before, and there will be an Equilibrium; that is, when you put the Beam into an horizontal Situation, and leave it to itſelf, it will remain ſo. If you diminiſh or encreaſe the Weight, the Bottom aſcends, or deſcends. But the Weight muſt be encreaſed, or diminiſhed, by half a Pound at leaſt, by reaſon of the Friction of the Bottom; there is often required a greater Difference. This depends upon the Friction, which may be diminiſhed by moving the Bottom upwards and downwards, before it is left to itſelf in an horizontal Situation.

The Height of the upper Surface of the Water above the Bottom, is, in this Experiment, as was ſaid, three Feet. The Weight

of a Pillar of Water, which has this Height, and whoſe Baſe is
* 1433. equal to the Bottom of the Cylinder, is two Pounds *; and the Experiment ſhews that the Preſſure of the Water againſt the Bottom is equal to this Weight, tho' it be preſſed by a ſmall Quantity of Water only.

When we conſider the Motion of the Bottom only, we muſt faſten the Machine, that it may not be lifted up altogether, and
* 1439.1440. this is done by putting the Weights P, P, upon it *.

Experiment 2.

1443. Pl. XLVI. Fig. 2. Taking away the Top with the Tube, the Cylinder A muſt be joined to the inverſe, truncated Cone N, which has towards its Bottom the Ring C with a Screw in it, whereby it is joined to the
* 1438. Cylinder A, as the Top was *.

Water muſt be poured into this Machine to the ſame Height above the Bottom, as in the laſt Experiment. As for what remains, the Experiment is performed after the ſame Manner as the other was, and the Event is the ſame; and the Preſſure, when there is the ſame Height of Water, is not altered by change of the Veſſel, and Quantity of Water. The Height of the Water is obſerved within the Veſſel.

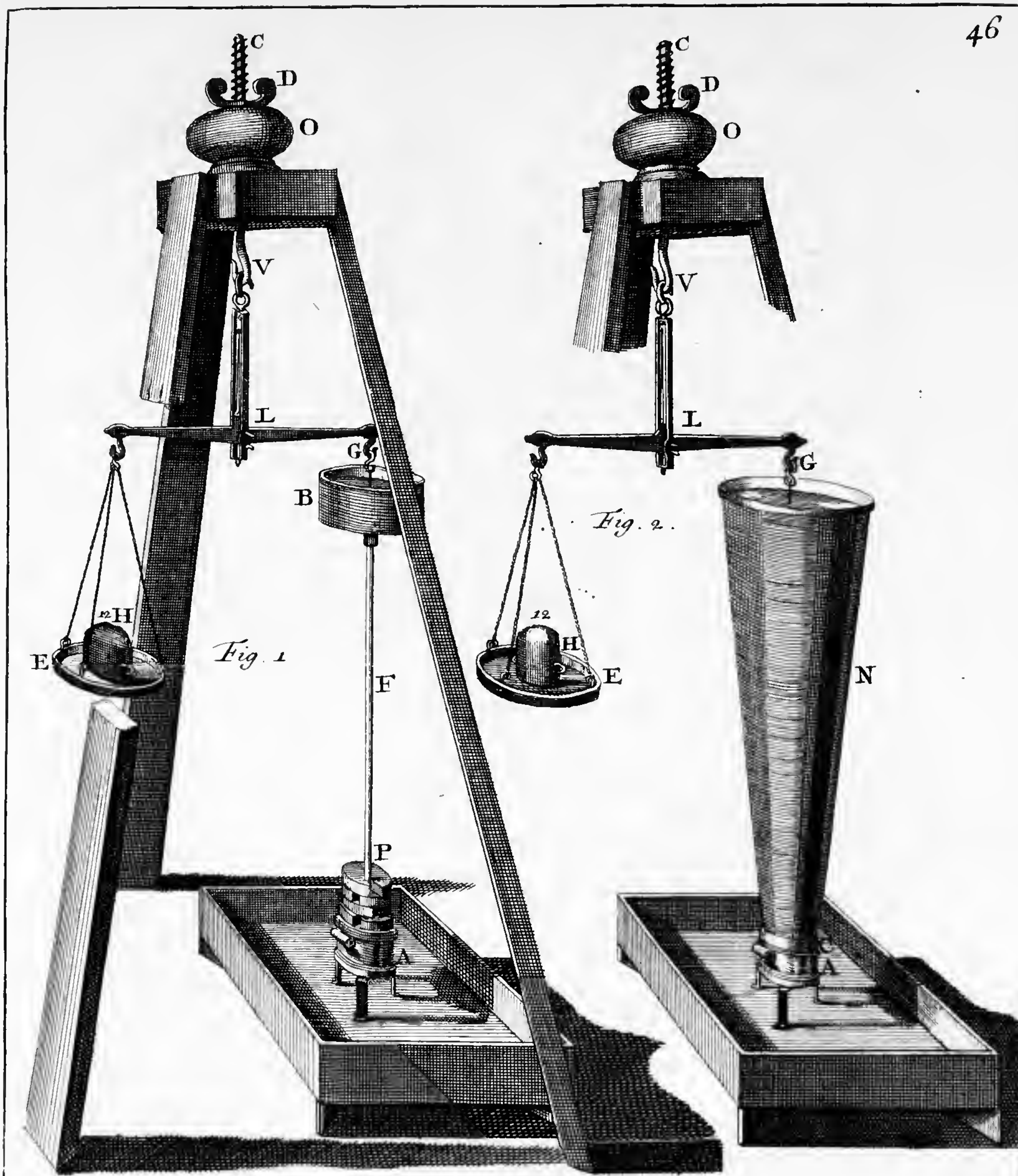
Experiment 3.

1444. Pl. XLVIII. Fig. 1. * 160. The Pillar C *, which has been often made uſe of already, is faſtened upon a Table; the Arm Q with its Hook *v*, is joined to it, the Balance L is hung upon it; whoſe Scale E is at a ſmall Diſtance from the Table, when the Beam is horizontal: upon the other Arm is hung the Glaſs Veſſel A, which has the Copper Ring *e e* round it, that the Handle B may be joined to it.

The Pillar D is alſo faſtened beneath the Table, by a Screw which goes thro' it.

This Pillar, by help of the Arm H, ſuſtains the wooden Cylinder G, which goes into the Veſſel A; but ſo, that it touches neither the Sides, nor the Bottom of the ſaid Veſſel, when the Balance is in Equilibrio. If there be poured Water into the Veſſel A, to any Height, ſo as to make an Equilibrium with the Weight P, put into the Scale E; this will be the Weight of all the Water, which would be contained in the Veſſel, the Cylinder being taken away, when the Height ſhould be the ſame as it was, before the Cylinder was taken away; and a ſmall Quantity of Water, whoſe upper Surface is raiſed, whereby the Preſſure upon the Bottom is encreaſed, ſuſtains a great Weight.

It



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It may be seen, by making use of the following Machine, that the lateral Pressure is equal to the Vertical.

A MACHINE,

Whereby the lateral Pressure of Fluids is shewn.

The Vessel D B is a wooden Parallelopiped, about three Feet 1446.
and an half high; in the lower Part, towards the Bottom, there Pl. XLVII.
is a lateral Aperture, where a Brass Ring, having a Screw, is fa- Fig. 3.
sten'd, that the Cylinder A, mention'd above *, may be fasten'd * 1433.
to it, when a Leather Ring is first put between; and the Feet are removov'd, which are screw'd to the lower Ring †. The Motion † 1434.
of the Bottom in the Cylinder is now horizontal. Two wooden Rulers are join'd to this Machine sidewise, one of which is represented at G H; the Ruler C C moves horizontally over these, which has a Piece in its Middle at F prominent sidewise; that the Bottom of the Cylinder may be thrust in by the Motion of it, which the Ruler presses a little below the Center. The Strings C *e*, C *e*, are fasten'd to this Ruler at C and C; these are stretch'd along the Rulers, G, H, and pass over the Pulleys T, T; at the Ends of these Rulers, as T, T, the Weights P, P, are hung upon them.

EXPERIMENT 4.

Pour Water into the Vessel B D so, that the Surface of the Wa- 1447.
ter may be rais'd three Feet above the Line, in which the Bottom is press'd. If each of the Weights P, P, be six Pounds, so as to be equal to twelve Pounds together, the Pressure of the Water will sustain the Weights; and the Bottom may be thrust in, and drawn out, with the same Ease.

The following Experiment proves the Force, with which Water 1448.
presses upwards, to be equal to that, with which it presses downwards, and sidewise.

EXPERIMENT 5.

In the middle of the upper Surface of the Stand E, there is a 1449.
Cylinder of about two Inches Diameter, upon which is plac'd the Pl. XLVII.
moveable Bottom of the Cylinder A so often mention'd *, so that Fig. 4. * 1413.
the Cylinder itself may be mov'd, the Bottom remaining; this is cover'd by its Top, and there is join'd to it the Tube F three Feet and an half long; to the Top of which is added the Funnel B, whose

whose Diameter is equal to the Diameter of the Cylinder A. Pour in Water, till it rises to any height in the Funnel. The Machine
*1439. 1440. is rais'd, the Bottom remaining; put the Weights P, P, P *, upon the Top, which together are equal to nine Pounds; these are sustain'd, together with the Weight of the whole Machine, by the Water in the Tube; but the Weight of the Machine, together with that of the Tubes and Funnel, wants but little of six Pounds.

The Force acting against the Top, is equal to the Weight of a Pillar of Water, whose Base the Top is, exclusive of the Hole to which the Tube corresponds, and whose Height is the Height of
* 1431. the Water above the inward Surface of the Top *; this Height is three Feet and an half, for we don't regard the Water in the Funnel; for by reason of the equal Diameters of the Cylinder A, and the Funnel, the Weight of this Water is exactly equal to the Action, which it exerts against the Top, whether its Quantity be greater, or less.

1450. If the Diameter of the Cylinder be encreas'd, the same Tube remaining, the Action against the Top will encrease in the same Ratio with the Top, so that a very great Weight may be supported, and even rais'd, by a very small quantity of Water.

Hydrostatical Bellows.

1451. Pl. XLVII. Fig. 5. Take two round Boards A B, A B, of 15 Inches Diameter, and join them together with a piece of Leather, so that they may make a cylindric Vessel somewhat like a Pair of Bellows, so that it may contain Water.

There is a Hole *l* in the upper Board, to which is fix'd a Brass Cylinder, cohering with a round Plate, and that has a Screw, whereby the Tube F is fix'd to the Machine which is as long as the Tube used in the former Experiment.

Experiment 6.

1452. Pour Water into this Bellows thro' the Tube, and the Water in the Tube will sustain the Weights P, P, P, P, P, all which together weigh 300 Pounds. These make the Water rise into the Funnel, but the Height of the Water in the Funnel is but small. The Weights will even be rais'd by continuing to pour Water into the Tube.

1453. Though these are Paradoxes, they follow from the Nature of Fluidity; every Drop which is at rest, endeavours to recede every
* 1418. Way with equal Force *; if therefore it be press'd on one side, it will

will press towards that part, because Action and Re-action are equal; and with that very Force it will endeavour to recede every Way. In the first Experiment, the Water which touches the Bottom, and corresponds with the Tube, sustains the Weight of the Column of Water contain'd in the Tube, and reaching quite to the Bottom; with this Force it presses the Bottom, and the Water next to it also, which, as it can't run out, acts upon the Bottom, and the Water next to it, with the same Force; which may be applied to the Water next to this also: wherefore, in all parts of the Bottom there is a Pressure equal to the Pressure in the place upon which the Water in the Tube act; and therefore the Bottom in this Case is as much press'd as if a Pillar of Water, of the same Height as the Water in the Tube, and of a Base equal to the Bottom, should lie upon it.

The fifth and sixth Experiments are illustrated by the same Reasoning. 1454.
For it is manifest that each of the Points of the Top is press'd upwards by the Water with that Force, with which the Water, which is in the Aperture of the Top, is press'd downwards by the superiour Water, which fills the Tube.

In the second Experiment, suppose that the Cylinder A should 1455.
be continued, so as to reach up to the Surface of the Water; by this means the external Water would be separated from the Water contain'd in this Cylinder, and then no Water but this interior Water wou'd press the Bottom, and the Bottom wou'd sustain it all. The Water in the Cylinder presses the Sides of the Cylinder, and the external Water presses the external Surface of the Cylinder, and the outward Surface is press'd in the same manner as the inward, and the Pressures against opposite Points are equal; so that if this Surface was taken away, these Pressures wou'd destroy one another; therefore it is no matter, whether there be such a Surface or not, so that taking it away (that is, taking away the Continuation of the Cylinder) the Action against the Bottom is no way alter'd.

In the third Experiment the Weight in the Scale, is not only 1456.
sustain'd by the Water in the Vessel, but also by the Action of the lower Surface of the Cylinder G against the Water; which Action is equal to the Action of the Water against this Surface, against which the Water presses in the same manner, as it acts against the Top in the 5th Experiment.

It is easily deduc'd from the Equality of the Action of a Fluid 1457.
towards every part, that the lateral Pressure, such as is shewn in Experiment

Experiment 4. is equal to that, which is directed upwards, or downwards; therefore it is manifeſt, that it by no means depends upon the Largeneſs of the Veſſel; and if we ſet aſide the Agitation, the whole Sea might be reſtrain'd with as much Eaſe as a Rivulet, if the Height of the Water ſhou'd be the ſame.

1458. Tho' all that has been ſaid depends upon the Weight of Fluids, their Actions muſt be diſtinguiſh'd from their Weight, which laſt
* 1411. is always proportionable to the Quantity of Matter *.

CHAP. III.

Of Solids immerſed in Fluids.

* 156. IT follows from what has been ſaid before *, that the different Gravity of Bodies, whether Solids or Fluids, ariſes from this, that they contain a greater or leſs Quantity of Matter in an equal Space.

Definition 1.

1459. *The Quantity of Matter in a Body being conſider'd in relation to its Bulk*, that is, in relation to the Space poſſeſs'd by it, *is call'd the Denſity of the Body.*

A Body is ſaid to have double, or triple, &c. the Denſity of another Body, when, ſuppoſing their Bulks equal, its Quantity of Matter is double, or triple, &c.

Definition 2.

1460. *A Body is ſaid to be Homogeneous, which has all its Parts of the ſame Denſity.* We uſe this Word in this Senſe, and do not regard any other Senſe, which may be given to it. Therefore alſo

Definition 3.

1461. *We call that Body Heterogeneous, whoſe Parts are not all of the ſame Denſity.*

Definition 4.

1462. *The Weight of a Body, conſider'd with relation to its Bulk, is call'd the ſpecifick Gravity of a Body.*

The ſpecifick Gravity is ſaid to be double, when under the ſame Bulk the Weight is double.

Fig. 1.

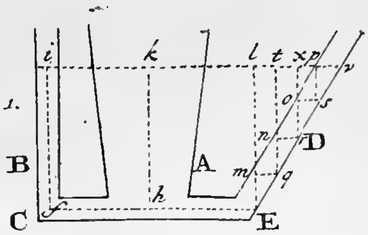


Fig. 2.

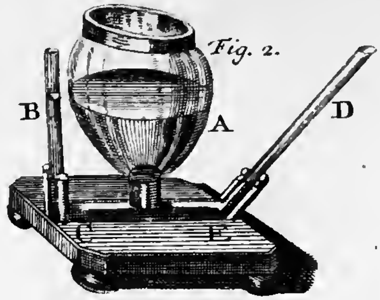


Fig. 5.

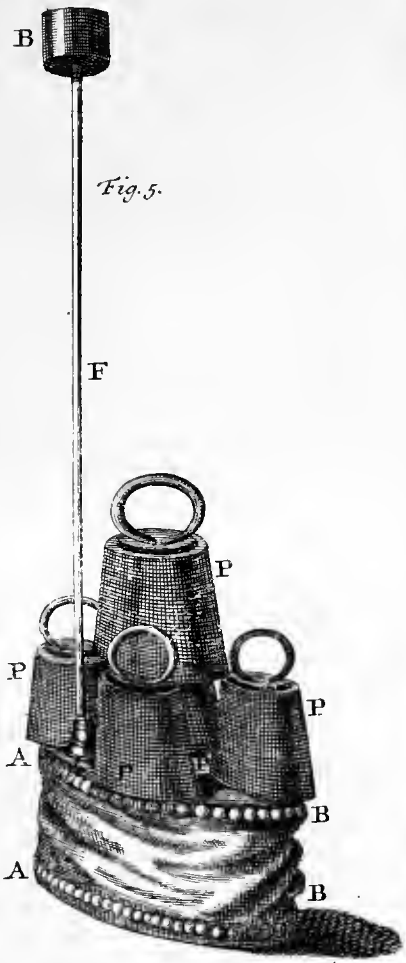


Fig. 4.

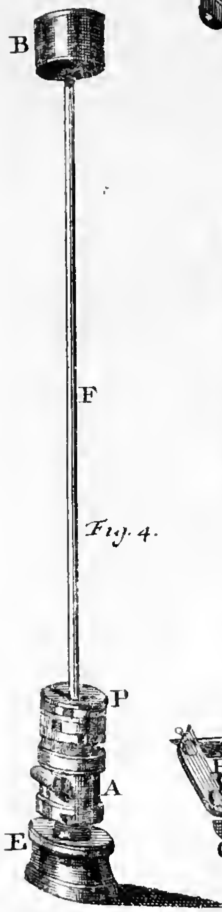
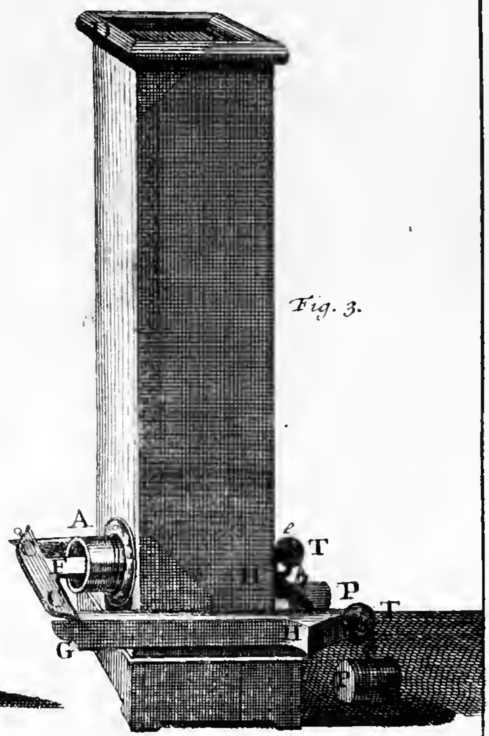


Fig. 3.



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Therefore *the specifick Gravities and Densities of Bodies, in homogeneous Bodies, are in the same Ratio; and they are to one another as the Weights of equal Bodies, in respect to their Bulk.* 1463.

If homogeneous Bodies are of the same Weight, their Bulk will be so much less as their Densities are greater, and under the same Weight the Bulk is diminish'd in the same Ratio in which the Density is encreased; therefore in this Case the Bulks are *inversely as the Densities.* 1464.

Hence we deduce a Method whereby, when in homogeneous Bodies there are given two of the three Ratios, of the Weights, Bulks, and Densities, the third may be discover'd.

The Weights are in a Ratio compounded of the Bulks and Densities. 1465.

The Bulks are directly, as the Weights, and inversely as the Densities. 1466.

Lastly, *The Densities are directly, as the Weights, and inversely as the Bulks.* 1467.

When a Solid is immersed in a Fluid, it is pressed by the Fluid on all Sides, and that Pressure increases in proportion to the Height of the Fluid above the Solid; as follows from what has been said in the foregoing Chapter; and which may also be proved by a direct Experiment. 1468.

Experiment 1.

Tie a Leather Bag S to the End of a Glass Tube B C, and fill it with Mercury; you may also make use of a Bladder; let this Bag be immersed in Water, but so, that the End C of the Tube may be above the Water; by the Pressure of the Water against the Surface of the Bag, the Mercury will rise in the Tube to *m*; and the Ascent of the Mercury follows the Proportion of the Height of the Water above the Bag. 1469. Pl. XLVIII. Fig. 2.

When a Solid is immersed in a Fluid to a great Depth, the Pressure against the upper Part differs very little from the Pressure against the under part; whence *Bodies very deeply immersed, are, as it were, equally pressed on all Sides:* But *a Pressure, which is equal on all Sides, may be sustained by soft Bodies, without any Change of Figure, and by very brittle Bodies, without their breaking.* 1470. 1471.

Experiment 2.

Take a Piece of soft Wax of an irregular Figure, and an Egg, and inclose them in a Bladder full of Water; the Bladder being exactly 1472. Pl. XLVIII. Fig. 3.

exactly shut must be put into a Brass Box A; we make use of the
* 1433. Cylinder mention'd before *, with the Top joined to it, but we
take away the moveable Bottom, and the Ring to which the Feet
† 1440. are fastened, and we place it inverted upon the wooden Ring †.
The Cylinder is covered by the wooden Top represented separately
at B, *(Fig. 4.)* but so, that the Top may bear upon the Bladder;
the Weight P, of an Hundred, or an Hundred and Fifty Pounds,
or even a greater Weight, is placed upon it, which neither breaks
the Egg, nor any way alters the Figure of the Wax.

1473. *Even the Figure of a Drop of any Fluid cannot be alter'd by the*
Pl. XLVIII. *Pressure of another Fluid, when it is equal on all Sides.* Let there
Fig. 5. be a Drop A of an irregular Figure, which is equally pressed on
1474. all Sides by another Fluid. *The Direction of the Pressure, in all
Points, is perpendicular to the Surface*; which if it be denied, we
* 319. must resolve the Pressure into two *, one of which acts perpendicular to the Surface of it, the other in a Direction parallel to it; which last does not act upon the Surface, and the Drop is press'd by that only, whose Direction is perpendicular to its Surface. Let the Point B be press'd; the small Drop press'd presses every way
* 1418. with equal Force *; and each of the smaller Drops, being press'd, press in the same manner; so that the Pressure immediately spreads thro' the whole Drop given; and a Particle as D, which in the Drop is equally press'd on all Sides, endeavours to recede along D E, with the Force with which it is press'd, that is, with the Force with which the Particle B is press'd outwardly: but we suppose the Particle D to be press'd along E D with an equal Force; therefore it cannot be moved. The same Demonstration may be applied to the Point F, and to any other Point whatsoever of the Surface; wherefore there can be no Motion in the Drop.

1475. *A Solid specifically heavier than a Fluid, being immersed in a Fluid to any Depth*, is driven downwards by a Pressure, equal to the Weight of a Pillar, made up of the Body itself, and the superincumbent Fluid. The Weight of a like Column, but which consists wholly of a Fluid, is the Force by which the Body is press'd
* 1431. 1468. upwards by the Fluid *. But when the Body is specifically heavier than the Fluid, this Force is less than that, and is overcome by it, and the Body *descends*.

1476. It is proved by a like way of reasoning, that *a Solid specifically lighter than a Fluid, and immersed in it, must ascend to the highest Surface of the Fluid.*

But

But *suppose a Solid of the same specifick Gravity with the Fluid, it will* neither ascend nor descend, but *remain suspended in the Fluid at any Heighth,* and the Fluid will sustain the whole Body; in which Case, by reason of the Equality of the specifick Gravities, the Fluid sustains a Weight equal to the Weight of the Fluid, which would fill the Space taken up by the Body. But a Fluid acts in the same Manner upon all equal Solids immersed to the same Depth, and will sustain them equally; therefore *every Body immersed loses a Part of its Gravity, equal to the Weight of the Fluid, which would fill the Space taken up by the Body.* 1477. 1478.

The Body does not indeed lose the Part of its Weight, which is sustain'd by the Fluid; but it descends in the Fluid or draws the String, which sustains it, as if it did really lose a Part of its Weight. 1479.

An Hydrostatical Balance.

To the Pillar C * we join the smaller Pillar G, the Ring E being put between †. To this last Pillar we apply the Arm A ‡, which is fasten'd by the Screw F ||. 1480. Plate XLIX. Fig. 1. * 162. † 163. ‡ 170. || 164.

We suspend the Balance *l* by two Strings, to hinder the horizontal Motion of the Beam; and for this purpose, we put the small Ring *i*, upon which the Beam hangs, into the small wooden Ruler B B, and sustain it by the Pin *b*, which goes thro' the Ruler and the Ring.

The Motion of the Ring is hinder'd by another Method also; if it be hung upon two Hooks, fasten'd to the Ruler B B, or a Brass Wire, as is represented in Plate 52. 1481.

The Strings, which sustain the Brass Wire, or the Ruler B B, are parallel, and go round Pulleys fasten'd to the Arm A; thence they are carried downwards, to Pulleys join'd sidewise to the Pillar near the Base, one of which is seen at S; the Strings are put round these also and become horizontal, and are fasten'd to the small wooden Ruler T; and this Ruler itself is join'd to the Hook of a six, or eight Pound Weight P. 1482.

The Beam is raised, or depressed, as you please, by moving this Weight. 1483.

The Scales are suspended by small Chains, instead of Strings; and have Hooks in their Centers underneath; and have three Feet, half an Inch high. 1484.

To the Hooks of the Scales are fasten'd the Brass Wires *a*, *a*; the lower Ends of which are bent into Hooks, such as *c*; but this is ſeen more plainly in Fig. 2. where the ſame Balance is repreſented.

1485. To the Pillar C is join'd the Board H L H, having a Border round it, which may be fix'd at different Heights; upon this Board the Scales are put, and when we would try whether there is an Equilibrium, the Balance muſt be raiſed a little, otherwiſe its Motion would be hinder'd by the Board. This Board has Holes at *m* and *m*; which anſwer to the Hooks of the Scales, and the Braſs Wires *a*, *a* go thro' them.

1486. But it often happens, that the Table, on which the Pillar C is put, is not exactly horizontal; in which Caſe the Holes of the Board do not anſwer exactly to the Hooks; to avoid which Inconveniency, ſome peculiar Things muſt be obſerv'd in the Conſtruction of it. The Arm D O, which ſuſtains the Board, is repreſented by itſelf; the Tail of it goes thro' the Aperture of the Pillar and is

* 167. faſten'd by the Screw O Q, as was ſaid of the other *. This Arm is perforated from *d* to *d*.

The Board H L H alſo, and its lower Surface, is repreſented by itſelf. There is a thin Piece of Wood I, which is moveable along its Surface between two Rulers; this can only be mov'd a little more than an Inch, and may be fix'd in any Part of this ſmall Space; and for this purpoſe a ſmall Braſs Plate *q* is join'd to the Piece of Wood, which has a Slit in it, which the Screw *o*, faſten'd to the Board H L H, goes through; by this Screw the Piece of Wood I is faſten'd. Upon the Middle of this there ſtands at Right Angles to it, and is join'd firmly the Copper Plate *n*, having the Screw *p* join'd to it.

When the Board is applied to the Arm D O, the Plate *n* goes into the Aperture *d d*, in which it may be mov'd a little more than an Inch; but it is faſten'd by the Screw *p*, the Nut *g* coming upon it, and the Plate *h* being put between, that the Wood may receive no Damage.

1487. When the Board is join'd to the Pillar C, if the Screws *o* and *p* are looſen'd a little, it may be remov'd from the Pillar, or mov'd towards it, by the Motion of the Plate *n* in the Slit *d d*; the Board may be mov'd ſidewiſe alſo, by the Motion of the wooden Plate I between the Rulers; and as, in theſe Motions, the Board has a parallel Motion, the Holes are eaſily diſpoſed in ſuch manner, that they may anſwer to the Hooks of the Scales.

Experiment

EXPERIMENT 3.

In this Experiment, we make uſe of the Hydroſtatical Balance, 1488.
juſt deſcribed; we have occaſion moreover for a Copper Cylinder C, Pl. XLVIII. Fig. 6.
nicely work'd, in the Center of whoſe upper Baſe, which could not be repreſented, there is a ſmall Hook. In the Center of the lower Baſe there is a Hole *a*, thro' which ſmall Leaden Balls are put into the Cylinder, whereby its Weight may be alter'd as you pleaſe; this Hole is ſtopp'd with the Screw *b*, whoſe Head goes into the Baſe ſo, as to make a Part of its Surface.

There is alſo a hollow Copper Cylinder E; which is open at 1489.
Top, and has a Handle F, that it may be ſuſpended by the Horſe-hair N. The inſide of it is made very ſmooth and even; and the other Cylinder C fills exactly the Hollow of this Cylinder; but that this Cylinder may not be hinder'd from being put in, and taken out, by the Air, there is a Screw *d* that goes into a Hole in the Center of the Baſe of the Cylinder E, that, when this Screw is remov'd, the Air may go in, and out, freely. A Horſe-hair M is faſten'd to the Screw *d*.

To the Hook of the Scale of the Balance is join'd the Horſe-hair 1490.
faſten'd to the Handle of the hollow Cylinder *, which is in this Pl. XLIX. Fig. 1.
Figure mark'd with the Letter N; and to the Horſe-hair faſten'd * 1489.
to the Bottom of this is join'd the other Cylinder †, repreſented by † 1488.
R. The Weight X is put into the oppoſite Scale, to make an Equilibrium. Then the Balance is raiſed *, and the Glaſs Veſſel * 1483.
V, having Water in it, is moved towards it; the Balance being let down again, and the Body R immerſed, the Equilibrium is deſtroy'd; becauſe part of R is ſuſtain'd by the Water: but the Equilibrium is reſtor'd, if N be fill'd with Water; that is, if ſuch a Quantity of Water be pour'd in, as would fill the Place taken up by R.

DEFINITION 5.

The Weight, which keeps a Body immerſed in a Fluid, is call'd its 1491.
reſpective Gravity.

And *this reſpective Gravity is the Exceſs of the ſpecifick Gravity* 1492.
of a Solid above the ſpecifick Gravity of a Fluid; becauſe a Solid loſes ſo much of its Weight as is equal to the Weight of a Fluid of the ſame Bulk.

Hence it follows that *all equal Solids, tho' of different ſpecifick* 1493.
Gravity, when they are immerſed into the ſame Fluid, loſe an equal Weight *. * 1478.

EXPERIMENT

EXPERIMNET 4.

1494. Alter the Weight of the Cylinder C, by encreasing, or dimi-
* 1488. nishing the Number of Leaden Balls contained in it *; and repeat the last Experiment, and the Event will be the same.

1495. Moreover, from what has been said, it follows, that *however the Densities of unequal Bodies differ among themselves, if they be immersed in the same Fluid, the Weights, lost by them, are in the Ratio of their Bulks.* For the Spaces which they take up in a Fluid, are in that Ratio.

1496. Therefore *Bodies of the same Weight, but of different Densities, lose unequal Parts of their Weight, when they are immersed in the sante Fluid,* because of the Inequality of their Bulks.

EXPERIMENT 5.

1497. Take two small thin Plates, of the same Weight, one of Tin S, the
Pl. XLIX. other of Lead P, and hang them by Horse-hairs upon the Hooks
Fig. 2. of the Balance mentioned *, and there will be an Equilibrium.
* 1480. Let down the Balance, that the Bodies may be immersed in the Water, contained in the Glasses V and V, and the Equilibrium will be destroyed.

1498. *The same Solid, immersed in Fluids of different Densities, loses a*
* 1478. *different Part of its Weight* *; therefore when two Bodies, of the same Density and Weight, are immersed in Fluids of different Densities, the Equilibrium between them is destroyed.

EXPERIMENT 6.

1499. This Experiment is made after the same Manner as the foregoing one; but the two Plates are of Lead. Which, if they are both immersed in Water, remain in Equilibrio; but the Equilibrium is destroyed, if whilst one is put into Water, the other is put into Spirit of Wine.

1500. When a Solid, specifically heavier than a Fluid, is suspended
* 1468. in it, this acts against that, on all sides, according to its Height *;
† 361. and the Solid re-acts equally against the Fluid †; therefore these Actions are the same, as if the Space, taken up by the Solid, were
1501. filled by the Fluid; so that *it matters not, in respect of the Gravity of the Fluid, whether a Solid specifically heavier than the Fluid be suspended in it, or a Quantity of the same Fluid be poured into it, of equal Bulk with the Solid.*

EXPERI-

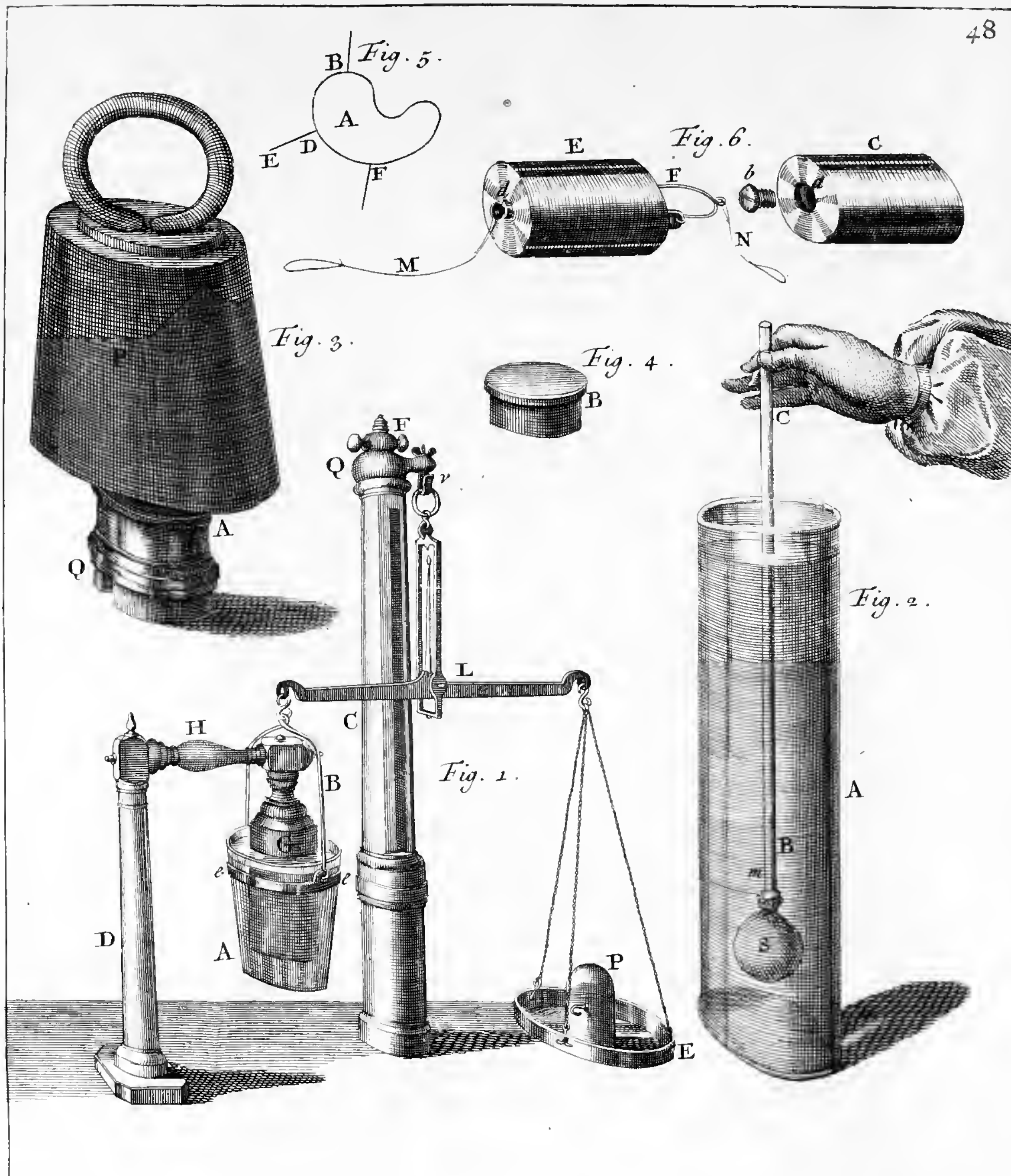
Fig. 5.
B
A
E
D
F
Fig. 6.
E
F
C
b
a
d
M
N
Fig. 3.
P
A
Q
Fig. 4.
B
Fig. 2.
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Fig. 1.
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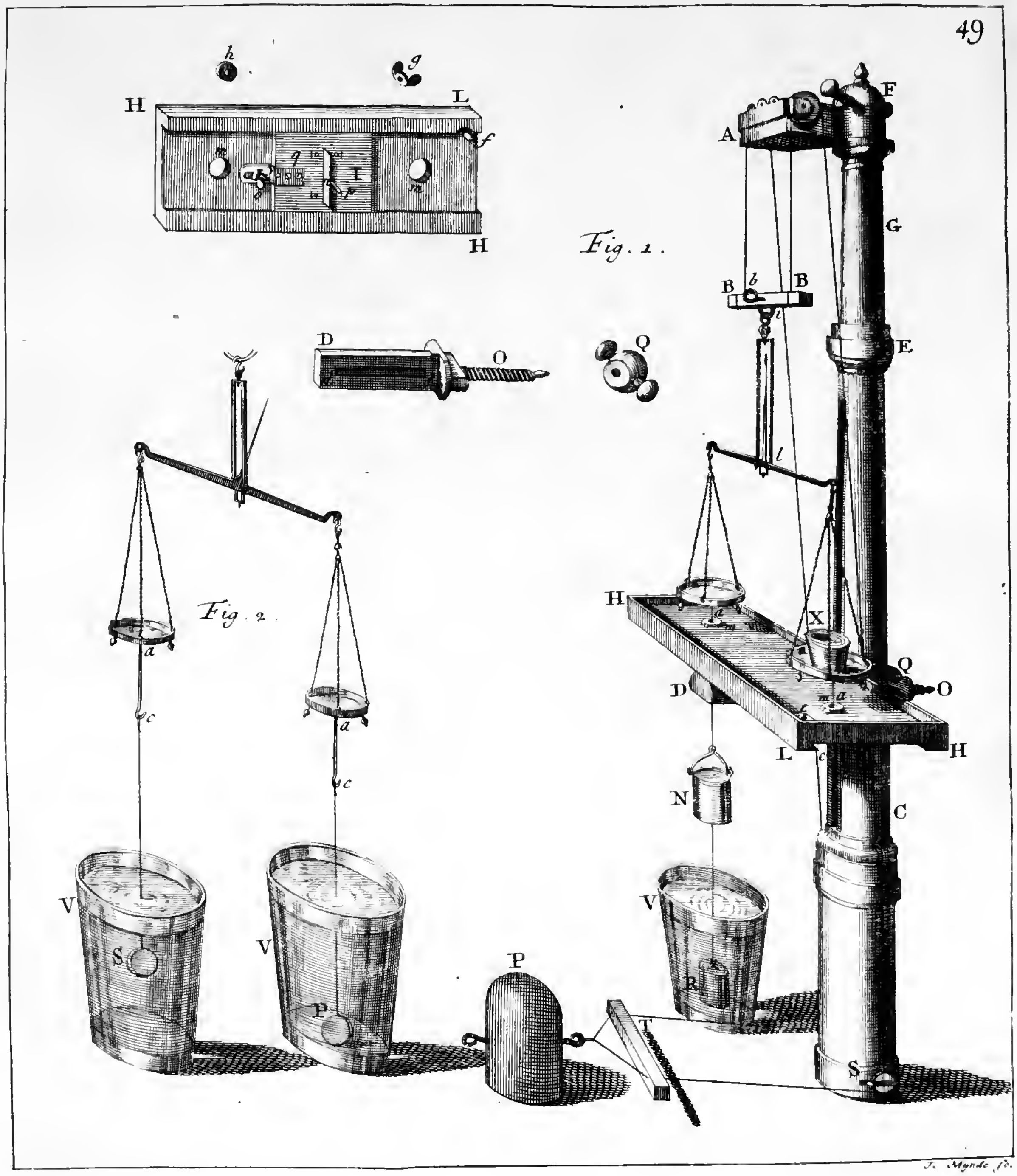

J. Mynde sc.

EXPERIMENT 7.

The Glaſs Veſſel V, having a Copper Ring round it, and an Handle, being almoſt filled with Water, is hung upon the Arm L of the Balance, and the Braſs Cylinder is immerſed in it, which is ſuſtained by an Horſe-hair, that it may not touch the Bottom of the Veſſel; an Equilibrium is made, by putting a Weight in the oppoſite Scale. This is deſtroyed when the Cylinder R is taken out, and reſtored, by pouring in Water, to fill the hollow Cylinder N. Theſe are the Cylinders above mentioned *; if the Cylinder R be put into N, it fills it exactly. If when we make the following Experiment, we would demonſtrate this alſo, this may be done conveniently, as we ſhall ſee preſently *. 1502. Pl. L. Fig. 1. * 1488. 1489. * 1505.

By comparing together N. 1478. and 1501. and the 3. and 7. Experiments, by which they are confirmed, it appears that *the Fluid acquires the Weight, which the immerſed Solid loſes.* The Force of Gravity is always proportionable to the Quantity of Matter, and is not changed by the Immerſion of a Solid in a Fluid; wherefore the Sum of the Weights of the Solid and Fluid is the ſame, before and after the Immerſion. 1503.

EXPERIMENT 8.

All Things are diſpoſed as in the third foregoing Experiment; and there is only this Difference; inſtead of the Ring E, the Arm Q * is faſtened between the Pillars C and G; and the hollow Cylinder is not made uſe of; but the Cylinder R is hung upon the Hook *c*, by an Horſe-hair. Upon the Hook *v*, of the Arm Q, we ſuſpend a greater Balance L, which we made uſe of before *; but in this Experiment, and the foregoing one, the Veſſel V is leſs, than in Experiment 3. of Chap. 2. of this Book. The Balance *l* is put in Equilibrio, the Weights *z* and *p* being put into the Scale I; of which *p* is equal to the Weight, which the Body R loſes in Water, and which together ſuſtain the Cylinder R. In the other Balance L, there is alſo an Equilibrium between the Glaſs V, having Water in it, and the oppoſite Scale with the Weight Y. *l* is raiſed, by moving the Weight P *; the Hook *v* is turned in ſuch manner, that the Veſſel V may come under the Cylinder R; to perform which, there is required a different Direction of the Arm Q, according to the different Length of the Beam L, which, before the Arm is fixed, may be varied at pleaſure. Let down the Balance *l*, that the Cylinder R may be immerſed 1504. Pl. L. Fig. 2. * 165. * 1444. * 1483.

merſed in the Water, contained in the Veſſel V; the Equilibrium in both Balances will be deſtroyed, and reſtored again, by removing the Weight *p* from the Scale I, into the Scale of the Arm L.

1505. If we would demonſtrate the foregoing Experiment; we muſt put the Balance *l* upon the Board H E H; then the Cylinder R would hang from the fixed Hook in the Veſſel V.

1506. A Body, ſpecifically heavier than a Fluid, and which deſcends in it, is carried downwards with a greater Force, than it is preſſed
* 1475. upwards, as was explained before *; the Difference of which Forces is the reſpective Gravity of the Body. The firſt Force partly conſiſts of the Weight of the Fluid lying upon the Body; and the Body may be immerſed to ſuch a Depth, that this Weight may be equal to the reſpective Gravity mentioned; if in this Caſe the ſuperincumbent Fluid be taken away, the Body will be ſuſtained by the Preſſure of the Fluid beneath it.

1507. If the Body be immerſed to a greater Depth, and the Fluid be likewiſe hindered from preſſing the upper Surface of the Body, (as the Preſſure, whereby a Body is driven upwards, encreaſes in
* 1468. proportion to the Depth to which it is immerſed *) it will be carried upwards with a greater Force, than it will deſcend by Gravity; wherefore, if it moves freely, it will aſcend.

Experiment 9.

1508. To the Glaſs Cylinder C, open at both Ends, let there be ap-
Pl. LI. F. 1. plied at the Bottom the Copper Plate F, a quarter of an Inch thick; if it be very plane and even, and the Edge of the Cylinder be made ſo ſmooth, that, when it is applied to the Plate, the Water may be kept out, and if the Plate be ſuſtained by a Thread, tied to the Hook *v* in the Center of the Plate, 'till it be immerſed in the Water to the Depth of about three Inches, it will be ſuſtained by the Water; which will appear by letting go the Thread. At a greater Depth, the Plate will ſtick faſter to the Cylinder; at a leſs, it will fall from it.

1509. The Depth, to which the Plate is immerſed, muſt be encreaſed, in proportion to the Thickneſs, and Denſity of it. If for example it were of Gold, the ſpecifick Gravity of Gold is to the ſpecifick Gravity of Water, as 19 is to 1; wherefore the reſpective Gravity
* 1492. of it is to the ſpecifick Gravity of Water, as 18 is to 1 *; therefore the Pillar of Water ſhould exceed the thickneſs of the Gold Plate, in its Height, eighteen Times, that it may be equal to the reſpective Gravity of the Gold; therefore it is requiſite, that the Height

Height of the Water, above the upper Surface of the golden Plate, be equal at leaſt to ſo many Thickneſſes of the Plate; if the Plate be not extended beyond the Baſe of the Cylinder: for if the Plate be greater, the Depth muſt be encreaſed.

EXPERIMENT 10.

The Cylinder A with its moveable Bottom, and covered by its 1510.
Top, and joined to the Tube F, as was before explained *, is im- Pl. L. Fig. 3.
merſed in Water; and the Bottom aſcends, when it is one Foot * 1433.
below the Surface of the Water; altho' by means of a Screw in the Center of the Bottom, this is joined to the Weight P, whereby the Gravity of the Bottom is ſo encreaſed as to exceed two Pounds, and beſides the Weights which are raiſed, the Friction is overcome alſo.

Every Body immerſed loſes ſo much of its Weight, as the Fluid weighs, which would fill the Space taken up by the Body in the Fluid *. Therefore, if *a Body* be *lighter than a Fluid*, and there- * 1478.
fore aſcends, it *will remain on the Surface, and the Part immerſed* 1511.
will be ſuch, that if its Place were filled up by the Fluid, this Fluid would weigh as much as the whole Body.

This may be alſo immediately deduced from N. 1415; for unleſs when the Body is at reſt, its Immerſion be ſuch, the horizontal Surface, which we conceive in the Fluid beneath the Body, is not equally preſſed every where.

The Rule mentioned concerning the Weight loſt * is univerſal; 1512.
a Body lighter than a Fluid is driven upwards, becauſe it loſes * 1478.
more than it has; but if it be retained, it immediately appears that the Action of the Fluid is the ſame as it is againſt a Body ſpecifically heavier than the Fluid; and in this Caſe the Rule may be applied to ſuch a Body.

EXPERIMENT 11.

This Experiment differs little from the firſt Experiment of this 1513.
Chapter. Inſtead of the Copper Cylinder, which fills the Cylin- Pl. L. Fig. 4.
der N, we make uſe of the wooden Cylinder *r*, which is of the ſame Bigneſs, and does alſo exactly fill N, when it is put into it. In the Center of the upper Surface of the wooden Cylinder there is a ſmall perforated, wooden Prominence. The Rectangle A is made of a Braſs Wire, to which is joined the little Ball *b* made of the ſame Metal, that the Weight may be encreaſed. This Rectangle hangs by a Horſe-hair, joined to the Baſe of the Cylinder N, and

and is suspended in Water. The Cylinder *r* is put into N, or into the Scale B; and the Balance *l* is brought to an Equilibrium.

r is taken away and is joined, being inverted, to the Rectangle A, by means of a small Hook at *d*, and which is put into the Hole in the Prominence mentioned of the Base of the wooden Cylinder. The Cylinder acquires the Situation, shewn in the Figure; and the Equilibrium is destroyed; but it is restored, as in Experiment 1 *, if N be filled with Water.

* 1488.

1514. Taking away the Cylinder *r*, before this is joined to A, the Equilibrium is destroyed; but it is restored if only part of N be filled with Water. Then the Bulk of the Water in N, is equal to the Bulk immersed, when *r* floats upon the Surface of the Water. But the Weight of the Water, which N wants of being filled, is equal to the Force, with which *r* is driven upwards, when, being joined to A, it is kept below the Surface of the Water.

1515. Hence it follows, that *the immersed Parts of Bodies, floating upon the Surface of the same Fluid, are to one another, as the Weights of the Bodies.* Therefore if, by superadding a Weight, the Gravity of a Body be changed, the Part immersed is encreased in the same Ratio;
1516. and *the Parts which descend into the Fluid by different Weights, are to one another, as these Weights*

Experiment 12.

1517. Plate LI. Fig. 2. The hollow Ball G, made of a thin Brass Plate, is joined to a Cylinder of the same Metal, which is also hollow, open at Top, and nicely made. The Ball has a Tail *c d* also, opposite to the Cylinder, at the End of which there is a small Weight *d*.

This Machine is lighter than Water, and being left to itself, whilst it floats the Cylinder acquires a vertical Situation. The Height of the Cylinder is divided into Parts, each of which is equal to half an Inch, and the Divisions are shewn by Circles, which are horizontal in the above-mentioned Situation of the Machine.

Small Leaden Balls are put into the Ball G, till the Surface of the Water reaches one of the Circles. Then the Weight is determined which would make the next Division come to the Surface of the Water; if, by putting in Leaden Balls, such a Weight be added again, and we continue to do it again and again in the same Manner, the Machine will descend equally each Time; that is, the Surface of the Water will come to the next Circle.

In

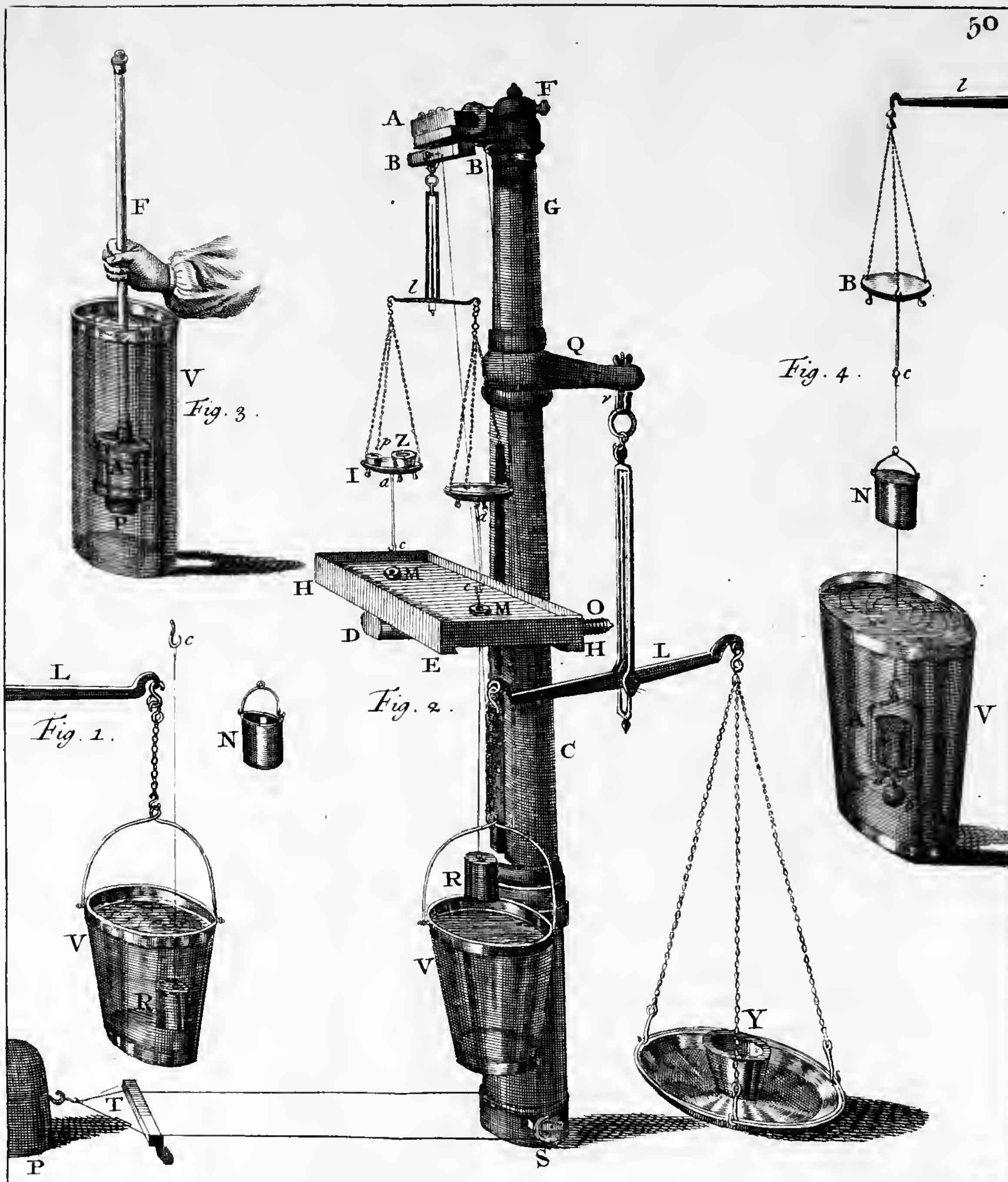
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.

In N. 1506. and 1507. confirmed by Exper. 9, and 10. we have ſeen, how a Body heavier than a Fluid is ſuſtained by it, and does, as it were, float; by a like Method a Body lighter than a Fluid may be kept at the Bottom; in that Caſe the Preſſure of the ſuper-incumbent Fluid is taken off; here we muſt take away the Preſſure of the Fluid againſt the lower Surface, whereby the Body is driven upwards. 1518.

EXPERIMENT 13.

The Braſs Plate *b c*, that is exactly flat and even, and faſtened to the Stand A, remains at the Bottom of the Glaſs Veſſel V; a like Plate *d e* is ſo joined to the wooden Cylinder *f*, having Cork L round it, as to make together with it a Body lighter than Water; this laſt Plate is put upon the firſt, that they may agree one to the other; and the Cork is kept down with a ſtick whilſt Water is poured on. The Cork being left to itſelf, does not aſcend, till, being moved from its Place, the Plates are partly ſeparated, so that the Water may preſs againſt the Plate joined to the Cork, and raiſe it up together with the Cork. The Surfaces of the Plates muſt be made very plane, and ſmooth. The Experiment may be repeated when the Water remains, if the Plates be preſſed together. 1519. Pl. LI. Fig. 3.

CHAP. IV.

Of exploring the Weights of Bodies.

WE have before obſerved * that the Weights of Bodies are explored by the Balance, an Inſtrument well known; and upon that Occaſion, we ſaid enough of its Properties, and the Particulars required to make it perfect. But nothing was ſaid concerning the Uſe of the ſame Inſtrument, when Weights are to be determined with great Exactneſs. 1520. * 179.

In what we ſhall deliver concerning the Compariſon of the Denſities, in the two following Chapters, an accurate Determination of the Weights is entirely neceſſary; but as the Demonſtrations of the foregoing Chapter ſupply us with the moſt accurate Method of doing this, it will be proper to ſpeak of it here. 1521.

We muſt in the firſt place get a Balance, very nicely made, which, when it is in Equilibrio, before it is loaded with Weights, may, as often as it is moved, return to an Equilibrium; we 1522.

 ſpeak

ſpeak of a ſmall Motion. It is moreover required, that the Equilibrium of this Balance, when it is loaded on both ſides with three or four Ounces, may be diſturbed by a fiftieth, or a ſmaller Part of a Grain.

The firſt Thing required is not eaſily obtained; and if this is wanting, we cannot have the Second.

1523. In the next place, we muſt have very exact Weights. It is moſt convenient, when we compare Weights, to expreſs the Weights by grains.

I make uſe of the following Weights; one of a thouſand Grains; one of 500 Grains; one of 400 Grains; two of 200 Grains; two of 100 Grains; two of 50 Grains; two of 30 Grains; two of 20 Grains; two of 10 Grains; to which are added the ſmaller Weights of ſix, five, four Grains, &c. But there is no ſmaller Weights among them than one Grain. Theſe Weights are required to be ſuch, that there may be an Equilibrium between them, as often as the ſame number of Grains is put into each of the Scales, the Weights laid upon them being varied at pleaſure.

The Balance and Weights muſt be had from a Workman; I will declare what elſe is to be added, to remove the Difficulties, which occur in the Uſe of it.

1524. Pl. LII. Fig. 1. * 1480. We ſuſpend the Balance, as was ſaid before *. The Beam is eight Inches long; the Handle of it has a Ring at *o*, that we may perceive more plainly, how the *Examen* (that is, the ſlender perpendicular Piece over the Axis) anſwers to the Index fixed in the upper part of the Handle. The Examen is not very ſlender, and its End, which is obtuſe, ſo moves, that its Diſtance from the End of the Index, which is ſimilar to it, may be ſmall; and as the Index and Examen are both of the ſame Thickneſs, the Equilibrium is perceived with great Exactneſs.

1525. * 1482. The Strings, which ſuſtain the Balance, go about the Pulleys, mentioned before *, and are faſtened to the Hook *v*. This Hook, by means of the Screw P, is moveable, that the Balance may be raiſed, or let down; but it can only move an Inch and a Quarter. When
1526. the Beam is to be moved further, the Tube S, to which is joined what is of ſervice in the Motion of the Hook *v*, and the Screw P, and which is faſtened by the Screw *q*, is moved along the ſquare Iron-Rod V K. The Tube S, with its Parts, is repreſented by itſelf in Fig. 3. and there is no difficulty in the Conſtruction of it.

1527. At the Angle E, of the Board H E H, there is a ſmall Braſs Tube, which, that it may be well faſtened, is joined to a ſquare

 Plate,

Plate, which goes into the Wood; the Tube appears at *f*, and the Plate at *e* (*Pl.* XLIX. *Fig.* 1.) the small brass Cylinder *b l* goes thro' this Tube, which is turned about its Axis, by means of the Head I.

The small Tube Q is moveable along this Cylinder; which sticks fast enough, to whatsoever part of the Cylinder it is applied; because the Ends are cut in, and are elastick. To this Tube is joined the Index T, which is moved horizontally, when, by turning the Head I, the Cylinder *b l* is moved about its Axis. 1528.

By the Hook *d*, of the Brass Wire *a d* *, the small Brass Cylinder *r s* is suspended, whose two Ends are perforated; this is more than four Inches long; and is wrapped round with Paper, that the Divisions, which will be taken notice of presently, may be marked upon it more conveniently. 1529. * 1484.

At present the solid L is removed, and the End *p*, of the Brass Wire *p n*, which is bent into a Hook, is put into the Hole *s*. 1530.

This Wire is about five Inches long, and at its lower End, the small Brass Ball *g* is joined to it, whose Diameter does not exceed a quarter of an Inch.

This Wire must be every where of the same Thickness exactly, and the Length of one Inch of it must weigh little more than four Grains.

p g is immersed in the Water contained in the Glass Vessel, and all Things are so disposed, that *p g* may be almost wholly immersed, when the Scales are put upon the Board H E H. 1531.

To the Hook *c* is applied the Brass Ball F; we suppose R removed, together with the Vessel in which it is immersed. The Weight of F must be such, that it may be in equilibrio with what is hung upon the opposite Scale, the Balance being so raised, that half of the Wire *p n* may be immersed. The Index T is applied to the point a, which must be first marked in the middle of the Cylinder *r s*, for the Beginning of the Divisions. 1532.

These Things being done, one Grain is put into the Scale d, and the Balance is raised; in that Motion the Brass Wire *p n*, is continually drawn out of the Water, whereby its Weight is encreased * so, that the Equilibrium is restored by raising the Balance to the Height of about two Inches. Then the Point *s* is marked, which answers to the Index T. Then the Grain is removed from the Scale d, and put into the other, and the Balance is let down, that the Equilibrium may be restored, and the Point *r* is marked. 1533. * 1478.

a

a r and a s are equal. But this may be repeated many times, that there may be no doubt about the Diſtance between r and s.

1534. Each of theſe Parts anſwers to one Grain, and they are divided into twenty ſmaller and equal Parts; each of which anſwers to 0,05 of a Grain; and they may be ſubdivided into five ſmaller Parts; which in the Uſe of the Machine may be done by the Sight, ſo that there will be no danger of any ſenſible Error.

1535. The Diviſions upwards and downwards begin at a. We call the Scale between a and r the aſcending Scale, and that between a and s the deſcending Scale.

The Method of Weighing Bodies.

1536. When the Weight of a Body is to be determined, the Balance is put in Equilibrio, and the Index is applied, as was ſaid in N.
* 1526. 1532. The Balance is let down *, that it may be at a ſmall Diſtance from the Board H E H; the Body to be tried is put into the Scale d, and the Weights into the Scale e; and when theſe are ſo determined, as to want ſomewhat leſs than two Grains of the Weight ſought, the Balance is raiſed by the Motion of the Tube S, till it wants
* 1526. but little of an Equilibrium *; then S being faſtened, by moving the
† 1525. Screw P, we bring the Balance to an Equilibrium very exactly †. The Index T ſhews what muſt be added, or taken away from the Weight, put into the Scale e. If for Example, the Index anſwers to the Diviſion 36 of the Deſcending Scale, and the Weights laid on be equal to 1095 Grains, we muſt add a Weight equal to thirty-ſix hundredth parts of one Grain, and the Body weighs 1095,36. Grains. If we had to do with the aſcending Scale, the Wire st being more immerſed would weigh leſs; and the Number, ſhewn by the Index, would be to be ſubtracted. In the laſt Example the Weight ſought would have been 1094,64. In practice we ſhould alſo have regard to what follows.

1537. pn muſt be oiled, before it is immerſed in Water, and then wiped with a Cloth; there will remain Oil enough on; the Balance muſt alſo be raiſed very ſlowly. Theſe Cautions are to be obſerved, that the Water may not ſtick to the Wire pn, when it is taken out; if this ſhould happen notwithſtanding, which will appear immediately, (for the Water will form itſelf into Drops, which, tho' ſmall, may be eaſily ſeen,) the Balance muſt be let down again, and raiſed more ſlowly, by which all Inconveniency may be avoided.

We

2

We muſt moreover obſerve, that the Index T muſt be applied to the divided Cylinder, when it is to anſwer to the Beginning of the Diviſions, or when we would examine, to what Diviſion it anſwers; but whilſt the Balance is moved, the Index is removed by turning the Head I. 1538.

When the Balance is come to an Equilibrium, it muſt be moved a little, once or twice; to ſee whether it returns to the Equilibrium freely; the leaſt Thing adhering to its Axis diſturbs the Equilibrium, which is remov'd by this Agitation. 1539.

We muſt take care that the Balance be not agitated too much, which may happen, when the Balance is raiſed; for the Board H E H, when it is once faſten'd *, ſhould keep its Situation. To hinder this, to the Pillar C there is join'd the Arm M †, to which is added the ſmall Braſs Plate *x y*, which is bent, and has a Tail in the Middle of it, going thro' the Arm that the Plate may be faſten'd by the Screw m. To the Ends of this, at Right Angles to it, the ſmall Braſs Rulers *t*, *z* are join'd, which are parallel to the Arm. 1540. * 1531. † 167.

When the Balance is raiſed, the Arm is ſo faſten'd, that the Rulers *t* and *z* may be but at a ſmall Diſtance from the Beam, ſo as to leave Room for a ſmall Agitation only.

CHAP. V.

Of comparing the Denſities of Fluids.

As the Denſity of a Body follows the Proportion of its Weight, the Bulk being given, by comparing the Weights of equal Bodies, we diſcover their Denſities *. Therefore *if any Veſſel be exactly fill'd with a Fluid, and this Fluid be weighed; and the ſame Veſſel be fill'd with any other Fluid, which is weighed alſo; the Weights will be as the Denſities.* But as this Method is liable to many Difficulties in practice, we ſhall not ſpend any Time in explaining it here. * 1463. 1541.

When the Preſſures of two Fluids are equal, the Quantities of Matter, that is, the Weights of Pillars, having equal Baſes, don't differ *; wherefore the Bulks, which are as *the Heights* of the Pillars, *are inverſely as the Denſities* †; whence is deduced a Method of comparing them in communicating Tubes; in which notwithſtanding the Baſes of the Pillars are not requir'd to be equal; that is, it matters 1542. * 1414. † 1464.

matters not whether the Tubes are unequal, or not, which does
* 1422. not alter the Height *.

EXPERIMENT I.

1543. Plate LI. Fig. 4. Pour Mercury into the Recurve Glaſs Tube A, to the Height *b c*; pour Water into one Leg from *b* to *e*; and let Oil of Turpentine be pour'd into the oppoſite Leg, 'till the two Surfaces *b*, *c*, of the Mercury are in the ſame horizontal Line; and let *c d* be the Height of the Oil; theſe Heights will be, as 87 is to 100, in which inverſe Ratio is the Denſity of the Water to the Denſity of the Oil of Turpentine; therefore theſe are, as $\frac{1}{87}$ to $\frac{1}{100}$, or as 100 to 87.

The Mercury is pour'd in, that the Fluids may not be mixed at the Bottom of the Tube.

1544. This Method is liable to Difficulties alſo. The ſmaller Differences are not well determin'd; it is difficult to diſcover by this Method the true Ratio between the Denſities of Rain-Water and diſtilled Water. And Mercury cannot be uſed for all Fluids, and then it is often difficult to keep the Fluids from mixing at the Bottom of the Tube.

The following Method is of all the moſt univerſal and accurate. It has for its Foundation what is demonſtrated of the Immerſion of
1545. a Solid heavier than Fluids. *When the ſame Body is immerſed in different Fluids, the Weights loſt by it, in them, are to one another,*
* 1463. 1478. *as the Denſities of the Fluids* *.

A MACHINE,

Whereby the Denſities of Fluids are compared.

1546. Plate LII. Fig. 1. * 1480. † 1524. ‡ 1532. We make uſe of the hydroſtatical Balance *, with all its Apparatus, explain'd above †.

The little Weight F ‡ is taken away, and inſtead of it we ſuſpend by an Horſe-hair the Glaſs Solid R.

This Solid may have a Cavity within it; and it is better for it to have ſuch a Cavity: for it is ſufficient, if the Solid be heavier than all Fluids, except Mercury, which this Method has no relation to but of which I ſhall ſpeak in the next Chapter.

1547. We determine ſome Weight at pleaſure; but it muſt be ſuch as may be ſometimes greater, and ſometimes leſs than the Weight, which R loſes in different Fluids. Our Weight is equal to 700 Gr.

The

The Brass Solid L, when suspended between *s* and *p*, is in Equilibrio with R, when not immersed, 700 Gr. being put into the Scale *e*; and the Brass Wire *p n* being immersed, as was said before *, L weighs 700 Gr. also. 1548.

* 1532.

Now if we take away the Weight of 700 Gr. put into the Scale e, and the Glass R be suspended in any Fluid, and there be an Equilibrium, by putting a Weight into one or other of the Scales *, the Density of the Fluid is discover'd; if in the Scale d there be added the Number of Grains, put into it, to the Difference of the 700 Gr. mention'd; if from the Scale, there be subtracted that Number from the same Number 700, and if in each Case, there be deducted from the Number so discover'd the Number, shewn in the descending Scale a *s*; or if it be added to it, if the Index answers to the ascending Scale a *r*: We have then the Weight lost by the Body; that is, the Weight expressing the Density of the Fluid *. 1549.

* 1536.

* 1545.

EXPERIMENT 2.

Things being disposed, as has been explain'd *, immerge the Glass Solid R in Water, suspending it upon the Scale d; we shall have an Equilibrium, if we put eleven Grains into the Scale e, and the Balance be raised, that the Index may answer to the fifty-sixth Division of the descending Scale; therefore from 700 we must subtract the Weight 11,56; and the Density of the Water is express'd by this Number 688,44 *. But if it had been Milk, we should have put ten Grains into the Scale d; which must be added to the 700, so that 710 expresses the Density of Milk. Sometimes I had this Number, sometimes a less Number, for all Milk has not the same Density. 1550.

* 1546.

* 1549.

Now let it be otherwise known, that a Cubick Foot of Water weighs 63 Pounds, 7 Ounces, 2 Drachms, and 2 Scruples: which we discover by determining the Weight lost in Water by a Body whose Capacity is known *. I made use of an hollow Copper Cube, whose Sides were exactly equal to six *Rhinland* Inches. The Weight mention'd is equal to 487360 Gr. whilst the Bulk of Water equal to our Glass R weighs 688,44 Grains; whence it appears that this Bulk must be multiplied by 707,92. that we may have a Cubick Foot; and 710 being multipled by this Number, we shall have the Grains in a Cubick Foot of Milk; and by this Method the Weight of a Cubick Foot of any Fluid whatever is discovered. 1551.

* 1478.

1552. I will now give another Method alſo of comparing the Denſities of Fluids, which is very compendious; but it can't well be made uſe of except in ſuch Fluids, as differ little in Denſity. It is of uſe chiefly in comparing the Denſities of different Wines; or different kinds of Malt Liquors.

This Method is alſo founded upon what is demonſtrated concerning Bodies immerſed in Fluids, but ſuch as are lighter than the Fluids.

1553. *If a Solid, which is lighter than the Fluids to be compar'd, be immerſed in different Fluids, the immerſed Parts will be inverſely as the Denſities of the Fluids*; for, ſince the ſame Solid is made uſe of, the Portions of the different Fluids, which would in every Caſe fill the Space taken up by the immerſed Part, would be of the ſame
* 1511. Weight *; therefore the Bulks, of thoſe Parts immerſed, would be
† 1464. inverſely as the Denſities †.

Another Machine,

By which the Denſities of Fluids are compared.

1554. Plate LI. Fig. 5. This Machine A is of Glaſs, and conſiſts of a hollow Ball, together with a cylindrick Tube, which is divided into equal Parts. Another Ball is join'd to this at the Bottom of it, Part of which is fill'd with Mercury, or very ſmall Shot, that the Tube may, by means of this Weight, deſcend vertically in Fluids, and remain in that Poſition: Care muſt be taken not to have too much Weight in the little Ball, for the Machine muſt be lighter than the Fluids to be compared together. The Machine deſcends to different Depths in different Fluids; and their Denſities are inverſely as the Parts im-
* 1553. merſed *, which therefore are to be compar'd together.

1555. A Thread is tied to the Machine at *a*; and it is immerſed in Water, the Weight of the lower Ball being increaſed, ſo that the Machine may indeed float, but have the greateſt Part of its Tube in the Water. The Machine, together with its Thread, is to be weighed exactly; mine weighed 550 Grains. The Machine, being put into Water, deſcends to *b*; therefore a Bulk of Water,
* 1511. equal to the immerſed Part of the Machine, weighs 550 Grains *, and may be expreſſed by that Number. The Thread mention'd is
† 1480. faſten'd to the Hook of the Scale of the hydroſtatical Balance †; the Machine remaining immerſed, put 20 Grains in the oppoſite Scale, and raiſe the Balance gently, (whereby the Tube will be drawn a little way out of Water, and become heavier,) 'till there be

be an Equilibrium, then the Surface of Water comes to the Point *d*. The Water sustain'd the Weight of the whole Machine, except 20 Grains; that is, it sustain'd 530 Grains; and the Weight of the Water, of the same Bulk with the Part then immersed, was equal to so many Grains, and is express'd by this Number; and the Bulk of the Part *d b* of the Tube was 20. If the Space *d b* be divided into 10 equal Parts, and the Divisions are continued upwards beyond *b*, and downwards beyond *d*, each Division will be equal to 2; and these Divisions may be again divided into smaller Divisions; and by observing the Division, to which the Machine descends in a Fluid, you will have the Bulk of the immersed Part. For Example, if the whole Tube stands out above the Water, the immersed Bulk will be 526; if it descends to the highest Division, the immersed Bulk will be 556; and the Densities of the Fluids, in which this happens, will be inversely as those Numbers, that is, as 556 to 526; and only the intermediate Densities can be compar'd by this Instrument. If the Ball was less in proportion to the Tube, it would serve for comparing together Fluids, whose Densities differ more than this.

When several Fluids are compared together, the Numbers which 1556.
express the Bulks of the immersed Parts are the Denominators of Fractions, which have 1 for their Numerator; and these Fractions express the Ratio of the Densities; for they are to one another inversely as the Denominators.

Experiment 3.

Let the Densities of Waters, containing different Quantities of 1557.
Salts, be to be compared, the Machine descends in one to the Division *e*; if it be immersed in another, it only descends to the Division *c*, their Densities will be to one another, as $\frac{1}{546}$ to $\frac{1}{542}$.

CHAP. VI.

Of the Hydrostatical Comparison of Solids.

IN homogeneous Bodies the Densities are in a Ratio compounded, of the direct Ratio of the Weights, and the inverse Ratio of the Bulks *; therefore *you will have the Densities*, that is, * 1467.
you will have Numbers that are to each other as those Densities, 1558.
by dividing the Weights by the Bulks.

The Weights of all Bodies may be compared by means of the Balance; the Bulks are found by weighing Bodies *in the same Fluid*;
* 1495 for the Weights, *lost* by them, are as their Bulks *.

A Machine,

Whereby the Densities of solid Bodies are compared.

1559. Here the hydrostatical Balance is to be used again *, together
Plate LII. Fig. 1, 2. with all its Apparatus †; as was done in comparing the Densities
* 1480. of Fluids ‡.
† 1524. Instead of the solid R we make use of the Glass Vessel N, in
‡ 1546. which the Bodies to be compared are put. Instead of the Solid L, we use the solid O, which is also of Brass; and is so determined, that, when the Vessel N is immersed in Water, there may be an
* 1532. Equilibrium, the Balance being in the Position before-mention'd *.
1560. The Body, whose Density is required, is put in the Scale d,
Fig. 2. and weigh'd *. The Situation of the Index T is to be changed in
* 1536. such manner, that it may answer to a, the Balance remaining in Equilibrio, whereby we determined the said Weight. This Body, other Things remaining, is put into the Vessel N, which is immersed in Water, as was said; and Weights are put into the Scale d, and the Balance is brought to an Equilibrium, and we have the Weight lost by the Body in Water; therefore we must divide the
* 1558. Weight of the Body by this, that we may have its Density * : that is, we divide the Weight in the Scale e by the Weight in the Scale d, correcting each by adding, or subtracting a Fraction, which the Scale shews.

Experiment.

1561. A Piece of pure Gold z, which was exposed to the Fire some Hours, and afterwards wash'd in Water, and wiped with a clean Cloth, that nothing extraneous should stick to it, was put in the Scale d, and nine hundred and sixty Grains in the Scale e. When
* 1536. the Balance was brought to an Equilibrium *, the Index answer'd to the seventeenth Division of the descending Scale; therefore the Weight of the Gold was 960,17 Grains.
* 1560. The Index being alter'd *, and the Gold being put into the Vessel N, other Things remaining as they were, the Equilibrium was destroy'd; fifty Grains were put into the Scale d, and the Balance being raised, that the Index might answer to the ninth Division of the descending Scale, the Equilibrium was restored; therefore

therefore the Weight deſtroyed was 49,91 *. By the Diviſion †, the Denſity was 1.9,238. Nearly $19\frac{1}{4}$. * 1536. † 1558.

I made the like Experiment with Silver. The Weight of a Piece of Silver, whoſe Surface was ſmooth, and well cleaned, was 439,15; the Weight loſt was 42,58; therefore the Denſity was 10,31. The Denſity of pure Silver is greater. 1562.

The Denſity of Mercury is diſcovered in the ſame manner as that of Solids is. A ſmall Glaſs Veſſel is put in the Scale *d*, and the Balance being in Equilibrio, the Index is placed as was mentioned *; pour what Mercury you pleaſe into this Veſſel, and find the Weight. Then pour the Mercury into the Veſſel N, and the other ſmall Veſſel is put into the Scale *d*, as before; and the Weight loſt by the Mercury in Water is determined. 1563. * 1532.

The Mercury ſhould be well cleanſed alſo, for there is a certain Greaſineſs which eaſily adheres to its Surface, which in ſome meaſure hinders the Water from applying itſelf cloſe to it, and then the Denſity found will be leſs than the true Denſity.

I found the Denſity of Mercury, pretty well cleanſed, to be 13,54 or 13,57, more and leſs; its Denſity, when very well cleanſed, was 13,62.

We here compare the Denſity of Solids with the Denſity of Water; and by means of this, with the Denſities of other Fluids. 1564.

The Denſity of Water is expreſſed by Unity; for ſuch would be the Denſity of a Body, which ſhould have the ſame ſpecifick Gravity with Water.

By this Method likewiſe we may diſcover the Denſities of Bodies ſpecifically lighter than Water; if they be ſo joined to the Veſſel, as to be drawn into the Water by its Weight. 1565.

By multiplying the Weight of a cubic Foot of Water * by the Number, expreſſing the Denſity of the Body, we have the Weight of a cubic Foot of the Body itſelf; which Determination of the Weight is in many Caſes of great Uſe. 1566. * 1551.

A Machine,

Whereby we may diſcover whether Money is good or bad.

Take the Machine A, like the Machine deſcribed in the preceding Chapter *; and let it have fixed to its Bottom the Ring DE; the Ball will, by its own Weight, be in part immerſed in Water. 1567. Pl. LI. F.6. * 1554.

The Money which we ſuſpect to be counterfeit is to be compared with good Money, of the ſame Weight, by firſt putting one 1568.

and then the other on the Ring; for if the Money ſuſpected be bad, the Machine will ſink to a leſs Depth.

That this may be of uſe to try ſeveral ſorts of Money, it muſt be ſo made, that, when a lighter Species of Coin is put upon it, the whole Ball may not be covered with Water; and then a Copper or Leaden Plate is applied, which being added to the Money, may make the Surface of the Water riſe about as high as the Middle of the Neck of the Machine.

1569. If we ſhould not have by us good Money, exactly of the ſame Weight with that which is ſuſpected to be counterfeit, we muſt make uſe of this Method. We muſt put upon the Ring a Piece of Money known to be good, and obſerve how far the Machine ſinks. Then we muſt put upon the Machine another Piece of Money, of the ſame Species, whoſe Weight differs a little from that of the firſt Piece, for example one Grain, and find the Difference of the Immerſion; let this be one Diviſion and an half. Now if from what goes before we know that the Money ſuſpected differs two Grains from the firſt Piece, we ſhall alſo know, that the Immerſion ſhould differ three Diviſions; and how far the Machine ſhould ſink, when the ſuſpected Money is put upon it; if it ſinks to a leſs Depth, this will ſhew that it is counterfeit.

By comparing the Denſities of Metals we may ſolve that celebrated Problem of *Archimedes* concerning mixt Metals.

1570. Let there be given a mixt Metal, made up of two known Metals; and let it be to be determined what Quantity of each goes to make up the Mixture, the Denſities of the Metals and the Mixture being given.

Pl. LII. F. 4. Let the Denſities of the Metals be A B, A D; that of the Mixture A C. Let alſo A L and L I be, as the Bulks of the firſt and ſecond Metals in the Mixture. And let us ſuppoſe the Rectangles A F, L H, A G to be formed.

The Weight of the firſt Metal in the Mixture may be repreſent-
* 1465. ed by the Rectangle A F *; then L H will repreſent the Weight
23. El. 6. of the ſecond Metal; and the Figure A B F M H I A ſhews the Weight of the whole Mixture; this is alſo repreſented by the
* 1465. Rectangle A C G I *; which therefore is equal to the Figure mentioned.

Taking away from each the common Figure A C N M H I, there remain the equal Rectangles B N, N H; whoſe Sides are reciprocally
* 14. El. 6. cally proportional *, F N is to N M, as N G is to N C; that is, as
† 34. El. 1. L I is to A L †; therefore by conv. and inv. F M is to F N, as A I is

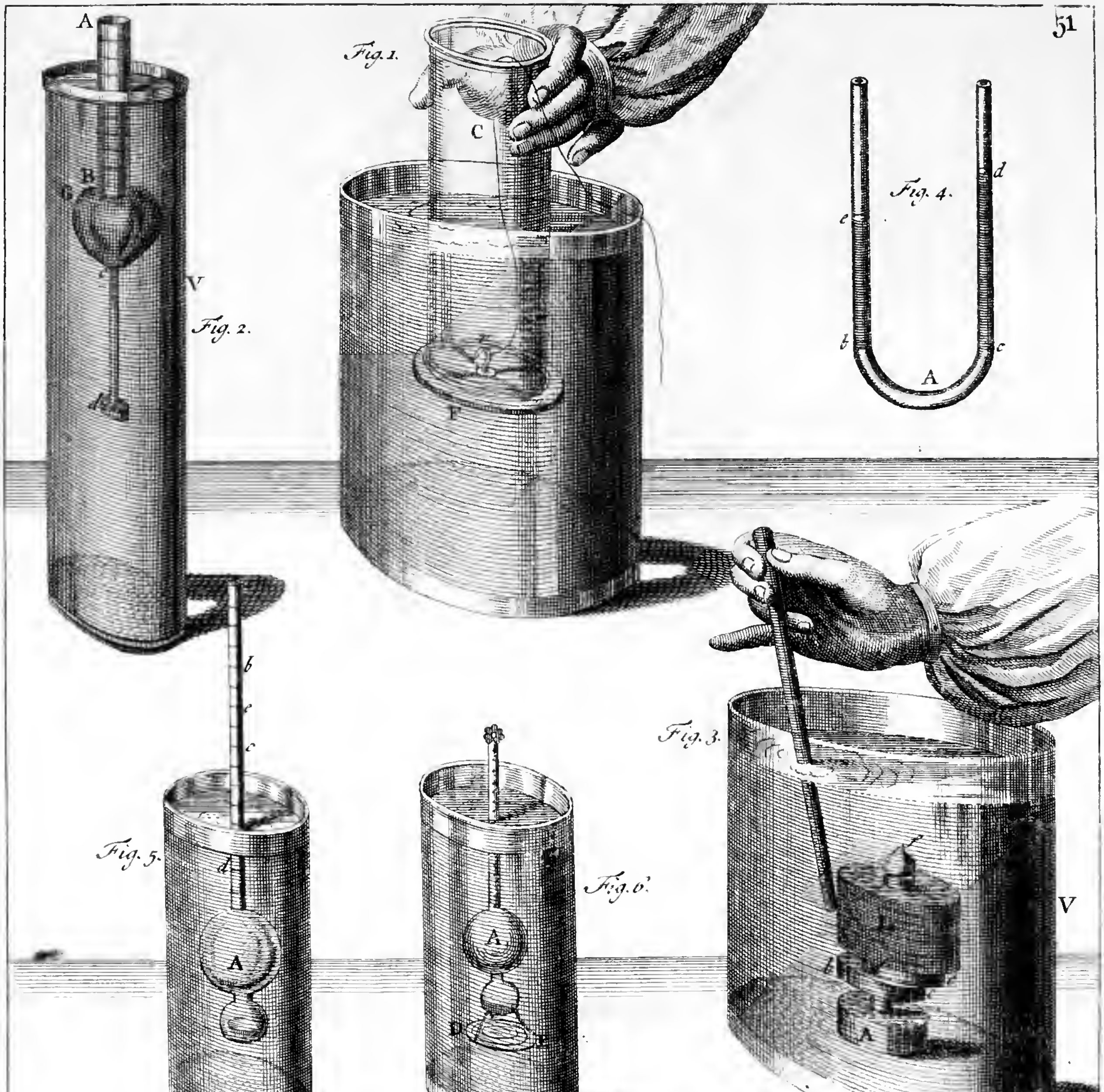
Fig. 1.
C
F
Fig. 2.
A
B
G
V
Fig. 4.
d
e
b
c
A
Fig. 3.
V
A
Fig. 5.
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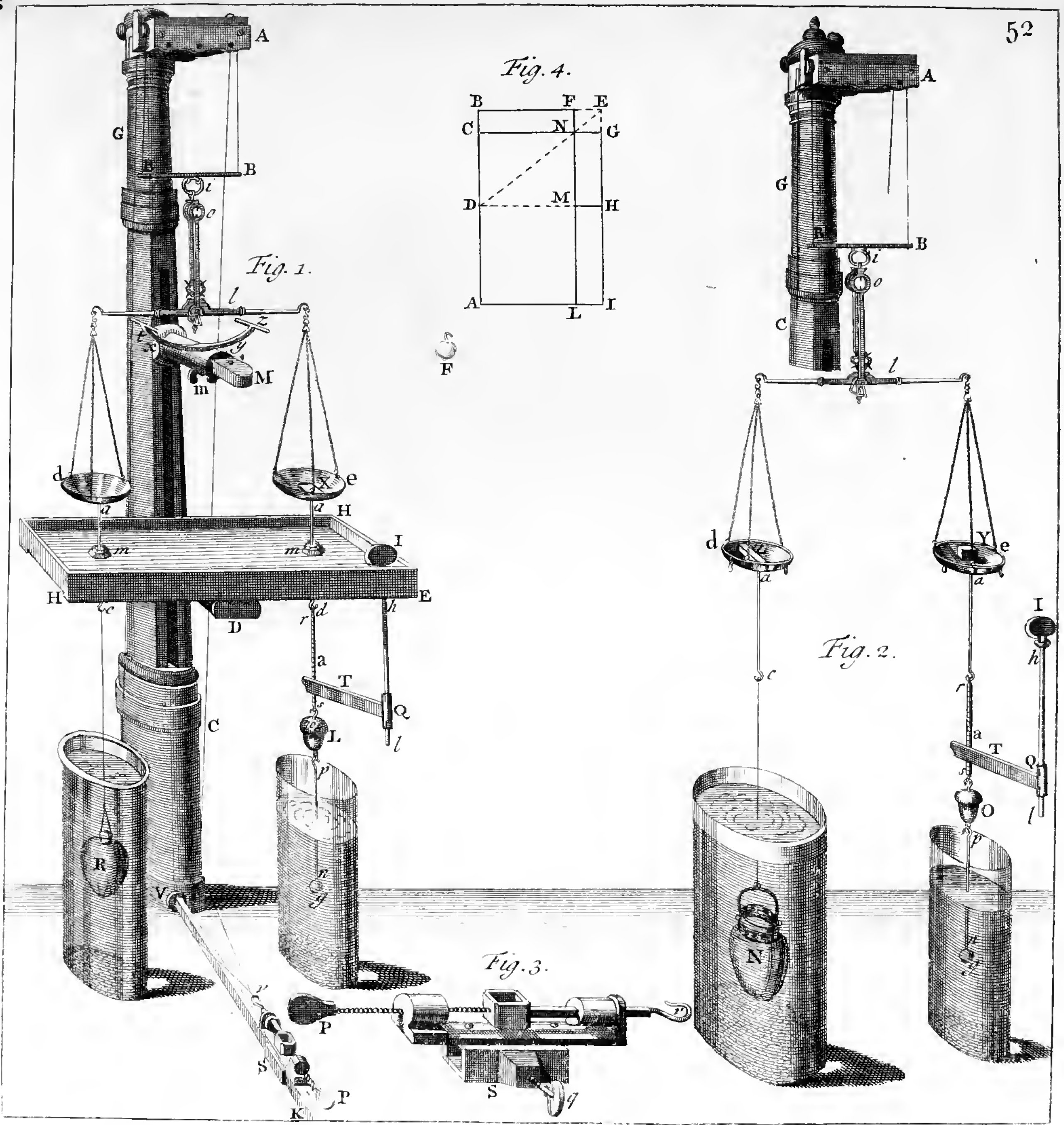
Fig. 6.
A
D
E
J. Mynde sc.



J. Mynde sc.

is to L I. *The Bulk of the Mixture is to the Bulk of the second Metal in the Mixture, as the Difference of the Densities of the first and second Metals is to the Difference of the Densities of the first Metal and the Mixture.* 1571.

But *the Weight of the whole Mixture is to the Weight of the second Metal in the Mixture*, in a Ratio compounded of the Densities of the Mixture and the second Metal, and the Ratio of the Bulk of the Mixture and the Bulk of the second Metal in the Mixture *; that is, *as the Product of the Density of the Mixture, by the Difference of the Densities of the Metals, is to the Density of the second Metal, multiplied by the Difference of the Densities of the first Metal and the Mixture.* 1572.

* 1465.

This Solution depends upon this Hypothesis, that each of the Metals in the Mixture keeps its whole Bulk; but if some of the Parts of the one enter into the Pores of the other, the Solution is not accurate. And who can affirm that there is a simple Apposition of the small Parts in every Mixture of Metals; the Thing has not yet been nicely determined by Experiments. Amongst the Experiments, which *Hook* made before the *English* Royal Society, there is one mentioned, by which it appeared that the Density of Copper was encreased by the Mixture of Tin, tho' a lighter Metal. 1573.

BOOK

BOOK III.

PART II. Of the Motion of Fluids.

CHAP. VII.

Of the Celerity of a Fluid arising from the Pressure of a superincumbent Fluid.

AN inferior Fluid is pressed by a superior, and that equally
* 1418. every way *, and it endeavonrs to recede every way with
1574. equal Force; therefore *if you take off the Pressure on one Side, the Fluid will move towards that Side; and which way soever the Pressure be taken away, it will move with the same Celerity*: which will be confirmed by Experiments to be mentioned in the Chapter, where we speak of spouting Fluids.

At the same Depth the Celerity is also every where the
* 1413.1414. same, by reason of the Equality of the Pressure *; but when the Depth is changed, the Celerity is also changed.

1575. I say that *the Velocity* mentioned *is communicated by the Pressure*,
1576. but that the Particles do not acquire it by falling: for *the first Particles that go out, do not move more slowly than those, that follow*: for the first Particles go the same way with those that follow, if they go out obliquely. Besides, not only those that descend go out, but those also that flow sidewise; and a Particle is moved by the Pressure of all the circumambient Particles, except those that go before it; and the Particles, which descend, don't so much acquire their Velocity by means of the superincumbent Particles, as by the Pressure of the lateral Particles; for they can't be accelerated by those that follow, and move with the same Velocity.

1577. Let there be a Vessel A, filled with a Fluid to the Height *a b*;
Plate LIV. Fig. 1. let the Water run out thro' a Hole *c d* in the Bottom. All the Particles following one another go out with the same Velocity. Let us conceive a Plate *c f*, answering to the Hole; this sustains the Weight of the Column *e m*; and is pressed downwards by this whole Action, and by its own Weight, whilst it is at rest: but when

when the Hole is opened the Action of the Column *e m* immediately ceases, except it be continually pressed by the circumambient Fluid. The Particles, which yield, are removed from the Action of the following Particles, if these be not driven on by a greater Force; but this can't be applied to the lateral Pressure, which yet produces the same Effect with the direct Pressure, since all Pressure produces the same Action towards all Parts *. * 1418.

It will plainly appear that the Velocity, with which the Plate *c f* goes out, is to be attributed to the lateral Pressure only, which the Fluid surrounding the Pillar *c m*, exerts upon it, if we suppose this Pressure to be taken away, and the Pillar *c m* to be inclosed in a Tube. This would then fall all together, like a solid Body, and the Motion of the Plate *c f*, would be like that of a separate Plate *n*; that is, it would go out of the Hole with the least Velocity, which does not happen *. 1578. * 1576.

But we discover the true Velocity, with which the Plate *c f* goes out of the Hole, setting aside the Retardations, by determining the Force with which it is thrown out. 1579.

This Plate, during the Action by which it is driven out, runs thro' its Height *d f*; let us suppose the Plate *n*, in falling, to run thro' its Height also; the Forces communicated to these equal Plates, in running thro' equal Spaces, are to one another, as the Intensities of the Pressures, whereby the Forces were communicated *; which Intensities are, as the Weight of the Pillar *c m* †; to the Weight of the Plate *n*. 1580. * 727. † 1577.

If we also suppose the whole Pillar *c m*, to fall alone by its own Weight, and to descend along the same Height *d f*, it will acquire a Velocity equal to that, which the Plate *n* acquired; and the Force, acquired by the Pillar, is to the Force of the Plate *n*, as the Weight of that to the Weight of this *; therefore the Forces of the Pillar and Plate *c f*, have the same Ratio to the Force of the Plate *n*; and they are equal to one another †. But the Forces, acquired in falling, are equal, when the Heights are inversely as the Masses ‡; that is, as the Weights ||; whence it follows that the Plate *n* goes out with a Force equal to that, which it would acquire in falling freely the Height *m d*. * 748. 156. † 9. El. 5. ‡ 790. || 156.

This Demonstration is very universal, whether the Plate be thicker, or thinner; it is always driven out with a Velocity equal to that, which a Body acquires in falling the said Height *m d*, and it acquires this Velocity, whilst it runs thro' the Space *f d*. But the Demonstration is founded upon this, *viz.* that the Pressure is 1581.

* 1577. equal every way *; therefore, tho' it be not confined to Particles of a determined Magnitude, yet can it only take place in Particles of ſuch a ſmallneſs, as to conſtitute a Fluid; but as very ſmall Particles are required for this, it follows that the firſt which go out, acquire their whole Velocity in a very ſmall, and entirely inſenſible Moment.

If the Particles, of which the Pillar *c m* conſiſts, ſhould go out ſucceſſively, the Efflux would not be continued; but the Plates would go out ſucceſſively, there being ſome little Space of Time betwixt the going out of each. But the Quantity, which goes out, in the Time in which a Body may paſs thro' the Height *d m*, does indeed exceed the whole direct Column, and the Exceſs is ſupplied by the lateral Fluid ſo, as to make the Efflux continuous without any interruption.

1582. In the Time, in which the firſt Plate, that goes out, acquires its Velocity, the adjacent ones alſo are driven on in ſuch manner, that many are in continual Motion, during the Efflux; to each of which, at their going out, ſo much Velocity is communicated as they want of that greateſt Velocity, above-determined, which the moving Cauſe can communicate to them.

1583. Hence it follows that *a Fluid, by means of the Preſſure of a ſuperincumbent Fluid,* (for even the lateral Preſſure depends upon this) *ſpouts out of a Hole, with a continued Jet, with a Velocity equal to that, which a Body acquires in falling from the Top of the Fluid to the Hole; ſetting aſide the Coheſion of the Parts,* as in this Demonſtration, which tho' it be ſmall, is yet to be obſerved in moſt Fluids; *by which* Coheſion *the Particles that are going out are retained,* whilſt the Fluid, that is flowing out, is ſeparated from what remains; and therefore they *are retarded*. But beſides this Retardation, which depends upon the Fluid, the Velocity of the Fluid is diminiſhed by many other outward Cauſes; of which I ſhall ſpeak in the next Chapter.

A Machine,

Whereby Experiments concerning ſpouting Fluids are made.

1584. Plate LIV. Fig. 2. The wooden Parallelopiped A B, which is 18 Inches long, and as wide, and above two Feet high, is filled with Water, and ſo placed, that its Bottom may be raiſed about a Foot above the horizontal Bottom of the wooden Trough C D, which is almoſt four Feet long, a Foot and a half broad, and five or ſix Inches deep.

At

At F at the Height of a Foot and an half above the Bottom of the Trough D, there is fixed an horizontal Brafs Pipe, the Diameter of whofe Cavity exceeds half an Inch; the anteriour Part of it is clofed by a Plate, in the Middle of which there is an Hole of the twelfth of an Inch diameter: this Hole is ftopped by a Stop R, whereby the anteriour Part of the Pipe is enclofed, and which is joined to it by a Screw: two fuch Pipes are faftened at E, near the Bottom of the Veffel A B, and at G; and this laft is as much above F, as that is below it.

Near the Bottom of the fame Machine there is alfo faftened a Cock N, with a Screw, that a Pipe may be faftened to it.

EXPERIMENT 1.

The Veffel A B is filled with Water in fuch manner, that the Heights of the upper Surface of the Water above the Bottom of the Trough C D, may be divided into two equal Parts by the Hole at F, each of which Divifions is equal to a Foot and an half in my Machine. The Water fpouts out of this Hole along F M, and the horizontal Diftance of the Point, to which it fpouts in the Bottom of the Trough D, from the Hole exceeds 34 Inches, which does not want two Inches of the Height of the Water above the faid Bottom: if it reached to the Diftance of 36 Inches, the Water would, in the Time in which a Body may fall from F to the Bottom of the Trough C D, run thro' a Space double of this Height *, with an equable Motion, and with the Celerity with which it goes out; and would therefore move with a Celerity equal to that, which a Body may acquire in falling from this Height †; but this Height is equal to the Height of the Surface of the Water above the Hole. But as it only reaches to the Diftance of about $34\frac{1}{2}$ Inch. the true Velocity of the Water wants about a twenty-fourth Part of the Velocity mentioned. 1585.

* 541.
† 376.

*Setting afide the Retardations, the Squares of the Velocities, with which a Fluid goes out of different Holes, are to one another as the Heights of the Fluid above the Holes**. And it appears by Experiments that the Retardations don't hinder this Proportion much, as long as the Heights do not exceed 30 Feet, or 35. In lefs Heights we make manifeft this Proportion by the following Experiment. 1586.

* 1583. 374.

EXPERIMENT 2.

The Machine above-mentioned is made ufe of here *; and we ought to confider this, that the Diftances, to which the Water 1587. Plate LIV. Fig. 1.

 fpouts

* 1584.

ſpouts in the Bottom of the Trough C D, whilſt it goes out of the Hole as E horizontally, ſuppoſing different Heights of the Surface of the Water, are Spaces horizontally run thro', with an equable Motion, in the Time in which a falling Body may run thro' I L, which is equal to the Height of the Hole above the Bottom
* 541. of the Trough *: and that therefore theſe Diſtances are as the
† 149. Velocities †.

Now if there be Water in the Veſſel A B, eight inches above the Hole at E, and the Diſtance to which it ſpouts, be meaſured, and more Water be poured in, till it be eighteen Inches high, and the Diſtance be meaſured again; theſe will be as 2 is to 3. The Squares of the Diſtances are here as the Heights of the Water, in which Ratio are the Squares of the Celerities.

CHAP. VIII.

Of Spouting Fluids.

1588. A *Fluid, ſpouting vertically out of a Hole*, ariſes up with that Celerity, with which it would come up *to the upper*
* 1583. 380. *Surface of the Fluid* *, yet it *never comes up* to that Height, and that for ſeveral Cauſes beſides the Coheſion of the Parts above
† 1583. mentioned †.

1589. 1. The Celerity, by which the Fluid aſcends, is diminiſhed every Moment, and the Column of the ſpouting Fluid conſiſts of Parts, which are moved to different Heights by different Celerities; all the Parts of a Column, which is every where of the ſame thickneſs, are neceſſarily moved by the ſame Celerity; the ſaid Column every where will be broader every Moment, as the Celerity of the Fluid is diminiſhed; which ariſes from the Impulſe of the Fluid following, and which from the Nature of a Fluid yields to every Impreſſion, and is eaſily moved every way; by that Impreſſion the Motion is retarded every where.

1590. 2. This Motion is alſo diminiſhed by the Fluid, becauſe, when it hath loſt all its Motion, it hangs in the upper part of the Column, and is ſuſtained for a Moment by the Fluid that follows, before it flows off on the Sides, which retards the Fluid that follows it, and that Retardation is communicated to the whole Column.

3. By

3. By the Friction againſt the Sides of the Hole, the Celerity of the Fluid is diminiſhed; which Friction is encreaſed, when the Fluid is brought thro' Pipes and Cocks. 1591.

4. Laſtly, the Air's Reſiſtance ſtops the Motion of Fluids. 1592.

It is manifeſt that the firſt Cauſe above-mentioned *, of the Retardation cannot be corrected. * 1589.

The ſecond † is corrected by ſomewhat inclining the Direction of the Fluid, as is ſelf-evident; and this is the Reaſon why *a Fluid riſes higher, if its Direction be a little inclined, than if it ſpouts vertically.* 1593. † 1590. 1594.

EXPERIMENT 1.

To the Machine above-deſcribed *, by help of the Screw at N, join the Curve Tube N O, from which the Water ſpouts up vertically through a ſmall Hole, by turning the Tube a little, which is eaſily done, by reaſon of the Screw at N, the Direction of the ſpouting Water will be inclined, and it will aſcend higher. But by this Inclination the Beauty of a Jet is often deſtroyed. 1595. Pl. LV. Fig. 1. * 1584.

As to the third Cauſe of the Retardation * it is to be obſerved, that the Friction is greater in proportion, when the Hole is leſs; for the Circumference, againſt which the Friction is, encreaſes as the Diameter, and the whole is encreaſed as the Square of the Diameter †; and the Quantity of the ſpouting Fluid encreaſes more than the Friction. It is alſo plain that the Friction encreaſes with the Celerity, wherefore *the Holes are to be encreaſed according to the Height of the ſpouting Water*, that whilſt the Friction is encreaſed from one Cauſe, it may be diminiſhed from another. 1596. * 1591. † 2. El. 12. 1597.

The Ends of the Pipes, from which the Water ſpouts, have commonly the Figure of a truncated Cone, as is repreſented at P; in which End the Water ſuffers a great deal of Friction, and is moved irregularly, and ſpouts up with that Irregularity. This may be amended *by covering the End of the Tube with a flat, ſmooth, and poliſhed Plate, which has a Hole in it,* whoſe Sides muſt be alſo well poliſhed; for then *the Water ſpouts higher*, and becauſe it riſes with a Motion entirely regular, it *is perfectly tranſparent.* 1598. Pl. LIV. Fig. 3. 1599.

EXPERIMENT 2.

Take the Tube above-mentioned P, as alſo the Cylinder Q, ſhut up at one End with a bored Plate; let theſe be ſcrewed on one after another, to the End O of the Tube N O; the Water remaining at the ſame Height in the Veſſel A B, the Water ſpouts out of 1600. Plate LIV. Fig. 2. 3.

the

the Cylinder Q to a greater Height, and there is a Difference of two Inches at leaſt in this ſmall Height.

1601. *The Pipes which bring the Water from a Reſervoir, muſt be very wide in proportion to the ſpouting Hole,* that the Water may move more ſlowly in theſe Pipes, and have no ſenſible Friction. *The Water-way, or Paſſage, of the Cocks muſt be very large alſo,* that the Friction may be diminiſhed.

EXPERIMENT 3.

1602. Pl. LIV. Fig. 2. To the Veſſel A B, at the ſame Height as the Pipe F, is fixed a Pipe which has a Cock in it; this is narrower and ſhut up with a Plate in the ſame manner as the Pipe F, and the Plate is bored in like manner, but the Hole is leſs; the Water-way of this Cock is a quarter of an Inch. The Water which goes through this Cock is brought thro' a narrower Space than that which moves through the Pipe F; this laſt is more tranſparent, and ſpouts to a greater Diſtance. If theſe Jets were directed upwards, the Height of the Jet which comes through F, would be double of the other; as is eaſily diſcovered from the Diſtances, to which the Water ſpouts.

1603. *The Reſiſtance of the Air has a ſenſible Effect upon the Motion of Fluids.* For the Air reſiſts Motion as all Bodies do; the ſpouting
* 709. Fluid acts upon the Particles of Air, by the Reaction of which *, the Motion of the Fluid is diminiſhed.

1604. Beſides this Reſiſtance there is alſo another, not to be overlooked, which is the Action of the Air againſt the ſpouting Fluid. It will appear in the following Book that Air has the Properties of Fluids. It encloſes the whole Column of the ſpouting Fluid, and reſiſts that part of its Motion, whereby it ſpreads itſelf ſide-ways,
* 1589. as it becomes wider *; and there is required a greater Force of the Fluid that comes after, than if this Reſiſtance was taken away; therefore the Air reſiſts by its lateral Preſſure alſo.

1605. The Reſiſtance ariſing from the Stroke of the Fluid againſt the Air encreaſes with the impinging Surface, that is, if the Celerities remain the ſame, it encreaſes with the Hole; in which Ratio alſo, the Quantity of the Matter moved encreaſes, and upon this account it is no matter of what Bigneſs the Hole is.

1606. The lateral Preſſure follows the Proportion of the Surface of the Column; the Matter moved, which (the Celerity being the ſame)
* 784. is in the Ratio of the innate Force *, is changed in proportion of the whole Column, that is, of the Square of its Surface; and therefore, if the Hole be encreaſed, the Force of the Fluid encreaſes

faſter

faster than the Cause retarding it; and for that Reason *in the greater* 1607.
Heights of spouting Fluids, that the lateral Pressure (which exerts a greater Action as it acts the longer) may be the better overcome, *greater Holes are required*; which we have also shewed before to be required in the same Case from another Cause *: where, as well as here, we supposed the greater Holes only necessary for the greater Heights, tho' the Demonstrations prove that these Holes, which are very necessary to the greater Heights, are in general to be preferred to others. I shall give the Reason of this Distinction.

* 1597.

Great Holes also hinder the Motion; for first there is a greater 1608.
Surface which is pressed upon by the highest part of the Fluid, which has lost all its Motion, and hangs on a longer Time, before it runs off at the Sides.

Secondly, Not only the Fluid which is directly against the Hole 1609.
runs out, but, that there may be a constant Supply, the neighbouring Fluid continually comes towards the Hole with an oblique Motion, and in going out it is carried with a compound Motion, whereby the Motion of the spouting Fluid is disturbed; the greater the Hole is, the greater is the Disturbance arising from that Cause.

In smaller Holes the Retardations prevail, which are diminished, when the Hole is made larger; but a Hole may be made so large that the Retardations may encrease with the Hole. Wherefore *in* 1610.
all Heights there is a certain Measure of the Hole, thro' which the Fluid will rise to the greatest Height possible. Yet one cannot give Rules to determine the Diameter of the Hole, because the Bigness of the Pipes of Conduct and their Inflections require it different; so that there would be a Variation *in Infinitum.*

But it is to be observed, that *the Height to which the Fluid can* 1611.
ascend, and also the Bigness of the Hole, have their Limits, which they cannot exceed.

For when the Celerity of the Fluid is too great, it strikes against the Air with so much Force, that it is dispersed into Drops; in which Case, by diminishing the Celerity, the Height, to which the Fluid spouts will be encreased; and the greatest Height, to which a Fluid can ascend, is different in different Fluids: and this Height in spouting Water, scarce exceeds an hundred Feet. The Diameter of the Hole, which answers to this greatest Height, scarce exceeds an Inch and a Quarter.

Fluids, which spout obliquely, are not retarded from so many Causes, 1612.
nor so much, as those that spout vertically. The second Cause of Retardation, above-mentioned *, has no place here, and the Effect of the first † is less. As for the rest, one may apply here what has

* 1590.
† 1589.

has been ſaid of Solids obliquely projected in Chap. xxii. of the
1613. firſt Book; and *a Fluid may be conſidered as innumerable Solids, following one another, and running thro' the ſame way.* In the Motion of a Fluid, the Way gone thro' is ſenſible, and what has been ſaid of Solids obliquely projected, may be reduced to Experiments by the help of Fluids; for doing which we muſt make uſe of Quickſilver, becauſe of the great ſpecifick Gravity of this Fluid in reſpect of others. But theſe Experiments are to be made by a particular Machine.

A MACHINE,

Whereby Experiments concerning Fluids ſpouting obliquely are made.

1614. Pl. LIII. Fig. 1. The wooden Trough A B C D E F H is four Foot long, and ten or twelve Inches broad; it is ſix or ſeven Inches high; the Bottom is made of a Board hollowed in half an Inch, to contain the Mercury the better.

In the End H, of the Side E, F, H, you have a Board H I ſix Inches wide, and two Foot high, which has in it a Slit *a t*. By this means you may fix to any Height upon the Board the wooden Parallelopiped *s*, which has a Screw fixed in its hinder Part.

This Solid is repreſented by itſelf at S (*Fig.* 2.) There is faſtened to it a cylindric Veſſel of Box-wood, which has a Groove round it to receive two Braſs Plates, one of which may be ſeen at *f e*; their Ends are joined together by the Screw *g*, ſo as to make the Box Veſſel immoveable; but this moves about its Axis, when the Screw is looſened a little.

In the Bottom of this Veſſel there is a cylindric Cavity *a b*, a quarter of an Inch Diameter; this communicates with a like Cavity *b c*, which terminates in the middle of the greater Cavity *c d*, whoſe Diameter is above half an Inch; into this is put the truncated Cone H (*Fig.* 3.) of Box, whoſe exteriour Surface anſwers to the interiour Surface of the Cavity in ſuch manner, that the Cone may turn about its Axis in this Cavity; whilſt it is held faſt by the Screw R, which goes thro' the Braſs Handle Q O.

The truncated Cone H is joined to the Cylinder I L at right Angles to it; and there is a bent Cavity *h i l* which goes thro' the Cone and Cylinder, of the ſame Diameter with the Cavity *b c*, and anſwering to it. But that is broader at L, that the Glaſs Tube N M may be put into it.

The Tube is a Foot and an half long, one End of which is ſeen at N M (*Fig.* 5.) which is joined to the Box Cylinder L I, which is hollowed at *l i h*, with a round Hole in the Form of a Gnomon, or Carpenter's Square; at *h c* the Cavity is greater, to receive the truncated Cone E D, that exactly fills it, and is moveable about its Axis by the help of the Handle E A.

The Cavity *h i* anſwers to the Cavity *d e*, which communicates with *f g*; this part of the Box has driven upon it an Iron Ferril B Q, in which is drilled a very ſmall Hole *g*, which when the Parts of the Machine are joined together, communicates with the Cavity of the Box P (*Fig.* 2.)

To prevent the Tube from breaking, the Ends L, L, of the Box Cylinders (*Fig.* 3, and 5.) together with the intermediate Tube, are applied to the wooden Ruler *m n* (*Fig.* 1.) whoſe lower End *m* has the Piece of Iron L P B Q fixed to it; the End L (*Fig.* 5.) of the Box Cylinder anſwers to the End M, of the Ruler M N (*Fig.* 6.) in which Situation the thicker part I of the Cylinder (*Fig.* 5.) anſwers to I (*Fig.* 6.) and the Screw Q preſſes the Cylinder B D at *o* (*Fig.* 5.) and faſtens it to the Cylinder L I.

All the Parts of the Machine may be ſeen joined at (*Fig.* 1.) Quickſilver being poured into the Veſſel *p*, ſpouts out of the Hole *g*, (*Fig.* 5.) When the Mercury is at the ſame Height in the Box, and you do not vary the Inclination of the Piece *n m*, the Mercury ſpouts with the ſame Celerity in any Direction *; but the * 1574.
Inclination of the Direction may be varied by moving the Handle *e a* E A in *Fig.* 5. The Angle that the Direction, in which the Mercury goes out of the Hole, makes with the Horizon, may be meaſured by help of the Quadrant of the divided Circle *q*, along which the Index *f h* is moveable, which by its Weight is always kept in a vertical Poſition. This Quadrant may be ſeen in *Fig.* 7. with its Index F H. It has two Rings behind, to receive the Handle E A, *Fig.* 5. When this Handle is vertical, the Index is againſt the 45th Degree, and the Direction of the Motion of the Mercury, which ſpouts out then, makes an half Right-Angle with the Horizon.

In *Fig.* 1. the Jets of Mercury in ſeveral Directions are repreſented: They become the more viſible by help of a wooden Plane G painted black, which the Mercury in its Motion does almoſt touch: upon this Plane muſt be drawn (what could not be repreſented here) the Ways which a Body (according to what is ſaid in N. 545.) runs thro', when it moves with the ſame Celerity according to Di-

 rections

rections which make different Angles with the Horizon. Alſo the Semicircle A L (*Fig.* 5. *Pl.* 19.) muſt be drawn upon this Plane.

There may be ſeveral other ſuch Planes, in which the ſame Things are drawn, according to the different Celerities.

This Plane ſtands upright near the middle of the Trough, and coheres with the Side E F H, ſo as to move backwards and forwards along the Trough.

The Celerity of the ſpouting Mercury is varied, as you change the Inclination of the Piece *n m*, and by lowering the Veſſel *p*, the Hole, through which the Mercury ſpouts, is ſet at an Height, anſwering to the loweſt Point drawn on the Plane. The Mercury will ſtop its ſpouting, when the Cavity *a b* (*Fig.* 2.) is ſtopped with the Plug D E, *Fig.* 4.

Experiment 4.

1615. Pl LIII. Fig. 1. The Parts of this Machine being joined and fixed together, in the manner above-deſcribed, incline the Piece *n m*, till the Height to which the Mercury ſpouts, when it aſcends in a Direction almoſt vertical, is nearly equal to the Diameter of the Semicrcle deſcribed on the Plane G. Let the Veſſel *p* be fixed at ſuch an Height, and the Plane G be ſo placed that the Axis of the Circumvolution of the Cylinder B D (*Fig.* 5.) may anſwer to the loweſt Point of the Semicircle above-mentioned. Howſoever the Direction of the Jet be inclined, its Amplitude will always be near the Quadruple of the Line B M in the Semicircle, A B L, (*Pl.* 19. *Fig.* 5.) There is a ſmall Difference, which chiefly ariſes from the Reſiſtance of the Air.

Experiment 5.

1616. The Machine being diſpoſed as in the foregoing Experiment, if the Mercury ſpouts in two Directions, and the Inclination of one of them exceeds an half Right-Angle as much as the other is under it, the Mercury will cut the horizontal Line, which is drawn from the loweſt Point of the Semicrcle on the Plane G, in Points but a little diſtant from one another.

Experiment 6.

1617. Every thing being diſpoſed as before, if the Way for any Direction of the Motion be drawn on the Plane, as was ſaid in the Deſcription of the Machine, and the Index *f h* agrees with the Diviſion

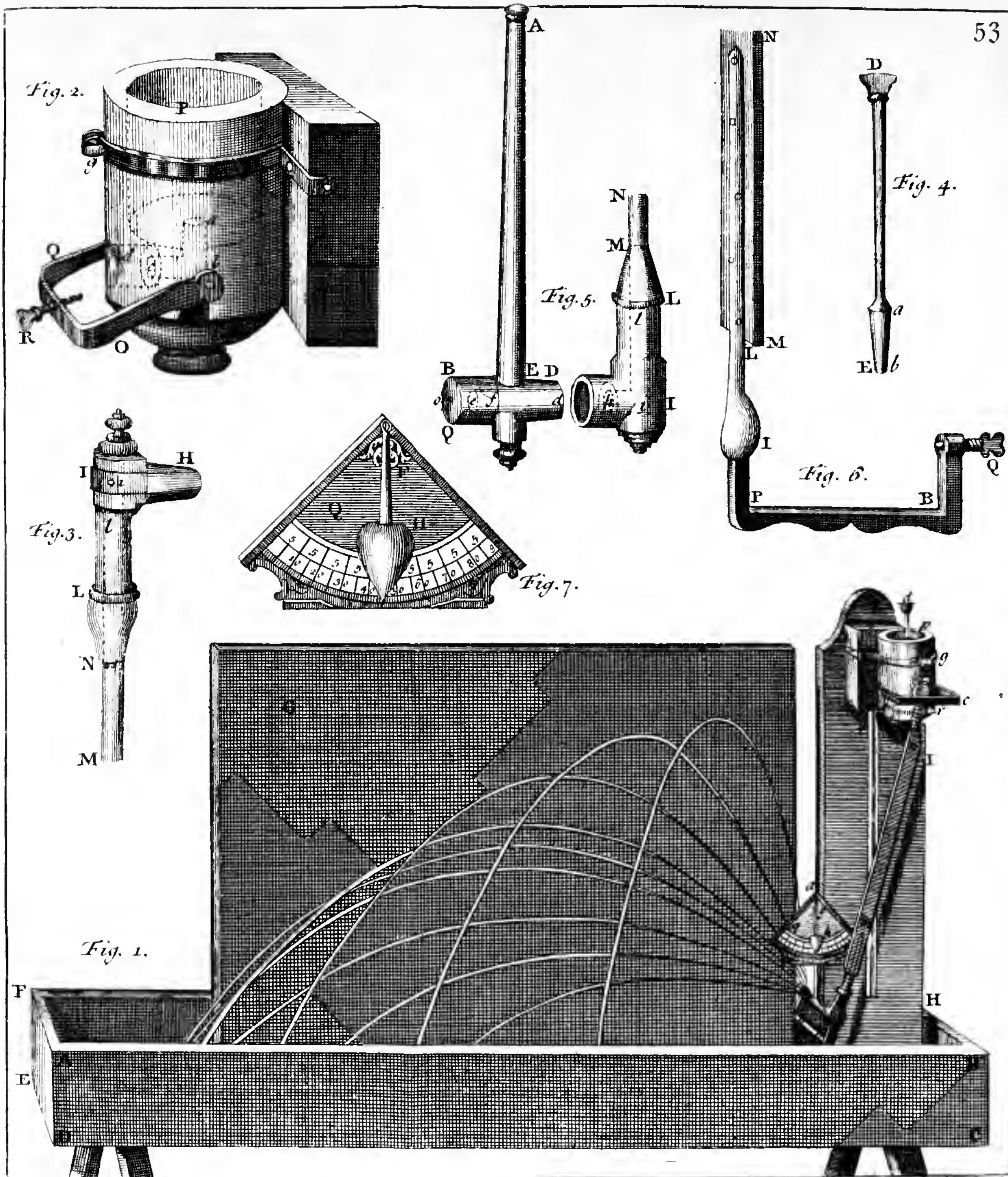


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
A
B
C
D
E
F
G
H
I
L
M
N
O
P
Q
R

viſion of the Quadrant denoting that Inclination, the Mercury in its Motion will nearly follow the Line drawn. If you draw the Ways for ſeveral Angles, by the Motion of the Handle *a e*, this may be obſerv'd ſucceſſively in theſe different Ways.

Experiment 7.

If another Plane as G be uſed, in which what is above-mention'd be drawn for another Celerity of the Mercury, the Experiments will ſucceed in the ſame Manner. 1618.

By the ſame Method, as we do by a Semicircle determine the Diſtance to which Bodies obliquely projected will fall, one may find the Diſtance to which a Fluid ſpouts out of the Side of a Veſſel, when the Veſſel is ſet upon an horizontal Plane; which Diſtance is different according to the different Height of the Hole, the upper Surface of the Liquid remaining the ſame.

Let A B be the Height of a Veſſel fill'd with a Fluid; let this Height be divided into two equal Parts at C; with the Center C and Diſtance C A deſcribe a Semicircle; *let there be a Hole at* E; laſtly, let E D be drawn perpendicular to A B, and terminated in the Circumference of the Semicircle at D. *Let the Fluid ſpout from* E *to* F *in the horizontal Plane, and the Diſtance* B F, *ſetting aſide all Retardations, will be double the Perpendicular* E D. 1619. Pl. LIV. Fig. 4.

Which will be demonſtrated, if we conſider, that the Fluid, with an equable Motion with the Celerity that it has coming out of the Hole, (in the Time that a Body can fall from E to B) runs thro' the Space B F *. In all Motion the Space run thro' follows the Ratio compounded of the Celerity and the Time †; and by multiplying this by that we have the Space run thro': that is, if this Operation be made for different Motions, you will have ſuch Quantities as will expreſs the Proportion of the Spaces gone thro'. If you make the Computation with the Squares of the Celerities and the Times, you will have the Ratio of the Squares of the Spaces gone thro'. A E here denotes the Square of the Celerity *; E B the Square of the Time †; therefore the Product of thoſe Lines expreſſes the Square of the Space gone thro' B F. But that Product is the Square of the Line E D ‖, which therefore, changing the Hole, encreaſes and diminiſhes in the ſame Ratio as the Diſtance B F. Suppoſe the Hole in the Center C; B G, the Diſtance to which the Fluid ſpouts, is equal to B A *, ſetting aſide all Retardations, and it is equal to double the Perpendicular, which at C 1620.

* 541. 1587.

† 120.

* 1586.

† 374.

‖ 13. 17. El. 6.

* 1583. 1585.

 may

may be drawn to A B in the Semicircle, which therefore obtains in all Holes, and E D will be the Half of B F.

1621. Hence it follows that *a Fluid, ſpouting from a Hole in the Center* C, *will go to the greateſt Diſtance poſſible.*

Experiment 8.

1622. We muſt here make uſe of the Machine, deſcribed in the foregoing Chapter *. Let the Water ſpout out of the Hole F, as in
Pla. LIV. Fig. 2. the firſt Experiment of the ſeventh Chapter; let it ſpout at the
* 1584. ſame time out of E, and out of G alſo; the Hole G is leſs than F, but the Hole E is at a greater Diſtance from the Surface of the Water, the Water ſpouts out of neither to the Diſtance, to which it ſpouts out of F.

1623. From what has been ſaid, it follows, That *the Water ſpouts to the*
Pl. LIV. Fig 4. *ſame Diſtance from the Holes* E *e, equally diſtant from the Center* C, becauſe in that Caſe the Perpendiculars E D and *e d* are equal.

Experiment 9.

1624. From F let there be conceived to be drawn an horizontal Line
Plate LIV. Fig. 2. which goes thro' H; H G and H E are equal, the Water will ſpout out of each Hole G and E to L.

CHAP. IX.

Of determining the Quantity of a Fluid, flowing out of Veſſels, and the Irregularities in this Motion.

1625. THE Quantity of a Fluid, which *in a given Time* flows *out of a given Hole*, encreaſes in proportion to the Velocity of the Fluid going out; this Velocity depends upon the Height of the Fluid above the Hole, and it is no matter to what part the Motion
* 1574. of the Fluid is directed *; and *ſetting aſide the Retardations, the Squares of the Quantities flowing out are in the Ratio of the Heights*
† 1586. *of the Fluid above the Holes* †.

1626. *In the Time in which a Body falling freely goes thro' the Height of the Fluid above the Hole, a Column of the Fluid flows out equal in*
* 1583. 376. *Length to twice that Height *, ſetting aſide the Retardations.*

1627. The Hole itſelf is the Baſe of the Column, and is given; if the Height of the Fluid above the Hole is known, the whole Column

is known; the Time alſo is eaſily determin'd by Experiments *; * 415. 883.
but having found what Quantity flows out in a known Time, one may know what Quantity will flow out in a given Time.

But if we compare what is here demonſtrated * concerning the 1628. Quantity of the Fluid going out, with what is demonſtrated con- * 1626. cerning the Preſſure of Fluids †, ſomething like a Paradox will fol- † 1431. low, tho' that be deduced from this. The Preſſure, which com- Pl. LIV. municates Motion to the Fluid going out of *c d*, is equal to the Fig 1. Weight of the Column *c m*; as we have ſeen above *. If this * 1577. Column were incloſed in a Tube, and ſhould fall by its own Weight only, like a ſolid Body, it would be driven downwards by this ſame Preſſure. Let theſe two equal Preſſures act during the Time, in which a Body falls the Height *m d*; to the Pillar *m c* this Preſſure will communicate a Velocity, whereby that Preſſure drives out the Fluid *. The Velocities, and Times are equal; but the Matter * 1583. moved in the laſt Caſe is double †. And therefore the whole Ef- † 1626. fect is double.

We deduce this Difference from what is obſerved above. If the 1629. Column *c m* ſhould act above, that would not be the Motion which really obtains †; but the lateral Preſſure is to be added, that † 1577. the Preſſure mention'd may be continued without Intermiſſion upon the Fluid, which goes out ||. This is the Action, which drives on || 1578. the moving Cauſe, and without the Help of which this cannot produce its Effect; therefore that is to be ſupperadded to this, that we may have the whole Cauſe, which exerts the Effect *, and of * 706. which this follows the Proportion.

In the firſt Moment only the Weight of the Column *c m* acts; but immediately after the lateral Preſſure acts, which I ſhall call the aſſiſtant Cauſe, by whoſe help the firſt Cauſe is preſerved in the ſame State; therefore as much as this Cauſe would loſe in acting, is ſupplied by the aſſiſtant Cauſe: but it would loſe in proportion to its Effect; therefore the Action of the aſſiſtant Cauſe is equal to the Effect which the firſt Cauſe, if it ſhould act alone, would produce. Therefore whilſt they act together, the Effect is double; but this is that which was to be illuſtrated.

But the Quantity of the Fluid, which we diſcover by the Com- 1630. putation mention'd *, does very ſenſibly exceed that, which really * 1627. goes out; and what is very remarkable, *The Experiments relating* 1631. *to the Velocities, and thoſe, in which the Quantities of Fluids, flowing out of Holes in a certain Time, are immediately meaſured, are not reciprocal*;

reciprocal; and this Quantity cannot be determin'd, from the known Velocity.

1632. The chief Cause of this Difference is the Irregularity of the Motion, spoken of above *; and which, though it be chiefly observed in great Holes, yet takes place in all; by this Irregularity the flowing out of the Water is hinder'd more, than its Velocity is diminished: The Base of the Column is less than the Surface of the Hole, as will be manifest, if the Column be measured at a small
1633. distance from the Hole. For this Reason, if the Water flows out through a short Pipe, for Example, of about an Inch long, a greater Quantity will go out, than there would through a Hole of the same Breadth, when the Tube is taken away.

* 1609.

1634. There is also an Error in the Measure of the Velocity. The Fluid, which passes out near the Sides of the Hole, suffers a Friction, and is retarded; which that Fluid does not, which flows out of the Center of the Hole: this last indeed is retarded by the lateral Fluid to which it adheres; but the Parts of a Fluid are easily mov'd amongst one another, and this Retardation is small in respect of the other; therefore also the Fluid is but little accelerated by the lateral Action of that, which flows out through the middle of the Hole, and this continually moves faster than that; yet the lateral Fluid is not separated from the middle Fluid. For though the Parts of Fluids are easily mov'd amongst one another, they are separated from one another with difficulty: therefore the middle Fluid, by its continual Efflux, carries the lateral Fluid with it, which tho' it moves slower, yet comes to the same Distance, or Height, with the middle Fluid.

But we can only judge of the Velocity from the Distance, or Height; but the Velocity, which is thus determin'd, wants a little of the Velocity, with which, the Fluid goes out of the middle of the Hole, because this in its whole Motion is retarded by the lateral Fluid, and other Causes. But this Velocity very much exceeds the Velocity of the lateral Fluid, as follows from what has been said; if any one therefore attributes the Velocity measured to the whole Fluid that runs out, he will determine a greater Quantity of the Fluid to run out in a certain Time, than what really does; but he will exceed the Truth less, than if he sets aside all the Retardations in determining the Velocity, and makes his Computation according to the Rule deliver'd in N. 1626.

1635. But *it appears by Experiments, that the Quantities of Water flowing out of equal Holes, in a certain Time*, if the Water be brought through

through wide Pipes, and goes out through a Hole in a Plate, *are in a Ratio little differing from the subduplicate Ratio of the Height of the Water above the Hole:* but as this Ratio can only take place very nearly, when the Heights differ very much, the Rule can be of no use.

When Computations shall be to be made concerning the Quantity of Water, which flows out of a given Hole, the Height of the Water above the Hole remaining the same, the Table underneath will be of use, which is not to be lengthen'd out to greater or less Heights. Upon what Experiment this depends, and what was to be observ'd in the Calculation of it, I shall shew in the *Scholium* annex'd to this Chapter. 1636.

I suppose the Water to flow out of a circular Hole, of the Diameter of half a *Rhinland* Inch; the Feet mention'd here are *Rhinland* Feet. 1637.

The Height of the Water.	*The Time in which a Cylindric Foot of Water runs out.*	*The Height of the Water.*	*The Time in which a Cylindric Foot of Water runs out.*
4 Feet - -	52,16 Min. S.	13 Feet - -	28,94 Min. S.
5 - - - - - -	46,66	14 - - - - - -	27,88
6 - - - - - -	42,59	15 - - - - - -	26,94
7 - - - - - -	39,43	16 - - - - - -	26,08
8 - - - - - -	36,89	17 - - - - - -	25,30
9 - - - - - -	34,78	18 - - - - - -	24,59
10 - - - - - -	32,99	19 - - - - - -	23,93
11 - - - - - -	31,55	20 - - - - - -	23,33
12 - - - - - -	30,12	21 - - - - - -	22,71

If the Holes are different, and the Height remains the same, the Quantity of the Fluid, which runs out in a certain Time, follows the Ratio of the Hole, if in all Points of the Hole the Fluid be carried with equal Velocity; which though it does not obtain, yet it appears by Experiments made with Water, that the Quantities, which really go out, differ but little from the Ratio mention'd. 1638.

Cæteris paribus, it is manifest that *the Quantities which flow out, are* as the Times: therefore *in general* these Quantities are *in a Ratio compounded of the Times, the Holes* *, *and the square Roots of the Height of the Fluid above the Holes* †. 1639.

* 1638.
† 1635.

In Vessels which are not supplied by the flowing in of the Fluid, the Celerity of the Fluid flowing out is continually changed; 1640.

to

to which Regard muſt be had, when you compare together the Times in which different Veſſels are emptied.

Here we conſider cylindric Veſſels; and what is here ſaid may be applied to any Veſſels that are of the ſame Bigneſs from Top to Bottom; we ſuppoſe the Fluid to flow out from a Hole in the Bottom.

1641. *The Times in which cylindric Veſſels of the ſame Diameter and Height are emptied, the Liquid flowing from uneqal Holes, are to each other inverſely as thoſe Holes.*

Let us ſuppoſe theſe Veſſels divided into very ſmall equal Parts, by Planes parallel to their Baſe; and the Diviſions of each Veſſel not to differ from one another: when we conſider the ſmalleſt Parts, one may ſuppoſe that the Celerity is not changed in the emptying of one Part. The Quantity of a Fluid which flows from a Hole, if the Height is not changed, encreaſes with the Hole, and a certain Quantity of a Fluid is emptied in ſo much ſhorter Time, as the Hole is greater; and this Time is diminiſh'd in the Ratio, in which the Hole is encreaſed. Whilſt the correſpondent Parts in the Veſſels are emptied, the Heights are equal; alſo the Parts themſelves, and therefore the Quantities of the Fluid, flowing out, are equal; therefore the Times are in an inverſe Ratio of the Holes; which, as it happens in all the correſpondent Parts, muſt alſo be
* 12 El. 5. referr'd to the Times of the whole Emptyings of the Veſſels*.

1642. *When the cylindric Veſſels are unequal, and equally high, they are emptied through equal Holes, in Times that are to one another as the Baſes of the Cylinders.* Let the Veſſels again be ſuppoſed to be divided into very ſmall Parts, and equal in Number in each Veſſel, in ſuch manner that the correſpondent Parts may have equal Heights, and may therefore be at equal Diſtance from the Bottom. When the correſpondent Parts are emptied, the Fluid flows out of both Veſſels, through equal Holes, and with equal Velocities; therefore the Quantities that flow out are as the Times; and conſequently the correſpondent Parts themſelves are in that Ratio of the Times, which are as the Baſes of the Cylinders: But the Times of the whole Emptyings are as the Times in which the correſpon-
* 12 El. 5. dent Parts are emptied*.

1643. Laſtly, *Let there be two cylindric Veſſels* E I, A D, *whoſe Baſes*
Pl. LIV. Fig. 5. 6. *are equal, but their Heights different*, for Example, as 1 to 4, *and let them be emptied through equal Holes.* Let theſe alſo be conceiv'd to be divided into very ſmall Parts, ſuch as H *i*, C *d*, by Planes parallel to the Baſe; and let the Number of theſe Parts be equal in each

each Vessel; and let those Parts be to one another as the Vessels, that is, as 1 to 4. All the Parts are emptied by an equable Motion, because the Parts are very small; the Celerities in the correspondent Parts are every where as 1 to 2 *, because the Heights of those Parts above the Bases are as the Heights of the Vessels, which are as the Squares of those Numbers. Whence it follows, that the Times, in which correspondent Parts are emptied, are to one another, as 1 to 2; because in twice the Time with a double Celerity, a quadruple Quantity is emptied. But as the Times are in the same Ratio for each correspondent Part, the Times in which the whole Vessels are emptied are also, as 1 to 2 *. If the Vessels are as 1 to 9, the Times will be, by a like Demonstration, as 4 to 3; and in general the Times are as the Celerities in which correspondent Parts are emptied, the Squares of whose Celerities are *as the Heights of the Vessels* *, in which Ratio *also are the Squares of the Times.*

* 1586.

* 12. El. 5.

* 1586.

EXPERIMENT I.

1644. Plate LV. Fig. 1, 2.

There are three Cylindric Vessels A, C, B of thin Metal, having equal Diameters, and whose Heights are, as 1, 3, and 4; each of them has a Lip in the Top to let the Water run out when it comes to a certain Height, which Lip must be reckoned the Top of the Vessel; in the Bottom of the Vessels A and B, which are as 1 to 4, there are equal Holes, and they are filled with Water; the Holes are opened in the same Moment; if the Water running out of B be received in the Vessel C, it will be filled in the same Time that A is emptied. C contains three quarters of the Vessel B; the Quarter which is left will also be emptied in the same Time as the Vessel A, which is evident to Sense; therefore A is emptied twice, whilst B is emptied once.

1645. *The Times in which any cylindric Vessels are emptied, are in a Ratio compounded of the Bases* *, *of the square Roots of the Height* †, *and of the inverse Ratio of the Holes* ‡.

* 1642. † 1643. ‡ 1641.

1646. Plate LIV. Fig. 6.

The cylindric Vessel may be so divided, that the Parts intercepted between the Divisions shall be emptied in equal Times, which will happen if the Distances of the Divisions from the Base be as the Squares of the natural Numbers; for the Times of the Emptyings of the Vessels, whose Heights are in that Proportion, are as the natural Numbers *, and the Differences of the Times are equal.

* 1644.

1647. The Time in which a cylindric Vessel is emptied, is as the Celerity with which the Fluid begins to run out *; therefore the Ce-

* 778. 743.

 lerity,

lerity, while the Fluid deſcends in the Veſſel, is diminiſhed in the ſame Ratio as the Time of the emptying of the Fluid remaining

1648. in the Veſſel, and *the Motion of a Fluid running out of a cylindric Veſſel, is equally retarded in equal Times.*

1649. *If thro' equal Holes a Fluid runs out of a Cylinder, and out of another Veſſel of the ſame Height (and in which the Fluid is always ſupplied ſo as to be kept at the ſame Height) in the Time in which the Cylinder is emptied, there runs out twice as much Water from*

1650. *the other Veſſel as from the Cylinder.* For, becauſe of the equal Heights of the Veſſels, the Celerities in the Beginning are equal; the Celerity of the Fluid, which comes out of the Veſſel that is always kept filled, is equable; the Celerity of the Fluid, which

* 1648. runs out of the Cylinder is equably retarded *. Therefore whilſt the Cylinder is emptying, there will flow twice as much Water out of the Veſſel as out of the Cylinder. For if two Bodies are driven with the ſame Celerity, and the firſt goes with an equable Motion, and the ſecond with a Motion equally retarded, and they move 'till this has loſt all its Motion, the firſt in that Time will run double the Space of the ſecond *; here the Fluid that runs

* 376. 377. 378. out may be looked upon as the Space gone thro', becauſe the Holes are equal.

1651. We have obſerved above, that the Coheſion of Parts retards the Motion of Fluids, we alſo obſerve the contrary in many Caſes; and tho' the Velocity, ariſing from the Preſſure, is the ſame every way, yet a Fluid is moved faſteſt, when it deſcends vertically; this, in its Motion, is continually accelerated by falling, adhering to the following Fluid, and drawing it along with it, it encreaſes the Velocity of the Fluid, flowing out of the Veſſel.

1652. *The Motion out of a Veſſel, whoſe lower End has a Tube joined*

Plate LV. Fig. 3. *to it, is accelerated much more.* Let E be ſuch a Veſſel; faſten'd to the Tube *c b*, both of whoſe Orifices we ſuppoſe equal.

1653. It immediately appears, that there cannot flow out thro' the Orifice of the Tube a greater Quantity of the Fluid, than what enters in thro' the upper Orifice; but to determine the Force, and, this being given, the Velocity, with which the Fluid comes into this Orifice, we muſt deliver the Cauſes, by which the Fluid is brought in.

1654. By the Preſſure of the ſuperincumbent Fluid, the Particles going out of the Veſſel, and into the Tube, there is communicated a Force, which they would each acquire, in falling from the Height

a

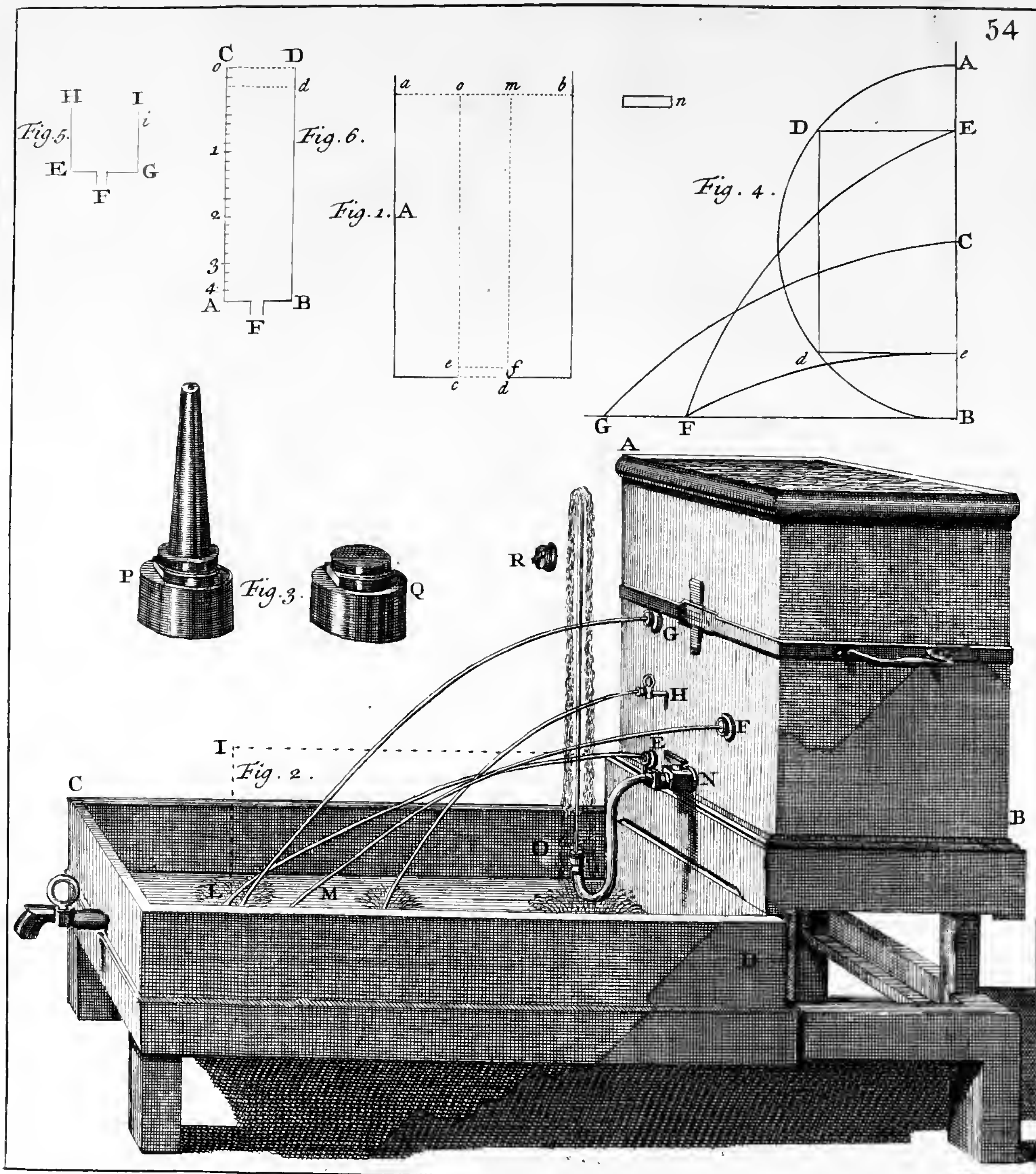


Fig. 5.
H
I
i
E
F
G
Fig. 6.
C
D
o
d
1
2
3
4
A
F
B
Fig. 1. A
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c
d
Fig. 4.
A
D
E
C
d
e
G
F
B
P
Fig. 3.
Q
R
Fig. 2.
A
B
C
I
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H
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L
M
D

J. Mynde sc.

a b *, but moreover these are also drawn downwards by the Weight * 1583.
of the Pillar contain'd in the Tube.

The Parts of the Fluid do not only cohere together in such manner, that all of them, which are in the Tube, make as it were one Body, but they also adhere to the Tube itself; for which Reason this remains fill'd with a continuous Fluid; and by reason of the Equality of the Orifices, it goes out of the Tube with the same Velocity, that it comes into it, and there is no Acceleration in the whole Descent along *b c*. But this is hinder'd by the re-action of the Particles, which are accelerated in their Entrance into the Tube by the Action of the lower Particles. 1655.

The Particles whilst they descend thro' the whole Length of the Tube, continually act by their whole Weight; the Effect of the Pressure, its Intensity, and the Space passed thro' remaining the same, is always the same *; therefore this Action of the Particles is * 727.
equal to the Force, which Gravity can communicate to them in the Descent thro' *b c*; which Force is always the same, whether the Particles run thro' this Space faster or slower *. * 754. 755.

1656. Now if we consider the whole acting Force, during any Time whatsoever of the Efflux, we have first the Force which the Particles would acquire in falling the Height *a b* *; we have besides * 1654.
the Force, which they would acquire in falling the Height *b c* †; † 1655.
which together are equal to the Force acquir'd in falling both Heights join'd together *, namely the Height *a c*. All the Par- * 754.
ticles flowing out act with a like Force, before they go out of the Tube; and this whole Action is consum'd in communicating Motion to these Particles; and is equal to the Force communicated *. Therefore, as all the Particles go out with the same Ve- * 700.
locity, that the Effect may be equal to the Action, they all necessarily go out with that Force and Velocity, which they would acquire in falling the Height *a c*; and with this Velocity at *b* they go into the Tube. The Velocity is diminish'd by the Friction against the sides of the Tube, oftentimes but a little. But it is diminish'd more, if the Height *a b* be less in respect of the Length of the Tube; also if the Tube be narrower, or longer.

Experiment 2.

1657. Pl. LV. Fig. 4. The Vessel E is equal and similar to the Vessel A, *Fig.* 1. and with its Tube is of the same height as the Vessel B, *Fig.* 2. The two Orifices of the Tubes are equal to one another, and to the Holes in the Bottoms of the Vessels A and B; that is, their

 Diameters

Diameters are equal to the third part of an Inch. Fill the Veſſels B and E with Water; and open the Holes at the ſame Time, and the Surface of Water at B will deſcend faſter than at E: But the Difference is but ſmall.

1658. *Let the upper Hole of the Tube, by which it communicates with the Veſſel, as alſo its Length, remain as before; and the lower Hole be open'd wider; then a greater Quantity of Water will flow out*, and the Water which goes into the Tube, will be more accelerated. In this Caſe thro' the upper Hole of the Tube, there flows out a greater Quantity of Water, than from an equal Hole four times the depth.

1659. If to this Caſe we apply the Reaſoning, which was applied to the foregoing Caſe; it will be manifeſt that, ſetting aſide the Cauſes of the Retardation, all the Particles have that Force, before they go out, which they can acquire in falling the Height *a c*; and that therefore they go out of the lower Orifice, with that Velocity, which a Body can acquire, in falling this Height. Whilſt the Particles go into the Tube thro' the upper Orifice, they are carried with a greater Velocity, and the Force of each exceeds that, which we have mention'd; but they loſe this again whilſt they draw the following Particles into the Tube, and communicate to them a like greater Velocity, which theſe alſo immediately loſe. The Force, with which we affirm'd the Particles to go out of the lower Orifice, is that, whoſe Effect remains, when the Particles have reach'd the lower Orifice, and this only is to be conſider'd here. The Quantity which goes out, ſetting aſide the Friction, is that, which, if the Height of the Veſſel were *a c*, and its Capacity remain'd the ſame, would run out of a Hole in the Bottom, equal to the lower Orifice of the Tube; namely, if the Tube ſhould always remain full; which will always happen, when the lower Orifice of the Tube does not very much exceed the upper: but how far that may exceed this, depends upon the Coheſion of the Parts of the Fluid.

EXPERIMENT 3.

1660. Plate LV. Fig. 5. The Veſſel F no way differs from the Veſſel E (*Fig.* 4.) but in having the lower Hole of its Tube bigger; and comparing together F, E and B (*Fig.* 2.) the Diameters of the three Veſſels are equal, and the Holes in the Bottom are equal, namely of the Diameter of four Lines, that is, a third part of an Inch; but the lower Hole *c* of the Tube, joined to the Veſſel F, is five Lines.

Fill

Fill with Water the Veſſels F and B; if the Water flows out of both Veſſels in the ſame Time, the Surface of the Water will deſcend faſter in the Veſſel F than in B. The Veſſel B is about 16 Inches high.

To theſe Experiments I ſhall ſubjoin two other remarkable ones concerning the Coheſion of Parts, whereby the Effect of this Coheſion are illuſtrated.

Experiment 4.

The two equal Syringes A, B, are by Screws join'd to the two 1661.
Tubes E *d a*, F *d b*, which are faſten'd together; the Axes of Pl. LV. F. 6.
theſe Tubes are in the ſame Plane, and cut one another at Right Angles, and the Tubes meet at *d*.

The Syringe A is fill'd with Water ting'd with Red, or any other Colour; B is fill'd with common Water, the Piſtons are joined by the Plate L, which is faſten'd by Screws. If the Piſtons be thruſt down together, the ting'd Water follows the Way E *d b*, the other the Way F *d a*; and there is ſcarce any ſenſible Mixture of the Waters, whilſt they croſs one another at *d*, and go in curve Lines.

Experiment 5.

This Experiment differs from the foregoing one only in one Cir- 1662.
cumſtance, but the Effect is quite different. The Axes of the Pl. LV. F. 7.
Tubes E *d a*, F *d b*, are not in the ſame Plane, but the Axis of the one does as it were touch the Cavity of the other in ſuch manner, that the Tubes are only join'd together in part at *d*. The Piſtons being now thruſt in, the colour'd Water, which in part paſſes freely thro' E *d a*, draws all the other colour'd Water along with it; whilſt the pure Water is in the ſame manner carried thro' F *d b*; they being ſcarce ſenſibly mix'd, tho' the Waters paſs thro' by one another at *d*.

This Experiment led the celebrated Author into an Error, who 1663.
made this Experiment, when he intended to try the foregoing one, and he drew a Concluſion contrary to both Experiments; namely, that the Particles of a Fluid continue their way freely, without any mixture between the Particles of another Fluid, tho' this laſt Fluid moves in a different Direction.

SCHOLIUM.

SCHOLIUM.

I Said that I would in this *Scholium* deliver the Experiment from which, and the Manner how the Table N. 1637. was calculated.

1664. *Mariotte* from an Experiment, many times repeated, the necessary Cautions being observed, found that out of a Hole, whose Diameter was $\frac{1}{4}$ of an Inch, the Water being kept 13 Feet above it, there flowed each Time, in one Minute, 28 Pints, 70 of which make a Cubic Foot. I speak here of the *French* Foot, which is to the *Rhineland* Foot, as 144 to 139.

This Experiment being given, we must discover in what Time a cylindric Foot can run out thro' a Hole of an half Inch Diameter, supposing the Height of the Water 13 Foot above this also, the *Rhineland* Measure being used.

1665. The Time, in which a certain Quantity of Water runs out, is the shorter, the greater the Quantity is, that runs out in a determined Time; therefore the Times are inversely as these Quantities, which, *cæteris paribus*,
* 1635. are in a sub-duplicate Ratio of the Heights *.

The Times are also the shorter, the greater the Holes are; that is, other Things being equal, they are in an inverse Ratio of the Squares of the Diameters of the Holes.

Lastly, *cæteris paribus*, the Times are directly as the Quantities that run out.

In the Experiment made by *Mariotte*, the Height of thirteen *French* Feet is to the Height of so many *Rhineland* Feet, in the Case of which we are speaking, as 174 to 139.

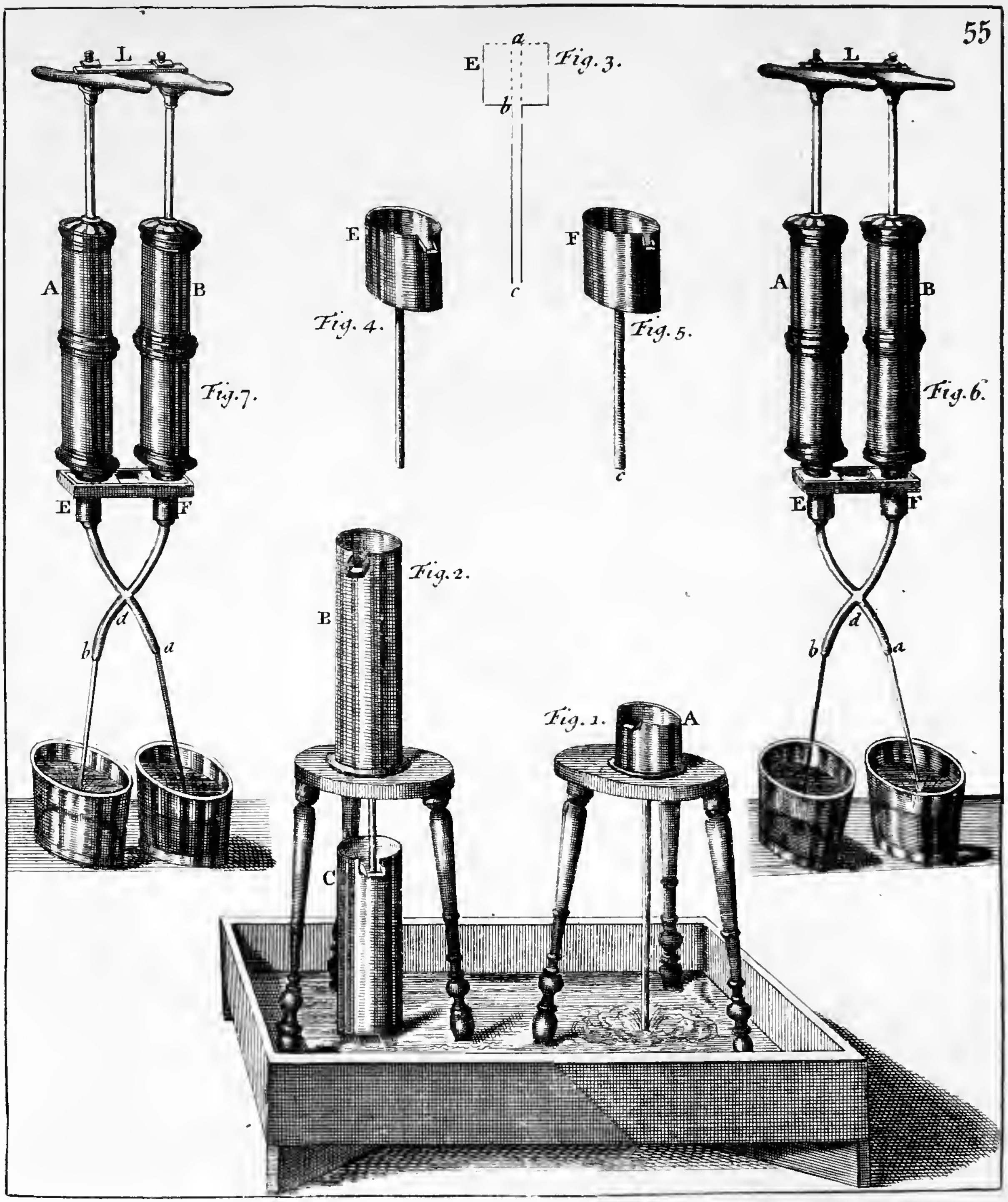
The Squares of the Diameters of the Holes are as 1 to 4, and as $\overline{144}^{q}$ to $\overline{139}^{q}$.

The Quantities of Water are, as 28 Pints to a *Rhineland* Cylindric Foot; which Quantities are in a Ratio compounded, of the Ratio of 28 to 70 or 14 to 35; that is, of the Quantity which flowed out to a *French* Cubic Foot, and the Ratio of a *French* Cubic Foot to the *Rhineland* Cubic Foot, as also of the Ratio of a Cubic Foot to a Cylindric Foot, or of 452 to 355.

Therefore the Time of one Minute, or 60 Seconds, is to the Time sought, in a Ratio compounded of these six Ratios, of $\sqrt{139}$ to $\sqrt{144}$, 4 to 1, $\overline{139}^{q}$ to $\overline{144}^{q}$, 14 to 35, $\overline{144}^{c}$ to $\overline{139}^{c}$, and 452 to 355.

The first, third, and fifth, are reduced to the Ratio of $\sqrt{144}$ to $\sqrt{139}$;
1666. and 60 *Seconds are to the Time sought, as* $4 \times 14 \times 452 \times 12$ *is to* $1 \times 35 \times 355 \times \sqrt{139}$. which Time is found to be 28,04 Seconds. Which Time being given, the other Things, which are observed in the Table N. 1637. are found, by seeking Numbers which are in an inverse sub-duplicate Ratio of the Heights.

CHAP.



L
A
B
Fig. 7.
E
F
d
b
a
a
E
b
c
Fig. 3.
E
Fig. 4.
F
Fig. 5.
c
B
Fig. 2.
C
Fig. 1.
A
L
A
B
Fig. 6.
E
F
d
b
a

CHAP. X.

Of the Running of Rivers.

DEFINITION 1.

THE Water that runs by its own Gravity, in a Channel open above, is called a River. 1667.

DEFINITION 2.

A River is ſaid to remain in the ſame State, or *to be in a permanent State, when it flows uniformly, ſo as to be always at the ſame Height in the ſame Place.* 1668.

DEFINITION 3.

A Plane, which cutting a River is perpendicular to the Bottom, and to the Direction of the Motion of the Water, is called the Section of a River. 1669.

When a River is terminated by flat Sides parallel to each other, and perpendicular to the Horizon, and the Bottom alſo is a Plane either horizontal or inclined, the Section of the River with theſe three Planes makes Right Angles, and is a Parallelogram.

In every River that is in a permanent State, the ſame Quantity of Water flows in the ſame Time thro' every Section. For unleſs there be in every Place as great a Supply of Water, as what runs from it, the River will not remain in the ſame State. And this Demonſtration will hold good, whatever be the Irregularity of the Bed or Channel, from which in another reſpect, ſeveral Changes in the Motion of the River ariſe; as, for Example, a greater Friction in proportion to the greater Inequality of the Channel. 1670.

The Irregularities in the Motion of a River may be infinitely varied, and Rules cannot be given to ſettle them: Therefore ſetting aſide all Irregularities, we muſt firſt examine the Courſe of Rivers; for unleſs the Laws of Motion be known in that Caſe, we have no certain Foundation for determining any thing; we muſt afterwards conſider what really happens in Rivers. 1671.

We ſuppoſe the Water to run in a regular Channel, without any ſenſible Friction, and that the Channel is terminated with plane Sides, that are parallel to one another and vertical; and alſo that the Bottom is a Plane, and inclined to the Horizon. 1672.

Let

Plate LVI. Fig. 1. Let A E be the Channel, into which the Water runs from a greater Receptacle or Head; and let the Water always remain at the ſame Height in the Head, ſo that the River may be in a permanent State. The Water deſcends along an inclin'd Plane, and is accelerated *; whereby, becauſe the ſame Quantity of Water flows thro' every Section †, *The Height of the Water, as you recede from the Head of the River, is continually diminiſh'd*, and the Surface of the Water will acquire the Figure *i q s*.

* 382.

1673.

† 1670.

1674. To determine the Velocity of the Water in different Places, let us ſuppoſe the Hollow of the Channel A B to be ſhut up with a Plane; if there be a Hole made in the Plane, the Water will ſpout the faſter thro' the Hole, as the Hole is more diſtant from the Surface of the Water *b i*; and the Water will have the ſame Celerity that a Body, falling from the Surface of the Water to the Depth of the Hole below it, would acquire *; which ariſes from the Preſſure of the ſuper-incumbent Water. There is the ſame Preſſure, that is, the ſame moving Force, when the Obſtacle at A B is taken away; for we ſuppoſe the Receptacle ſo capacious, that in this Caſe the lateral Preſſure may act upon the Water, which enters into the Channel.

* 1583.

Then every Particle of Water enters into the Channel, with the Celerity that a Body would acquire in falling from the Surface of the Water to the Depth of that Particle. This Particle is moved along in an inclin'd Plane in the Channel, with an accelerated Motion; and that in the ſame manner, as if, in falling vertically, it had continued its Motion to the ſame Depth below the Surface of the Water in the Head of the River *.

* 393.

If you draw the horizontal Line *i t*, the Particle at *r* will have the ſame Celerity as a Body falling the Length *i* B, and running along B *r*, can acquire; which is the Celerity acquir'd by the Body in falling thro' *t r* *. Therefore *the Celerity of a Particle may be every where meaſur'd, drawing from it a Perpendicular to the horizontal Plane, which is conceiv'd to be drawn along the Surface of the Water in the Head of the River; and the Velocity, which a Body acquires in falling thro' that Perpendicular, will be the Celerity of the Particle*, which is greater the longer the Perpendicular is; and *it is not encreas'd by the Preſſure of the ſuper-incumbent Water*, which cannot encreaſe the Celerity of the Water, which has a greater from another Cauſe, than can ariſe from this Preſſure: in like manner, as a Body following another Body, cannot act upon that which goes before, with a greater Celerity.

* 393.

1675.

1676.

From

From any Point as *r*, draw *r s* perpendicular to the Bottom of the River, which will measure the Height or Depth of the River; if this be continued upwards, so as to come to the Line *i t* at *t*; it plainly appears, that the Celerities of the Particles in the Line *r s*, are so much the less, the nearer they are to the Surface of the River, *and that the lower Water is mov'd faster than the upper Water.* 1677. 1678.

Yet *the Celerities of those Waters, as the River runs on, continually approach nearer and nearer to an Equality.* For the Squares of those Celerities are as *r t* to *s t* *, the Difference of which Lines, as you recede from the Head of the River, is continually lessen'd, because of the Height *r s* *, which is also continually diminish'd, whilst these Lines are lenghten'd. Now as this obtains in the Squares, it will much more obtain in the Celerities themselves, whose Difference therefore is diminish'd as they encrease. 1679.

* 1675. 374.

* 1673.

If the Inclination of the Bottom be chang'd at the Head of the River, so as to become *y* Z, and *a greater Quantity of Water flows into the Channel, it will be higher every where in the River, but the Celerity of the Water is no where chang'd.* For this Celerity does not depend upon the Height of the Water in the River; but, as has been demonstrated, upon the Distance of the moved Particle from the horizontal Plane of the Surface at the Head continued over the said Particle, which Distance is measur'd by the Perpendicular *r t* or *s t*; but these Lines are not chang'd by the Afflux of Water, provided that the Water remains at the same Height in the Bason or Head. 1680.

Let the upper Part of the Channel be stopp'd up by an Obstacle, as X, *which descends a little way below the Surface of the Water*; the whole Water which comes cannot run through; therefore it must rise up: but the Celerity of the Water below this Cataract is not encreas'd *; and the Water that comes on is continually heap'd up, so that at last it must rise so as to flow over the Obstacle or the Banks of the River. But *if the Banks be rais'd, and the Obstacle be continued, the Water would rise above the Line i t*; but, before that, the Celerity of the Water cannot be encreas'd: In which Case the Height of all the Water in the Head will be encreas'd; for, as we suppose the River in a permanent State, there must continually be as great a Supply of Water to the Head, as there now runs from it down the Channel; but, if less Water runs down, the Height must necessarily be encreas'd in the Head, 'till the Celerity of the Water flowing under the Obstacle be so much encreas'd, that the same 1681.

* 1680.

Quantity of Water ſhall run under the Obſtacle, as uſed to run in the open Channel before.

1682. All theſe Things, as we have already ſaid, if we abſtract from all the Irregularities, are true; and the leſs the Irregularities are, the more will the true Motions agree with what we have ſaid; of which, as alſo of the Changes which really happen in Rivers, I ſhall now ſpeak.

1683. The Earth is ſpherical, and heavy Bodies tend towards its Center; yet this Figure is not accurate, and the more depreſs'd Places are cover'd with Waters, which being collected make Seas and Lakes. In receding from theſe, the Surface of the Earth is rais'd up to a certain Height, and is again depreſs'd, towards other Seas, or Lakes. Beſides theſe Heights, which are extended a great way, by Mountains in many places, whether near to the Sea, or remote from it, there is caus'd a more ſenſible Inequality in the Surface of the Earth.

1684. In many places upon the Surface of the Earth, eſpecially in mountainous Places, there are Springs of Water; the Water by its Gravity deſcends from the higher Places to the lower; many Rivulets meet, and by continually deſcending, hollow the Surface of the Earth, and make a River, and to which frequently Water flows alſo, from neighbouring ſubterraneous Places, thro' imperceptible Paſſages; and thus the River acquires Force in its Motion. The Water in its Deſcent often meets with Obſtacles, and turns aſide, and goes on in its Courſe. Hence ariſe the Inflexions of Rivers. The Water continuing to deſcend, at laſt reaches the Sea, and runs into it, often paſſing over a vaſt Tract of Land.

1685. The River, that reaches the Sea, in its whole Courſe, runs in the moſt depreſs'd Places, and the Surface of the Earth is rais'd up in receding from it ſidewiſe; wherefore leſs Rivers on the right and left Hand, tend to the greater Rivers, as theſe do to the Sea.

1686. It is certain, that *the Channels of all Rivers were not at firſt made hollow* towards the Sea, *by Nature* in ſuch manner, as we have
* 1684. explain'd it *; oftentimes Waters being collected together in a low Place, and endeavouring to break out in ſeveral Places, ſo as would be of detriment to the neighbouring Inhabitants, were brought thro' Channels, made by Men, to the lower Places, whence they might eaſily run out afterwards, and of themſelves continue their Channel.

1687. From what has been ſaid it is manifeſt, that many Changes muſt have happen'd in Rivers in a long Courſe of Time, and it is alſo

alſo plain, that theſe Changes ought at length to ceaſe in many Rivers.

But to illuſtrate what belongs to this Subject, we muſt enquire 1688.
what Cauſes may change the Velocity of a River, and what will follow from this Change of Velocity.

We have ſeen that the Water in a River, as we recede from its 1689.
Head, is continually accelerated * ; but this is only true, when we * 1675.
ſet aſide the Retardations : but in all Rivers there are many Cauſes of Retardation, which always act contrary to the accelerating Force, and their Effects encreaſe, as the Velocity of the Water encreaſes.

The Water is only *accelerated, as long as the accelerating Cauſe* 1690.
overcomes the Impediments. But *when the Retardation is equal to the* 1691.
Acceleration, the River flows with an equable Motion. If the Retardation then encreaſes, from ſome new Cauſe, the Velocity is diminiſh'd; and we *oftentimes* obſerve, that *the Water in a River runs* 1692.
ſlowly, in a Place which is remote from the Head of the River, and to which Place the Water could not come without running down from a great Height.

In this Caſe what we ſaid before is not true, *viz.* that the Water 1693.
in a River is not accelerated when its Height is encreas'd * ; if for * 1680.
Example the River runs four Feet in a Second, that is, if the Water moves with that Velocity, which a Body acquires in falling from an Height of about three Feet, it will be accelerated, if the Surface be rais'd four Inches ; as follows from what is explain'd above *. * 1674.

Whence we deduce this Concluſion, that *the Water is often ac-* 1694.
celerated in a River that runs ſlowly, if a new Quantity of Water flows into it, or if the Channel be narrow'd ; for this raiſes the Surface, ſo as to encreaſe the Preſſure upon the lower Water.

But we muſt not conclude that the Velocity is always encreas'd, 1695.
when the Surface is rais'd ; for if there be not a ſufficient Height, it will not encreaſe the Velocity * ; the Water will be rais'd, when * 1676.
the Channel remains the ſame, without an Addition of Water, if there ariſes any new retarding Cauſe *. * 1670.

In the River, which we conſider'd before *, we ſuppos'd the 1696.
Bottom inclin'd to the Horizon ; yet *Water may be mov'd along an* * 1672.
horizontal Channel, if it deſcends into it from an higher Place. 1697.
The Surface of the Water, ſetting aſide the Retardations, would be horizontal *in that Channel,* if it ſhould be of the ſame Breadth every where ; becauſe the Water would keep its Velocity. But the retarding Cauſes always diminiſh the Motion ; therefore *the*

Surface becomes inclined; it is higher at the Head of the Channel, than in any other Place, and the Height decreaſes as you go from the Head.

1698. For the Preſſure at the Head of the Channel ſhould overcome the Reſiſtance along the whole Channel, becauſe there is no accelerating Force: and the Preſſure, in any other Place, ſhould only overcome the Reſiſtance along the remaining Part of the Channel.

Pl. LVI. F. 2. Let E F be the horizontal Bottom, the Water is higher at E *a* than at L *b*; becauſe at E *a* there is required an Encreaſe of the Action equal to the whole Reſiſtance, which is to be overcome between E and F, whereas at L *b* that Reſiſtance only is to be deſtroyed, which obtains between L and F.

1699. But there cannot be a leſs Height in the River, towards F, un-
* 1670. leſs the Velocity be encreaſed *; wherefore a greater Difference between the Heights E A and L *b* is required, than if only the Reſiſtance were to be overcome.

1700. But *if, in the Motion along an horizontal Channel, the Water moves very ſlowly, the Surface will be inclined a very little.*

1701. We ſaid that Rivers made themſelves a Channel. If the Water paſſes over ſandy, or clayey Places, it continually wears away ſome of the Particles, and carries them along with it; Experience in-
1702. forms us, that Water hollows Stones themſelves, by a Motion continued for a long time. It is very evident, that *all Rivers have undergone many Changes*, before their Channels have acquired their due Magnitude; but who can determine the Number of them? It would indeed be of no uſe to enquire into them: it will be more uſeful to examine Rivers, as they now are, and thoſe indeed, which run over ſandy or clayey Soils; for ſcarce any thing ſenſible happens to them, when they run among Stones, till after a long time.

The Water continually rubs off ſandy, or clayey Particles, and this Action is encreaſed, when the Velocity of the Water is en-
1703. creaſed; whence it follows, that *the Water*, in running along ſuch Places, becomes turbid, and *by continually corroding the Channel, thruſts forward the Sand.* This Sand continually falls by its own
1704. Weight; and, *the Velocity of the Water continuing ſome Time, the Water becomes ſo turbid, that it lays down as much Sand, as it raiſes in the ſame Time.*

1705. *If the Velocity be* then *encreaſed, it raiſes a greater Quantity than what falls by Gravity; if the Velocity be diminiſhed, the contrary obtains.*

Beſides

Besides the Velocity, two other Causes also, *the Weight of the Water, and its Impetus, encrease the Quantity of Sand, which is moved out of its Place.* 1706.

DEFINITION 4.

We call that a regular River, the Matter of whose Channel is equable; whose Bottom is either equably inclined, or horizontal; and whose parallel Sections would all be similar and equal, if the Water were every where at the same Height. 1707.

DEFINITION 5.

We call that Line, which in all the Sections passes thro' the Point, in which the Velocity of the Water is greatest, the Thread of the River. 1708.

If the River be regular, the Thread is equally distant from both Banks, by reason of the like Causes of Resistance on either Side; if the River be not regular, the Thread often comes nearer to one Bank than the other, neither does it keep a regular Course in respect of these. 1709.

In a regular River the greatest Corrosion is in the middle of the Bottom, for this Place answers to the greatest Velocity, and its Action is encreased by the Weight of the whole Water. 1710.

The Sand is dispersed towards the Sides, and there is a double Cause of the Change of the Figure of the Channel; yet the River remains regular. But if the Water runs uniformly in such a River, all Things will be so ordered, that not only the Quantity of Sand that falls continually will be equal to that, which is raised; but this will also obtain in every part of the Channel; then the Changes will cease. 1711.

If there comes in new Water, the Corrosion is often encreased *, but only in that Place, where this Encrease is; the lower Places are not altered †. But in time these are altered also, if this Encrease be continually supplied uniformly; for when the first Place is so hollowed by the Corrosion, that the Velocity of the River in that Place is no longer encreased, the Encrease of Velocity is transferred to the next lower Place, which is hollowed also; thus the Corrosion is propagated successively as far as the Sea, and it ceases, the Channel itself being every where encreased. 1712.

* 1694. 1705.

† 1704.

We have considered a regular River, which moves in a Right Line; but a River must not always be ranked among those that are 1713.

Pl. LVI. F. 3. are irregular, becauſe its Courſe is bent; for if the two Directions make a very obtuſe Angle, as B C, C D, the Motion is often bent without any Corroſion; although this may ſeem to follow from the Impetus of the Water againſt A. By reaſon of the acute Angle, which the Bank makes with the Direction of the Water, its Action upon it is ſmall; the Water, deſcending along C D, coheres to that which follows, and draws it along with it in its Deſcent, and takes it away from A, diminiſhes its Impetus, and often entirely deſtroys it; and this is the Caſe of which we are

1714. now ſpeaking. It ſometimes happens that there is a Corroſion, by which the Bank acquires a certain Figure, which being given, there is an Equilibrium between the Impetus of the Water and the Force, which takes away the Water; in which Caſe the Corroſion of the Bank only laſts for a Time.

1715. If the River be irregular, it will be liable to many Changes. Such is a River, whoſe Banks are hollow in one Place, and prominent in another; whoſe Breadth is different in different Places; in whoſe Bottom there are Inequalities; and laſtly, whoſe Direction alters too ſuddenly.

1716. Let us ſuppoſe a River A D; whoſe Bank is hollowed between

Pl. LVI. F. 4. A and C; this is the Conſequence of this Figure. The Velocity, by reaſon of the Friction, would be ſmall in the Line A C, if the Water moved along the Bank; but now, becauſe it is removed to B, the Velocity between A and C is greater, and the Water comes to the Angle C with a certain Impetus; which Thing would happen alſo, if there were only a Prominence of the Bank at C, and no Excavation at B.

1717. But it is a general Rule, that *where the Water comes to the Bank with a certain Impetus*, there it is moved in a Vortex, and *the Bottom is hollowed*; therefore the Depth will be greater at C; there will alſo be a like Excavation at D.

1718. From the unequal Breadth of the River in different Places, it follows that there will be Changes of the Velocity, whilſt the Water proceeds in its Courſe; and that therefore it will continu-

* 1705. ally move the Sand from one Place to another*, and that the River will undergo continual Changes.

1719. The Inequalities in the Bottom retain the Water; whilſt it runs againſt them, the Velocity is diminiſhed, and the Sand is laid

* 1705. down in theſe Places *, which encreaſes the Obſtacle; and *a very*

1720. *ſmall Matter has often been the Origin of an Iſland which has ariſen*

in

in a River, which has in itself been the Cause of greater Changes afterwards.

It plainly appears, that there can be no Inflexion in the Course of a River, without an Action against the opposite Bank, as F; which Action indeed is sometimes taken away, if the Angle of Inflexion be very obtuse *, but not if it be a right or an acute Angle. 1721. * 1713.

When there is a Corrosion in any Part of the Bank, the Thread approaches nearer to this Bank; the Motion towards the opposite Bank is retarded, the Sand is laid down *, and there is an Inundation caused. 1722. * 1705.

From all this it is manifest, that there may be such Changes in a River, as may make is change its Channel, by corroding the Bank in some particular Place in such manner, as to open itself a way to the lower Places; and the Fate of the old Channel will depend upon the Velocity, with which the Water shall penetrate into this Place. 1723.

In this River, now divided, the Water is immediately retarded, 1724.
and its Bottom is rais'd; whereby the Channel is diminish'd: and 1725.
thus it frequently happens that the Mouths of a River are multiplied. And oftentimes, when there runs a great Quantity of Water into the new Channel, the old one is quite stopp'd up.

With regard to their Mouths, Rivers are liable to other Changes also; there is continually a Change of Velocity at the Mouths of Rivers, as the Sea, by its Recess or Access, assists, or hinders the Efflux of the Water into the Sea. 1726.

In the Access of the Sea the Velocity of a River is diminish'd, and the Sand falls down *; which, the Velocity being encreas'd in the Recess, is again carried to the Sea; then the River is not alter'd. This takes place in Rivers, which have a sufficient Quantity of Force to sustain the Impetus of the returning Sea; they are retarded indeed, but they don't bend their Motion to a contrary Part. 1727. * 1505.

But if the River has not so great Force, its Motion is too much retarded, and the Sand is heap'd up in a greater Quantity: in this case, before this Sand can be carried to the Sea, there continually flows thither from the higher Places a turbid Water, which lays down as much Sand, as it takes up *, and what had fallen to the Bottom it there leaves; and thus the Bottom is continually rais'd. 1728. * 1704.

The continual Change of the Bottom mention'd *is the Cause of the Dilatation of the Mouths, and the reason why Rivers often open to themselves new Mouths.* 1729.

Yet

1730. Yet this Change of the Bottom will not take place in a weaker River, which has a very wide Mouth: For then the Sea in its Recess not only carries back the Sand, which it brought, but that also, which the River supplied in a small Quantity.

1731. In Winter, the Snows are heap'd up upon the Mountains, and other high Places, from which Rivers run down. About the End of this Season, and the Beginning of Spring, when the Snows are melted, the Surfaces of the Rivers are rais'd, and run out of their Channels. The Water being mov'd faster carries a greater Quantity of Sand along with it; but the Water, dispers'd sidewise be-
1732. yond the Banks, lays down the Sand; and thus *every Year, those Places are rais'd, which are overflowed by River Waters in the Winter.*

1733. There have been many ways invented by Men, to remove the Inconveniencies arising from the Changes of Rivers. They have made Dams at the Sides of Rivers, to keep the Waters in their Channel, in Winter and Spring; the Event of which Caution was often this, that the Sand, which would have cover'd the Places next to it, being now restrain'd, is accumulated in particular Places from many Causes, and often becomes the Cause of the greatest Inconveniencies in a length of Time.

1734. Dams are made in the Channel itself to remove other Inconveniencies; I will shew their Effects, and thence it may easily be deduc'd in what Cases they may be of service.

1735. Plate LVI. Fig. 5. Let A B be such a Dam, plac'd obliquely, that its Direction may in part conspire with the Motion of the Water; by this Dam the Action of the Water is remov'd from the Bank C D, and is encreas'd against the opposite Bank, which is hollow'd at E. The Water moves slowly at the Angle F, being hinder'd by the Dam; for this reason it lays down its Sand *, which is continually accumulated in that Place.

* 1705.

1736. The Water, contain'd in the Angle G, is not at rest; that which moves along B H, carries along with it the lateral Water, with which it coheres, and this is follow'd by that, which at the Angle adheres to the Dam; and as, by the Motion of this Water, the Surface of the Water is depress'd at the Angle, the Water returns along the Bank with a contrary Motion. In this slower Motion the Sand continually falls down, and at last fills up the Angle itself.

C H A P.

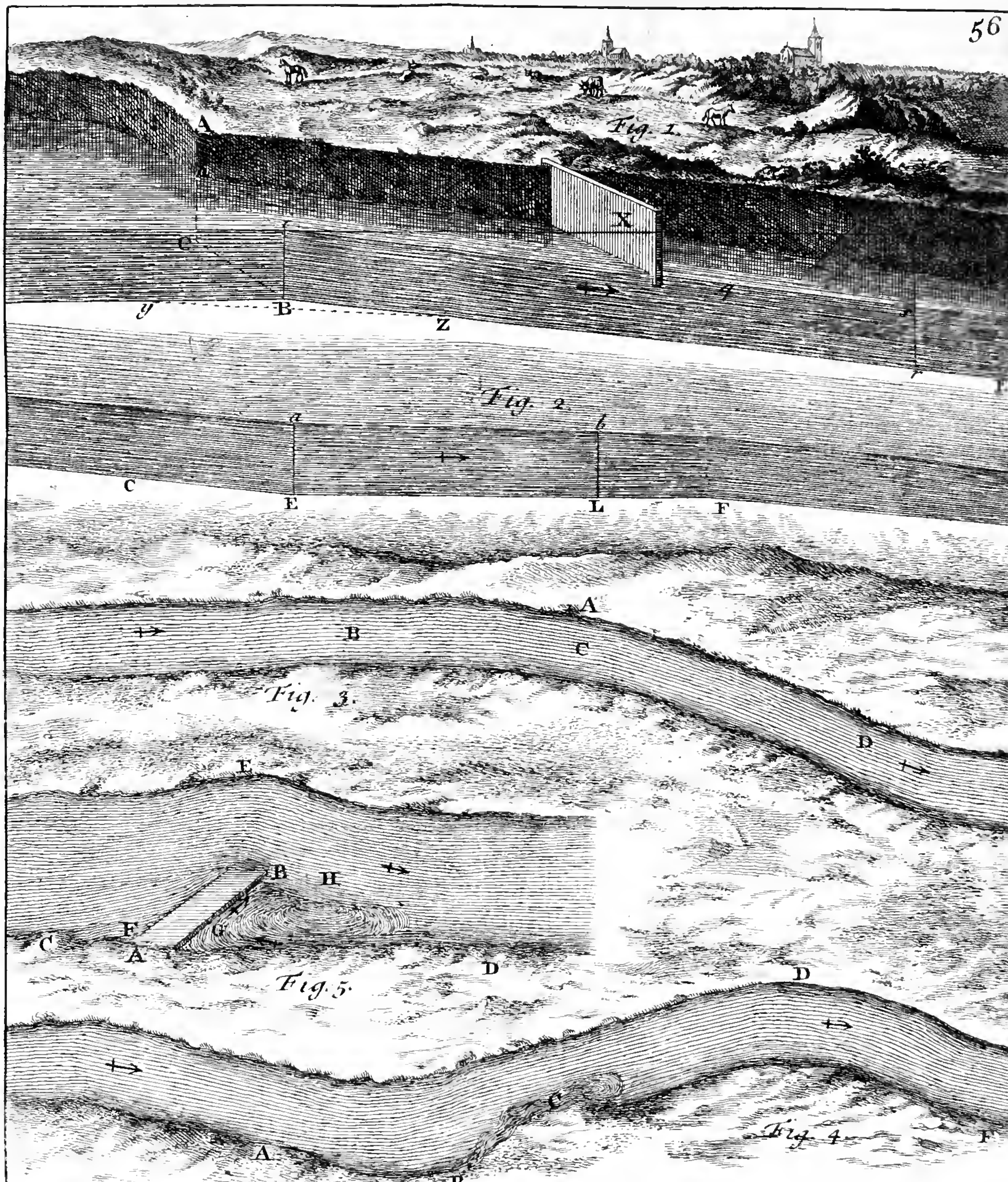


Fig. 1.
A
a
C
X
q
y
B
Z
r
Fig. 2.
a
b
C
E
L
F
A
B
C
D
Fig. 3.
E
B
H
F
C
A
G
D
Fig. 5.
D
C
A
B
F
Fig. 4.

J. Mynde sc.

CHAP. XI.

Of the Motion of the Waves.

THE Surface of the ſtagnant Water is plain, and parallel to the Horizon *; if it becomes hollow at A, upon any Account whatever, this Cavity is ſurrounded with the Elevation B B. This rais'd Water deſcends by its Gravity, and, with the Celerity acquired in deſcending, it forms a new Cavity; by which Motions the Water aſcends at the Sides of this Cavity, and fills the Cavity A, whilſt there is a new Elevation towards C; and, when this laſt is depreſſed, the Water riſes anew towards the ſame Part; whence there ariſes a Motion in the Surface of the Water, and a Cavity, which carries an Elevation before it, is mov'd from A towards C. 1737. Plate LVII. Fig. 1. * 1413.

DEFINITION 1.

This Cavity, with the Elevation next to it, is call'd a Wave. 1738.

DEFINITION 2.

The Breadth of a Wave is the Space taken up by a Wave in the Surface of the Water, and meaſur'd according to the Direction of the Wave's Motion. 1739.

The Cavity, as A, is encompaſs'd every way, with an Elevation, as was ſaid, and the Motion above-mention'd expands itſelf every way; therefore *the Waves are moved circularly.* 1740.

1741. Plate LVII. Fig. 2. Let A B be an Obſtacle againſt which the Wave, whoſe Beginning is at C, does run; we muſt examine what Change the Wave ſuffers in any Point, as E, when it is come to the Obſtacle in that Point. In all Places thro' which the Wave runs, whilſt it goes forward its whole Breadth, the Water is rais'd; then a Cavity is form'd, which is again fill'd up, which Change while the Surface of the Water undergoes, its Particles go and come thro' a ſmall Space. The Direction of this Motion is along C E, and the Celerity may be repreſented by that Line; let this Motion be conceived to be reſolv'd into two other Motions along G E and D E, whoſe Celerities are reſpectively repreſented by thoſe Lines *. By the Motion along D E the Particles do not act againſt the Obſtacle, and after the Stroke continue the Motion in that Direction, with the * 1155.

the ſame Celerity; and this Motion is repreſented by E F, ſuppoſing E F and E D to be equal to one another, by the Motion along G E the Particles come directly againſt the Obſtacle, and the Water which cannot go forward beyond the Obſtacle, and is puſh'd on by that which follows it, yields that way where there is the leaſt Reſiſtance, that is, aſcends: And this Elevation, greater than in other Places, is caus'd by the Motion along G E, becauſe 'tis by that Motion alone that the Particles come againſt the Obſtacle. The Water, by its Deſcent, acquires the ſame Velocity with which it was rais'd; and the Particles of Water are repell'd from the Obſtacle with the ſame Force in the Direction E G, as that with which they came againſt the Obſtacle. From this Motion, and the Motion above-mention'd along E F, ariſes a Motion along E H, whoſe Celerity is expreſs'd by the Line E H, which is equal to the Line C E; and by the Reflexion the Celerity of the Wave is not chang'd, but it returns along E H in the ſame Manner, as if, taking away the Obſtacle, it had mov'd along E *h*. If from the Point C, C D be drawn perpendicular to the Obſtacle, and then produc'd, ſo that D *c* ſhall be equal to C D, the Line H E continued will go through *c*; becauſe the Triangles C D E, *c* D E, agree in every thing. And as this Demonſtration holds good in all the
1742. Points of the Obſtacle, it follows that *the reflected Wave has the ſame Figure on that ſide of the Obſtacle, as it wou'd have had beyond*
1743. *the Obſtacle, if it had been taken away. If the Obſtacle be inclin'd to the Horizon*, the Water riſes and deſcends upon it, and ſuffers a Friction, whereby *the Reflexion of the Waves is diſturb'd, and often wholly deſtroy'd*. This is the Reaſon why very often the Banks of Rivers do not reflect the Waves.

1744. *When there is an Hole*, as I, *in an Obſtacle*, as B L, the Part of the Wave which goes through the Hole, continues its Motion directly, and expands itſelf towards Q Q, and *there is a new Wave form'd, which moves in a Semicircle, whoſe Center is the Hole.* For the raiſed Part of the Wave, which firſt goes thro' the Hole, immediately flows down a little at the Side, and then by deſcending makes a Cavity, which is ſurrounded with an Elevation on every Part beyond the Hole, which moves every way in the ſame
* 1737. manner, as was ſaid concerning the Generation of the firſt Wave *.

1745. In the ſame manner a Wave, to which an Obſtacle, as A O, is oppos'd, continues to move between O and N; but expands itſelf towards R in a Part of a Circle, whoſe Center is not very far from O.

Hence

Hence we may easily deduce what must be the Motion of a Wave behind an Obstacle, as M N. 1746.

1747. *Waves are often produc'd by the tremulous Motion of a Body, which also expand themselves circularly*, tho' the Body goes and comes in a right Line; for the Water which is rais'd by the Agitation, descending, forms a Cavity, which is every where surrounded with a Rising.

1748. *Different Waves do not disturb one another, when they move according to different Directions.* The Reason of which Effect is, that whatever Figure the Surface of the Water has acquir'd by the Motion of the Waves, there may in that be an Elevation and a Depression, as also such a Motion as is requir'd in the Motion of the Waves.

Whoever has, with Attention, consider'd the Motion of the Waves, will find that all these things agree with Experiments.

To determine the Celerity of the Waves, another Motion, analogous to their Motion, is to be examin'd. Let there be a Fluid *in the Recurve cylindric Tube* E H, and let the Fluid in the Leg E F be higher than in the other Leg by the Distance *l* E; which Difference is to be divided into two equal Parts at *i*. The Fluid, by its Gravity, descends in the Leg E F, whilst it ascends equally in the Leg E H; and so, when the Surface of the Fluid is come to *i*, it is at the same Height in both Legs, and that is the only Position in which the Fluid can be at rest: But, by the Celerity acquir'd by descending, it continues its Motion, and ascends higher in the Tube G H, and in E F it is depress'd quite to *l*, except so much as it is hinder'd by the Friction against the Sides of the Tube. The Fluid in the Tube G H, which is higher, also descends by its Gravity; and so the Fluid in the Tube rises and falls, 'till it has lost all its Motion by the Friction. Pl. LVII. Fig. 3. 1749.

The Quantity of the Matter to be mov'd is the whold Fluid in the Tube; the moving Force is the Weight of the Pillar *l* E; this pressing Fluid is mov'd in the same manner as the rest of the Fluid, and, in respect of this, is at rest; therefore it acts upon the Fluid in motion as if it were at rest, and presses the lower Fluid with its whole Weight *. But the Height of this pressing Fluid is always double the Distance E *i*; which Distance, therefore, encreases and diminishes in the same Ratio with the moving Force. But the Distance E *i* is the Space to be run thro' by the Fluid, that from the Position E H, it may come to the Position of Rest; which Space therefore is always as the Force which continually

* 371.

acts upon the Fluid. But we have demonſtrated that it is from a like Cauſe that all the Vibrations of a Pendulum, oſcillating in a
* 414. Cycloid, are perform'd in the ſame Time *; and therefore here alſo, *whatever be the Inequality of the Agitations, the Fluid always goes, or comes, in the ſame Time.*

1750. *The Time in which a Fluid, thus agitated, aſcends, or deſcends, is the Time in which a Pendulum vibrates, whoſe Length*, that is, the Diſtance between the Center of Oſcillation and Suſpenſion, *is equal to half the Length of the Fluid in the Tube*, or to half the Sum of the Lines E F, F G, G H. This Length is to be meaſur'd in the Axis of the Tube.

1751. Let ſuch a Pendulum vibrate in a Cycloid, in the manner ex-
Plate LVII. plain'd above *. Let the Pendulum P C and the Arc A D be of
Fig. 3. 4. the ſame Length †; in the Point A the Direction of the Curve is
* 410. † 411. perpendicular to the Horizon, and the Body endeavours to deſcend along the Curve with its whole Weight: But this Weight is to the Force acting upon the Body, plac'd at P, as A D, or P C, is to
‡ 414. P D ‡. Now let the Fluid be in ſuch a Poſition that *i* E may be equal to P D; the Weight of the whole Matter to be mov'd, that is, of the whole Fluid, is to the Weight *l* E (which is the Force acting upon the Fluid in that Poſition) as the Length of the Fluid in the Tub to the Line *l* E, in which Ratio alſo the Halves of thoſe Quantities are, that is, P C to P D. Therefore in the Pendulum the Weight of the Matter to be mov'd is to the Force acting upon it at P, as in the Tube, the Weight of the Matter to be mov'd is to the Force acting upon it in the Poſition E H. Therefore the pendulous Body and the Fluid, in this Caſe, are acted upon by equal Forces, and this always obtains where the Spaces run thro' by the Fluid in Agitation, and by the Body in Vibration, are equal; therefore, in this Caſe the Agitation and the Vibration are
‖ 1749. perform'd in the ſame Time, and not only in this Caſe, but always ‖. But, as the ſmall Vibrations in a Circle do not differ from the Vibrations in a Cycloid, the Demonſtrations will agree to them alſo.

Experiment.

1752. Take a Cylindric recurve Tube, as E F G H; let the Length of
Plate LVII. the Legs be one Foot, and the Bore of the Cylinder half an Inch;
Fig. 3. pour Mercury into this Tube, and having made a Pendulum, whoſe Length is equal to half the Length of the Cylinder of Mercury in the Tube; if the Mercury be agitated in the Tube, it will aſcend and deſcend in the ſame Time as the Pendulum will go and come.

To

To determine the Celerity of the Waves from what has been said, we must consider several equal Waves that follow one another immediately, as A B, C D, E F, which move from A towards F, the Wave A B runs its Breadth, when the Cavity A is come to C; which cannot be, unless the Water at C ascends to the Height of the Top of the Waves, and again descends to the Depth C; in which Motion the Water is not agitated sensibly below the Line *h i*; therefore, this Motion agrees with the Motion in the Tube above-mention'd, and the Water ascends and descends; that is, the Wave goes through its Breadth, whilst a Pendulum of the Length of half B C performs two Vibrations *, or whilst a Pendulum of the Length B C D, that is four times as long as the first, performs one Vibration †. 1753. Pl. LVII. Fig. 5. * 1750. † 418.

Therefore, the Celerity of a Wave depends upon the Length of the Line B C D, which is greater, according as the Breadth of the Waves is greater, and as the Water descends deeper in the Motion of the Waves. 1754.

In the broader Waves, which do not rise high, such a Line as B C D does not much differ from the Breadth of the Wave; and in that Case, *a Wave runs through its Breadth, whilst a Pendulum, equal to that Breadth, vibrates once.* In every equal Motion, by multiplying the Time by the Celerity, you have the Space gone through *, whence it follows, that *the Celerities of the Waves are as the square Roots of their Breadths*: For as the Times in which they go thro' their Breadths are in that Ratio †, the same Ratio is requir'd in their Celerities, that the Products of the Times by their Celerities may be as the Breadth of the Waves, which are the Spaces gone thro'. 1755. * 119. 120. 1756. † 418. 1754.

All these things must be only look'd upon as nearly true, because the Motion of the Waves differs something from the Motion in the Tube; which Error is in part taken off, because the Length of the Pendulum is measur'd along the inclin'd Lines B C and C D. 1757.

BOOK

BOOK III.

PART III. Of the Actions of Fluids in Motion, and their Resistances.

CHAP. XII.

Of the Impetus of Fluids in Motion.

1758. IN the last Chapter but one we took notice of some things relating to the Actions of Fluids; but we only deliver'd these things, that we might deduce the Changes arising thence in the Motion of a River.

1759. But I shall now speak of measuring *the Impetus of a Fluid, which runs against a Body*. It is manifest, that this Impetus *is the Pres-*
* 123. *sure* *; which will be determin'd by determining the Reaction of
† 361. the Body †. This Reaction destroys the Motion, in a Time equal to that, in which it is communicated to the Fluid; therefore *the*
1760. *Impetus of which we are speaking*, is equal to the Pressure, which
‡ 713. communicated Motion to the Fluid ‡, which was determin'd
‖ 1577. above ‖, and which *is equal to the Weight of a Column of the Fluid, whose Base is the Hole thro' which the Fluid runs out, and whose Height is the Height of the Fluid above the Hole.*

EXPERIMENT.

1761. We make use of the Pillar C explain'd in the first Book *; the
Pl. LVII. Arm A is join'd †; upon which is put the smaller Pillar G ‡; and
Fig. 6. lastly upon this is put the Arm E, which differs from the Arm,
* 162. mention'd in N. 170, in this, that instead of the Brass Plate, and
† 173. the Pulleys, it has a wooden Ruler fasten'd to it, to which is ap-
‡ 163. plied a Copper Ruler *a a*, to which is join'd *b b* ‖. An Ivory Cy-
‖ 763. linder D is suspended by two Threads, passing over the Hooks
* 1102. *g g* *; the Threads are fasten'd to the Pins *m m*, which are fix'd to the Arm A, by the turning about of which the Cylinder is rais'd, or depress'd, and kept horizontal.

The

The wooden Veſſel I F H L, a Foot long, ſix Inches broad, and as many deep, has its Brim cut in at *c d*, *c d*; where the Water runs down when the Veſſel is fill'd; that the Surface of the Water, which flows out of a Hole, of which I ſhall ſpeak preſently, may be kept at the ſame Height for ſome Time, by a continual pouring in of Water. I have given the inward Dimenſions, and meaſur'd the Height from the Bottom to the Lines *c d*, *c d*. 1762.

But that the Motion in the Water, ariſing from the Infuſion, may not diſturb the Efflux thro' the Hole, there are two tranſverſe Diviſions, *e e e e*, *f f f f*, four Inches high, the firſt of which is join'd to the Bottom, the other riſes above the Surface of the Water; they are parallel, and are an Inch and an half diſtant from one another; the Water is pour'd in at M. 1763.

A Copper Plate, with a Screw in it, is fix'd at O; to this, a Leather being put between, is join'd the Pipe P, which has a Screw that fits the other Screw; the Pipe is clos'd by a Plate which has a Hole in it: The Diameter of the Pipe in my Machine is half an Inch, or 0,50 Inch, and the Diameter of the Hole is 0,43 Inch. This Veſſel is ſo plac'd, and the Cylinder ſuſpended, that the Center of the Baſe of the Cylinder may anſwer to the Hole, ſo that the Water, ſpouting out of the Hole horizontally, may run directly againſt the Cylinder. 1764.

Things being thus diſpos'd, the Veſſel is fill'd with Water, which runs directly againſt the Baſe of the Cylinder, and repels it in ſuch manner, that the Threads acquire an oblique Poſition; and the Body is ſuſtain'd in the ſame Poſition, during the Efflux, which is continued uniformly; for Water is continually pour'd on in ſuch Quantity, as to run down ſideways at *c d*, and *c d*. 1765.

The Cylinder by this Action is remov'd from its Poſition, in which it can reſt, by the Quantity *i v*, which is equal to an Inch and a Quarter, the Line *i g* being equal to twenty-nine Inches. 1766.

To ſhew that this Experiment agrees with the foregoing Propoſition*, many things are to be conſider'd; for the retarding Cauſes diminiſh the Motion of the Water running againſt the Body, ſo that the true Action exceeds but a little the half of that, which is diſcover'd, if the Computation be made without conſidering the Retardations. Therefore the Effects of the retarding Cauſes will be to be examin'd; but we ſhall firſt determine the Action of the Water againſt the Body. 1767.

* 1760.

The Cylinder D is drawn by three Powers; by its own Weight downwards; by the Threads obliquely; and laſtly it is preſs'd horizontally 1768.

zontally

zontally by the Action of the Water. These Actions are to one
* 317. another as the Sides of the Triangle $i\ v\ g$ *; wherefore the Weight of the Body is to the Impetus of the Water, as $i\ g$ is to
† 1766. $i\ v$; that is, as 116 to 5 †.

1769. The Weight of the Cylinder was six Ounces, wanting two Drachms; therefore the Impetus of the Water was equal to 119 Gr.

1770. Now this Action must be compar'd with that which the Ruler shews; which can't be done without a new Experiment; because it must be determin'd how much the Water was retarded; and we must discover the Height which the Water wou'd have, above the Hole, in a Vessel, from which, setting aside the Retardations, it shou'd go out with a Velocity equal to that, with which it came against the Body in the Experiment; and this is the true Height of the Pillar mention'd in N. 1760.

1771. I weigh'd the Water, which, in the Time of ten Seconds, ran out of the Vessel, whilst it flow'd with that Velocity which it had in the Experiment; the Weight was forty one Ounces and a Quarter.

* 1551. From the known Weight of a Cubic Foot of Water *, we infer that a Cylinder of Water, the Diameter of whose Base is one Inch, and whose Height is equal to a Foot, weighs 2659 Gr. Whence, by the Ratio, we discover that in the Time mention'd there flow'd out of the Vessel a Pillar of Water, whose Base the Hole was, and Length 40,3 Feet.

Therefore the Water flow'd out with a Velocity equal to that, with which 4,03 Feet are run thro' in a Second; which is the
* 883. 374. same as a Body acquires in falling from an Height of 3,1 Inches *; but this is the Height of a Column, whose Base is the Hole, and
† 1759. whose Weight, according to what is demonstrated before †, is equal to the Impetus of the Water against the Cylinder. The Weight of this Column is 127 Gr.

1772. In this Computation, we suppos'd all the Water to be mov'd with the same Velocity, but the Celerity of all the Parts was not
* 1634. equal *; if the Impetus were as the Velocity, it wou'd be determin'd exactly, by this Computation; for it wou'd be sufficient to determine the mean Velocity: But

1773. *the Impetus is as the Square of*
† 1760 1586. *the Velocity* †.

1774. Therefore, if the mean Velocity be attributed to all the Particles, a less Impetus is discover'd than the true one. If three Particles being given, the Velocity of the first of which is four, of the second five, of the third six, the Action be suppos'd as the Square

Square of the Velocity, the Sum of the Actions may be exprefs'd by 25, and 36. and 49, that is, by 110. But if we attribute to all the mean Velocity fix, the Sum of the Actions will only be equal to 108.

We find therefore that the Impetus of the Water in the Experiment exceeded 127 Grains, perhaps by three or four Grains and no more; but we have feen that this Impetus was only equal to an hundred and nineteen Grains *, which Difference we attribute to the Agitation of the Cylinder during the Experiment, by which it appear'd that the Action was not exactly direct. 1775. * 1769.

We have here confider'd the Action of a Fluid againft a quiefcent Obftacle, if the Obftacle be in motion, the Impetus depends upon the refpective Velocity *, and follows the Ratio of the Square of the refpective Velocity †; that is, the Intenfity of the Preffure, acting againft the Obftacle, follows this Ratio; but *the Action of the Fluid, againft an Obftacle mov'd the fame Way with the Fluid, follows the Ratio compounded of the* faid *duplicate Ratio of the refpective Velocity, and the Ratio of the Velocity of the Obftacle* ‡. 1776. * 948. † 1773. 1777. ‡ 725.

If in this Cafe the Velocity of the Fluid be given, its Action againft the Obftacle is the greateft of all, as is demonftrated in the following Scholium, *when the Velocity of the Obftacle is equal to a third Part of the Velocity of the Fluid*; two Thirds of whofe Velocity then give the refpective Velocity of which we are fpeaking *. 1778. * 918.

SCHOLIUM.

The Demonftration of N. 1778. *concerning the greateft Action from an Impetus againft an Obftacle in motion.*

LET A B be the Velocity of the Fluid; C B the Velocity of the Obftacle; the refpective Velocity with which the Fluid runs againft the Obftacle will be A C *. The Action of which we are fpeaking is as the Product of the Square of the Line A C multiplied by C B †. 1779. Pl. LVII. Fig. 7. * 918. † 1777.

Let A E be a Parabola, whofe Vertex is A; Axis A D; A B a Tangent in the Vertex.

The Abfcifs A D, or C E, follows the Ratio of the Square of the Ordinate D E, or A C *; therefore the Rectangle C G follows the Ratio of the Product of the Square of A C by C B †; that is, it follows the Ratio of the Action of the Fluid againft the Obftacle, carried with the Velocity C B ||. A Point, as C, is fought, when this Rectangle is the greateft of all. * *La Hire Sect. con. B.* 3. Prop 1. † 23 El. 6. || 1777.

Let us fuppofe the Point fought to be given between C, and *c*; as we recede on either fide from the Point fought, the Rectangles become lefs;

 and

and there is none given on one ſide but there will be one equal on the other ſide. Let there be two ſuch equal Rectangles C B G E, *c* B *g e*; and let the Diſtance E *e* be infinitely ſmall; taking away the common Rectangle *c* B G F, there remain the equal Rectangles C *c* F E, G *g e* F; &

* 14. El. 6. F *e*: F E : : E C : F G*, or E G = C B.

But F *e*: F E : : D L : D E = A C.

Therefore E C : C B : : D L : A C.

† La Hire. Sect. con. B. 2. Prop. 20. But the Subtangent D L is double A D = E C†; therefore A C is double C B; therefore is equal to a Third of the whole A B, which was to be demonſtrated.

C H A P. XIII.

Of the lateral Action of Fluids in Motion.

1780. WE have ſeen before* that Fluids preſs equally every way, at the ſame Depth. But the Demonſtration olny belongs to Fluids at reſt.

* 1418.

1781. *If a Fluid be moved along an horizontal Tube, or one inclined downwards, ſetting aſide all Retardations,* it preſſes downwards with its own Weight; but beſides this, and a ſmall lateral Preſſure, ariſing from thence, *the Tube ſuffers* nothing *from the Action of the Fluid, with what Velocity ſoever it be carried.*

1782. For if we ſuppoſe the Fluid to ſpout out of a circular Hole, ſetting aſide the Motion ariſing from Gravity after it goes out, it will make an horizontal Cylinder, which does not preſs laterally; if this be ſurrounded by a Tube, it will move in the ſame manner as before, that is, it will not act againſt the Surface of the Tube.

1783. But *if it be retarded in the Tube,* whether by means of the
1784. Friction, or by being ſtraiten'd at its going out, *the Fluid preſſes laterally*; and *the lateral Preſſure is equal to the whole retarding Force, which acts beneath the Point, which is preſſed*; that is, if A B be the Tube, thro' which the Fluid is moved from A to B, the Preſſure againſt E, will be equal to the whole Action, which retards the Fluid between E and B, and at its going out at B.

Plate LVII. Fig. 8.

1785. The Action which moves, or retards a Fluid, is proportional to the Height of a Column of the ſame Fluid, which can exert ſuch an Action; therefore we may expreſs that Action by this Height, and alſo meaſure it in the Experiment. If the Force acting upon the Fluid, whilſt it paſſes along E, be equal to the Weight of a Column

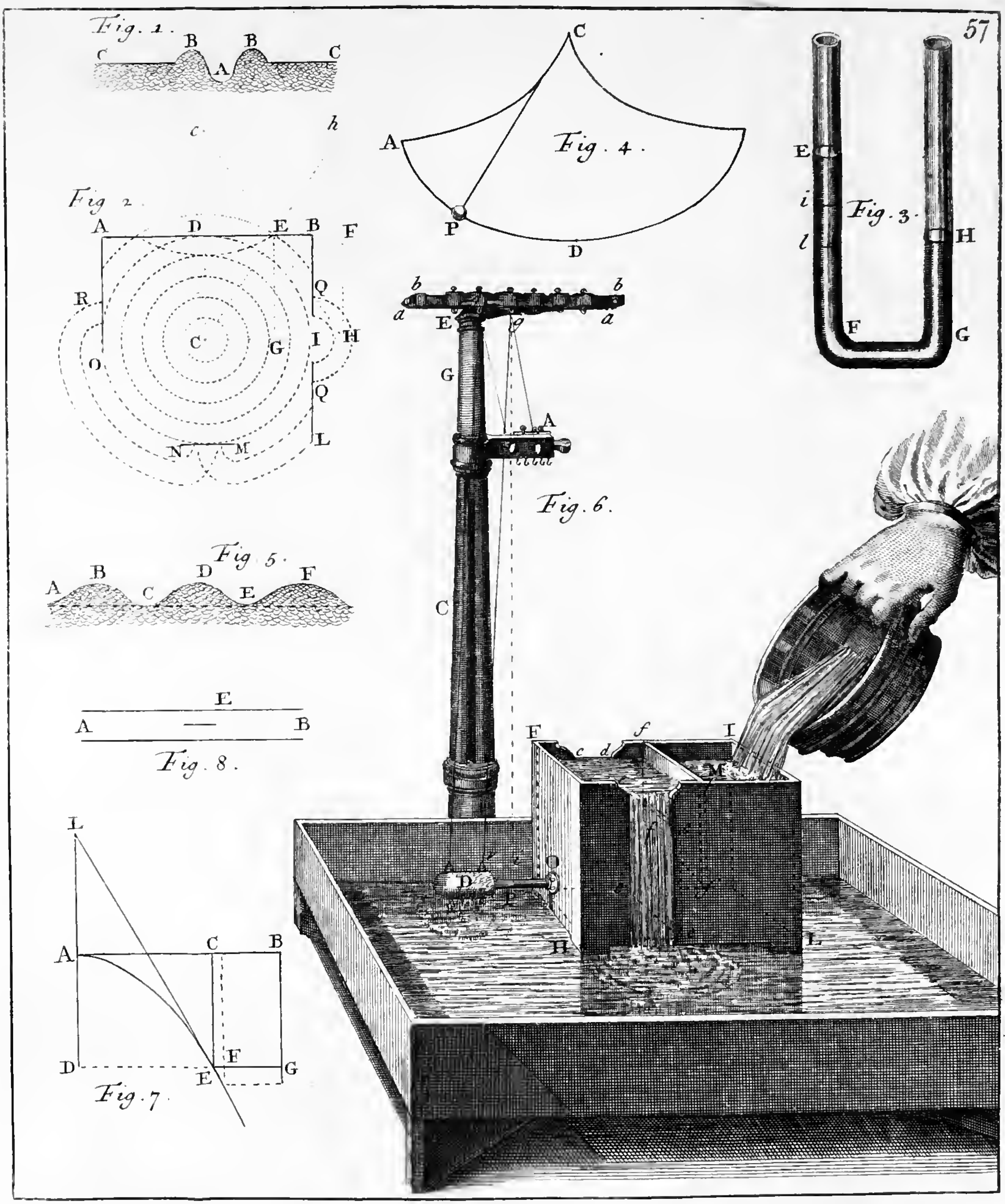


Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.

Column of the Fluid, which is five Inches high, and the Fluid be mov'd with a Velocity equal to that, which a Body acquires in falling from an Height of two Inches, the Preſſure againſt the Point E will be equal to the Weight of a Column of three Inches: For if the Fluid ſhou'd be at reſt, the Preſſure wou'd be equal to five Inches; but it does not preſs with that Action, with which it is mov'd *; ſo that two Inches muſt be ſubſtracted. * 1781.

In this reaſoning we ſuppoſe the whole Fluid, which paſſes along the Tube, to be mov'd with the ſame Velocity, which indeed is not true; the Effect of the Retardations alſo, acting above E, at E is different, according to the different Retardation below E, and chiefly at the going out. Therefore the Circumſtances may be infinitely varied; wherefore in moſt Caſes, it can't be determin'd by Computation, what will follow from the Rule deliver'd *; as will appear from the following Experiments. 1786. * 1784.

A Machine,

Whereby Experiments, concerning the lateral Preſſure of Fluids in motion, are made.

The principal Part of this Machine is the Veſſel, mention'd in the foregoing Chapter *, to which is faſten'd at O the Pipe P, about four Inches long, made ſmooth within, whoſe Bore is half an Inch. To the Middle of it, at top, is join'd the Vertical Glaſs Tube L, which communicates with the horizontal Pipe P; *(See Fig. 2.)* in which alſo L repreſents the Glaſs Tube, which we will call the Index. 1787. Pl. LVIII. Fig. 1. * 1762.

Several Pipes, of the ſame Capacity with the ſaid Pipe P, may be join'd to it; which that it may be done conveniently, every Pipe is ſolder'd to a broader Pipe *b*, into which the End *a* of the Pipe P is thruſt. 1788. Pl. LVIII. Fig. 2.

Several ſuch Pipes are repreſented at B, C, and D; that repreſented at B, has a Side Pipe E faſten'd to it, of the ſame Capacity as B, to which it is join'd at Right Angles to it; the Glaſs Index *l* ſtanding vertically upon it, in the ſame manner as the Index L. 1789.

There muſt be two Stops as *c*, to cloſe the Ends of the Tubes; ſuch a Stop is made of a ſmall Tube which has one End clos'd by a Plate. 1790.

1791. In the ſame manner the ſmall Tubes *d* and *e* are cloſ'd by Plates, but theſe have Holes in them; that the Aperture, thro' which the Water ſpouts, may be varied.

EXPERIMENT I.

1792. Pl. LVIII. Fig. 1. * 1787. † 1765. To the Veſſel F L join the Tube P *. Pour in Water, ſo that it may run over, as was ſaid above †. During the Efflux thro' *a* no Water appears in the Tube F; but at the very Moment, when this Aperture is cloſ'd, the Water aſcends in L to the Height of 5,19 Inches.

1793. This Height was meaſur'd from the external upper Surface of the Pipe P; wherefore we muſt add the Thickneſs of the Metal, but we muſt ſubſtract the Aſcent of the Water ariſing from the Attraction of the Glaſs *; for this Reaſon we neglect theſe two Differences, and in what follows we ſhall meaſure the Height in the ſame manner. * 82.

1794. In this Experiment there is no ſenſible Friction between the Point, to which the Glaſs Index is applied, and the Aperture, thro' which the Water goes out; and there is no lateral Preſſure from this ſmall Friction, becauſe the middle Water moves faſter, carrying along with it the lateral Water; but this Action is directed obliquely towards the Axis of the Pipe P, and therefore deſtroys the Effect of the Friction. In other Caſes it only diminiſhes it.

1795. The Motion of the whole Water is very much retarded, in this Experiment; but the retarding Cauſes act chiefly at the Entrance into the Tube *; that is, above the Point, in which the Preſſure is meaſur'd, and upon which thoſe Cauſes produce no Effect †. But to compare this Experiment with the following, the Velocity, with which the Water goes thro' the Pipe, will be to be determin'd, both in this and in the other Experiments; and this is determin'd by attributing to all the Particles the ſame Velocity; but as this does not accurately agree with the Motion, ſome Corrections muſt be made in the Concluſions. * 1609. 1632. † 1784.

1796. I determin'd the Velocity of the Water in the Pipe, by receiving in a glaſs cylindric Veſſel the Water, which flow'd out in ten Seconds. A Pound of Water in this Veſſel was two Inches and three Eighths high. The ſame Water in the Pipe P continued wou'd take up a Space of 11,55 Feet, as is eaſily deduc'd from the known Weight of Water. *. * 1551.

1797. But the Water, which run out in the ſaid Time, took up in the Veſſel a Space of 8,13 Inches; a tenth Part of which in the Pipe, wou'd wou'd take up a Space of 3,95 Feet; which therefore wou'd be the Space run thro' in one Second by the Water going out, if it kept its Velocity; but this Velocity is equal to that, which a Body acquires in falling an Height of almost three Inches, as is deduc'd from what is said before*. * 883. 374.

EXPERIMENT 2.

To the Pipe P, I applied the small Pipe *e (Fig. 2.)* the Diameter of the Aperture, thro' which the Water then ran out, was to the Diameter of the Pipe P, as 43 to 50; and the Water, which flow'd out in 10″ took up, in the Vessel above-mention'd *, an Height of 6,13 Inches; whence we infer'd that the Water in the Pipe P, had a Velocity equal to that, which a Body acquires in falling 1,7 Inch. The Height of the Water in the Index was 2,06 Inches; and this Height is the Measure of the lateral Pressure. 1798. Pl. LVIII. Fig. 1. * 1796.

Therefore the whole Action which acted upon the Water, whilst it passed along the Glass Index, which is equal to the Sum of the lateral Pressure and the moving Cause *, exceeds the Weight of a Column of three Inches and three Quarters; which Measure of the Action shou'd be encreas'd †, because the Middle Water moves faster than the Side Water ‡. 1799. * 1785. † 1794. ‡ 1634.

EXPERIMENT 3.

We change the small Pipe, and instead of *e* make use of *d* *(Fig. 2.)* the Diameter of whose Aperture is 0,26 Inches. The Height of the Water, which flows out in 10″, is in the Vessel 2,19 Inches. Therefore the Velocity with which it moves along the Pipe, is equal to that, which a Body acquires in falling from an Height of 0,22 Inches. In the Glass Index the Height is 4,94 Inches; the Sum is equal to 5,06. If we encrease this last Action a little *, we find that the Water, in this Case, was retarded but a little, in its Entrance into the Tube †. 1800. Pl. LVIII. Fig. 1. * 1794. † 1785. 1792.

We find the like Variations if the Fluid be retarded by an Inflexion of its Way. 1801.

EXPERIMENT 4.

Taking away the small Pipe, whereby the Aperture of the Pipe P was narrow'd, I applied the Pipe C, bent to a Right Angle, *Fig.* 2. The Water receiv'd in the Vessel had the Height 5, 97; therefore it was carried with a Velocity equal to that, which a Body 1082. Pl. LVIII. Fig. 1.

acquires in falling from an Height of 1,6 Inches. In the Index the Height was 2,56 Inches; the Sum is equal to 4,16 Inches.

Experiment 5.

1803. Pl. LVIII. Fig. 2. I applied the ſmall Pipe *d* (*Fig.* 2.) which was made uſe of in the third Experiment, to the End of the Pipe C, and every thing which was obſerv'd in the third Experiment, obtain'd here alſo.

Experiment 6.

1804. Pl. LVIII. Fig. 2. This Experiment differs little from the fourth; the Angle, which the Pipes P and C make, which is a Right one in the ſaid Experiment, is in this 135 Degrees. But this was the Effect. The Water receiv'd in the Veſſel was 6,38 Inches high. Therefore the Velocity of the Water in the Pipe was equal to that, which a Body acquires in falling an Height of 1,84 Inches. In the Index the Height of the Water was 1,87; therefore the whole Action was 3,7 Inches.

1805. From all this it appears, that the Effects of the retarding Cauſes
1806. can't eaſily be foreſeen; but in general *the Retardation is diminiſh'd in the going in, when it is encreas'd in the going out.*

1807. In bent Pipes the lateral Preſſure is greater in the Place of the Inflexion from the centrifugal Force, but it can't eaſily be reduced to a certain Rule, as may be infer'd from the following Experiments.

Experiment 7.

1808. Plate LVIII. Fig. 3. Inſtead of the Pipe C, which we made uſe of in the fourth Experiment, we uſe the Pipe B, (*Fig.* 2.) to which alſo is join'd the Pipe E at Right Angles to it, of the ſame Capacity as B and P. Upon this Pipe B is plac'd the Glaſs Index *l*, like L, as we have
* 1789. ſeen above *.

In this Experiment the Water flow'd out of the Pipe E, this being quite open, ſo that this Experiment coincides with the fourth; the Event was alſo the ſame: the Velocity indeed was ſomething leſs, but the Difference was ſo ſmall, that I cou'd perceive no Difference in the Height of the Water in the Index L. In the Index *l* it was higher, and the Difference was 0,96 Inches.

Experiment 8.

1809. Plate LVIII. Fig. 3. Every thing remaining the ſame, I applied the ſmall Pipe *d* (*Fig.* 2.) to the Aperture of the Pipe E, ſo that the Experiment coincides with the fifth Experiment; the Event was alſo the ſame, and

and ſcarce any Difference cou'd be perceiv'd in the Height of the Water in the Indices L and *l*.

C H A P. XIV.

Of Hydraulic Machines.

IT has been demonſtrated in the 21ſt Chapter of the firſt Book, that the Reſiſtance of a Body ariſing from Gravity, muſt be conſider'd in a different way, when a Body is to be ſuſtain'd, than if it were to be raiſed. In the ſame manner when we conſider Machines, by which Water is rais'd from a lower Place to one that is higher, thoſe Things are not ſufficient, which are demonſtrated in the firſt Part of this Book, concerning the Preſſures of Fluids. I ſhall not treat of particular Machines; but I ſhall explain ſuch things, as may have reference to any Machines whatſoever, which are deſign'd for the ſaid Uſe. 1810.

The Deſign of all ſuch Machines is, that the greateſt Quantity of Water may be rais'd to a certain Height, in a certain Time with a given Action. 1811.

I don't regard the external Cauſes that diminiſh the Effect; I ſhall only conſider that Action, which really moves the Water, without regarding that, which is conſum'd, whilſt the Defects of the Machine are overcome, or any other Obſtacles whatſoever are remov'd. 1812.

We reduce the Propoſition, of which we are ſpeaking, to the moſt ſimple Caſe, if we conceive an horizontal Pipe A B C D join'd to a vertical Pipe I B E F. We ſuppoſe the Plane M L to be mov'd in the horizontal Pipe, and indeed without Friction, and without loſs of Water, with which we ſuppoſe both the Pipes to be fill'd. We alſo ſuppoſe the Action, which hinders the Deſcent of the Water, or which drives it upwards, to be applied in the Direction N P perpendicular to this Plane. 1813. Pl. LVIII. Fig. 4.

Moreover, we ſuppoſe the Pipe A B C D to be immers'd in Water in ſuch manner, that the Surface of the Water may anſwer to A B. Laſtly, we ſuppoſe that the Water is to be rais'd above I F.

There are different Actions, whereby the Machines, made to raiſe Water, are mov'd; I ſhall examine them in their Order, conſidering them as acting againſt the Plane I L, along N M. By this Method 1814.

Method we ſhall come to a general Theory, which may be applied to any Machines whatever.

But we muſt firſt conſider general Things, which belong to any Actions whatever.

1815. The Preſſure of the external Water, contain'd in A L M C, deſtroys the oppoſite Preſſure of the Water, which fills the remaining Part of the horizontal Pipe, and the Action, applied along N M, ſuſtains the Water contain'd in the vertical Pipe I B E F.

1816. If the Capacity of both Pipes be the ſame, *the Power, which*, being applied along N M, *ſuſtains the Water* in the ſaid Pipe, *at the Height* I F, *above which it is to be rais'd*, is equal to the Weight of the Column of Water E I. And this Power, *as long as it acts alone, cannot* drive one Drop out of the Pipe, nor *raiſe one Drop to the Height requir'd.*

1817. But *if the Action applied be encreas'd*, ſo as to be equal to the Weight of a Pillar of Water, whoſe Baſe is the ſame as the foregoing one B E, but its Height *e g*, exceeding the firſt by the Quantity *f g*, *the Water is thrown out* of the Pipe *with a Velocity* with which it can go from *f* to the Height of the Point *g*; that is, with a Ve-
* 380. locity, which a Body can acquire in falling thro' *g f* *; with which Velocity it is mov'd along the whole Pipe; and *which follows the*
† 381. *ſubduplicate Ratio of the Encreaſe of the Action itſelf* †.

1818. Hence it follows, that the Action, applied to the Machine, produces two Effects; and that it may be reſolv'd into two Parts. The firſt raiſes the Water to the determin'd Height; and this Part of the Action alone does nothing at all, with regard to the Uſe of the Machine. The ſecond Part of the Action applied drives out the Water; and the Quantity of the Water rais'd depends upon the Magnitude of this Part of the Action.

1819. As long as we conſider the ſame Plane L M, the firſt Part of the
* 1431. Action is always the ſame, howſoever the Vertical Pipe be chang'd*; but if the Capacity of this Pipe be chang'd, the ſecond Part ſhou'd be varied, that the ſame Quantity of Water may be rais'd to the ſame Height in the ſame Time; therefore we muſt ſee which is the beſt.

1820. If the Orifice I F be chang'd, and the ſame Quantity of Water
Plate LVIII. Fig. 5. 6. be driven out, the Velocity of it will follow the inverſe Ratio of the Orifice, and the Height *f g* will follow the duplicate Ratio of the
* 381. Velocity *. Therefore, *the Part of the Action, mention'd laſt, is in-*
1821. *verſely as the Square of the Orifice*; that is, it is diminiſh'd, when the

the Orifice is encreas'd; in which Caſe the whole Action, which follows the Ratio of the Height *e g*, is alſo diminiſh'd.

Let the Water be to be rais'd to the Height E F, or *e f*, of ten Feet; and let us ſuppoſe it to be driven thro' any Orifice whatſoever with a Velocity, with which it wou'd run thro' almoſt three Feet and a Quarter in one Second, which Velocity a Body acquires in falling from an Height of two Inches *; and *f g* is equal to this Height. Therefore the whole Action which drives forwards L M, may be expreſs'd by the Number 122; for there are ſo many Inches in *e g*. 1822. Fig. 4. * 883. 374.

If the Orifice I F be doubled, *f g* is reduc'd to a Quarter *, and *e g*, that is, the whole Action, is only equal to 120,5. 1823. Fig 5. * 1821.

If the Orifice be reduc'd to half, the Velocity of the Water going out is doubled, *f g* becomes eight Inches, and the whole Action is equal to 128. Fig. 6.

In this Computation we ſuppoſe all the Water, which paſſes thro' the Orifice of the Tube, to flow down its Sides in the ſame Time in which it paſſes thro', which does not happen when the Orifice is very wide, or the Velocity very ſmall; wherefore the Action, determin'd by the foregoing Rule *, muſt be encreas'd in ſome Caſes, more or leſs, according to the different Figure of the Orifice. We ſuppoſe the Orifice circular; becauſe the Friction, which can never be avoided in Practice, tho' we have no regard to it now, is the leaſt of all in this Figure; therefore, ſuppoſing this to be the Figure, we ſhall examine the Efflux. 1824. * 1821.

The Velocity of the Plane L M being given, the Velocity is given, with which the Water wou'd go out, if the Orifice were equal to this Plane; and, any other Orifice whatſoever being given, the Height *f g* may be determin'd, by proceeding as in the foregoing Example. 1825.

If *f g* be leſs than three Eighths of the Diameter of the Orifice, the whole Action will be to be encreas'd, that it may be proportionable to *e n*, a Portion of which *f n*, is diſcover'd by this Rule. The Square of three Eighths of the Diameter of the Orifice, multiplied by *f g*, is equal to the Cube of the Height ſought *f n*, as we ſhall demonſtrate in the firſt Scholium following. 1826.

Let us ſuppoſe in the foregoing Example, in caſe N. 1823, the Diameter of the Orifice to be ſix Inches, we ſhall have *f n* equal to 1,36 Inches; and except the whole Action, which in the ſaid N. 1823, we determin'd 120,5, be encreas'd, ſo as to be 121,36, the Quantity of Water requir'd will not run out. 1827.

1828. Although we may often overlook ſmall Matters in Practice, yet it is of ſervice in many Caſes to know them; and oftentimes of the greateſt Uſe.

From the Rule, deliver'd in N. 1821, we deduce this general
1829. Concluſion, that *greater Orifices are to be preferr'd to ſmaller*; becauſe the ſame Quantity of Water is driven out by a leſs Action, *cæteris paribus*.

But ſome Things muſt be obſerv'd concerning the Determination of the Orifice in ſome peculiar Caſes.

1830. If the Orifice I F be indeed greater, but at a little Diſtance from
Pl. LVIII. the narrower Part of the Tube, which is terminated at G O; ſo
Fig 7. that the Water, with the Velocity with which it paſſes thro' G O, can aſcend higher than thro' *i g*, all the Water, which goes out, does not aſcend with the ſame Velocity above the Line E F, but the Velocity is greater in the Middle, as appears from the Protuberance of the Water there; in this Caſe we muſt reaſon as if we had to do with an Orifice of a mean Magnitude between I F and G O.

1831. It often happens alſo in Machines, that the Way, thro' which
Plate LVIII. the Water is brought, is narrow'd, before the Water comes to the
Fig. 8. Orifice, as by putting in a Diviſion G O, which has a Hole at P.
1832. If the Diſtance G I be leſs, thoſe Things, which we deliver'd juſt
* 1830. before, muſt be applied to this Caſe alſo *.

1833. But if the Diſtance G I be ſuch, that all the Parts have equal Velocity at I F, we need not regard the Diviſion: The Force of the Particles muſt indeed be encreas'd, that they may paſs thro' P; but this is again conſum'd, in communicating Motion to the upper Water. As long as the whole Effect is the ſame, the whole Action is the ſame; but the Effect is diſcover'd, by conſidering the Height to which the Water is rais'd, and the Velocity with which it goes out; the greater this is, the more Force is loſt; and it appears from what is ſaid before, that this properly is the Reaſon, why greater
1834. Orifices are to be prefer'd: But *the Changes, which happen in the Velocity, before the Water comes to the Orifice, are not to be conſider'd.* For if the Way be inlarg'd, and afterwards diminiſh'd, or if it be firſt narrow'd, and afterwards again inlarg'd, there are contrary Changes in the Forces of the Particles; and as theſe Changes mutually deſtroy each other, their Effects are neglected in the Calculations.

1835. Theſe Changes, in regard to the Way, which is narrow'd, very much encreaſe the Friction, and therefore ſhou'd be diligently aoided;

avoided; but this Impediment is external, whereas we consider the Thi ngabstractly.

We have hitherto consider'd continued Motion, although we have said nothing of the Manner how Motion may be continued in a Pipe; for we do not speak of particular Machines. Let us now suppose a Machine to act by Fits; and let us conceive two equal, and similar Machines, and that a Plane as L M is driven forwards by each Machine with an equal Action. Let these Planes be driven alternately, thro' equal Spaces, in equal Times, so that these two Actions may be equal to one, which acts continually. 1836. Pl LVIII. Fig. 4.

These two acting alternately, will not produce the same Effect as one, which shou'd act continually, tho' all Things agree. For the Times will be different, or the Actions, acting alternately, which are applied, must be greater. 1837.

For in this Case the Motion of the whole Water, contain'd in the Tubes, ceases every Agitation, and must be renew'd again; so that Part of the Action, applied to the Machine, is consum'd, whilst this new Motion is communicated. This Part of the Action, which is lost, is varied, all Things being alike, according to the different Space, run thro' by the Plane L M, every Agitation; and the Proportion being kept, that is so much the greater, as this Space is less. 1838.

The Ratio likewise of the Part of the Action lost, to the whole Action, differs according to the different Nature of the Action, which is applied to the Machine; therefore it will be sufficient to observe in general, that *the Machines, which drive out Water by Fits, are very imperfect.* 1839. 1840.

This Defect is diminish'd, if the two Pipes, in which Planes as L M are mov'd, press the Water upwards in the same Pipe, as is observ'd in many Pumps; for then, if when the Motion of one Plane L M ceases, the other be mov'd immediately, the Motion of the Water is continued upwards in the common Pipe; but as the Motion can't be continued uniformly, that is, without any Retardation, some Part of the Action is always lost in restoring the Motion. 1841.

These general Things being premis'd, we must now speak of the Actions themselves.

The Actions, which are applied to hydraulic Machines, may be refer'd to four Classes. *To Fire, Air, Water, and lastly the Force of Men, or Animals.* 1842.

I shall not speak of Fire here; those are peculiar Artifices, whereby the Action of Fire is applied in the raising of Water; and the 1843.

Examination of them wou'd lead us away from the Subject entirely; and cause us to mention many Things, necessary for this End, which belong to the following Book.

1844. We must say but little of Air also, we have not treated of this yet. The Action of it is often join'd with the Action of Fire; and it is often applied alone.

1845. This obtains, when Sails are join'd to a Machine, which, being expos'd to the Wind, turn round, and communicate Motion to the Machine.

1846. When we make the Computations, we must determine the Intensity of the Action of the Wind upon the Surface of the Sail, which, I question whether it can ever be done accurately by Mechanics. For not only the Impetus of the Particles of Air upon the Surface itself, is to be consider'd; another Pressure must be superadded to this. The Air, which passes at the Sides, carries along with it the Air which is behind the Sail, and diminishes the Density of that, which remains; whence there follows an Action from a peculiar Property of the Air, which is call'd Elasticity, of which we shall speak in its Place. This Action is different according to the different Velocity of the Wind, the different Velocity of the Sail, and the different Breadth of it: Moreover, the Sail must have its End bent after a certain manner, that the Air may in some measure be retain'd, from which there follows a Pressure, which acts contrary to that, which acts upon the rest of the Sail; but the Diminution arising from thence is small, if compar'd with the Encrease, which follows from the Efflux of the Air at the Extremity of the Sail being hinder'd. I treat of these Things briefly, because we may easily err in determining the Action of the Wind.

1847. We said that Water was used also, in raising of Water. The Water for this End must be such as flows in a River, or a Rivulet, naturally.

1848. But *Water* may be applied two Ways; it *acts either by its Gra-*
1849. *vity, or its Impetus.* In the first Case, sometimes, the Water is receiv'd in Buckets, which by descending raises other Water; yet often in this Case, as always, when Water acts by its Impetus, Wheels are made use of, that have Ladle Boards fasten'd to their Circumferences, which moving by means of the Weight of the Water, running down upon them, or being driven on by the Impetus of it, make the Wheel turn round.

1850. When a Machine is mov'd by the Weight of the Water, we are to consider a Pressure whose Intensity is not alter'd, by the Ma-
* 371. chine's being in motion *. But if the Wheel is turn'd by the Impetus

petus of the Water, its Motion must be consider'd; for the Impetus depends upon the respective Velocity †. 1851.
† 946.

Therefore we refer these Actions to two different kinds, of which I shall presently speak separately. 1852.

There remain the Actions of Men and Animals; in these Actions we must consider the Intensities, and the Velocities. 1853.

The Velocity of the Pressure being chang'd, its Action is chang'd *; therefore if the Action be determin'd, the Intensity is varied, when the Velocity is chang'd; and indeed that is diminish'd, as this is encreas'd †. But the Force of a Man, or Animal, is not so determin'd, that we can refer it to the Rule. 1854.
* 723.
† 725.

If the Velocity be chang'd a little, the Intensity of the Action is not chang'd; if it be chang'd very much, the Intensity is not chang'd in the inverse Ratio of it. 1855.

Therefore, as to the Actions of Men and Animals, we must thus determine; as long as the Velocity of the Agent is chang'd but a little, the Actions belong to the Kind mention'd before *; but if we seek the greatest Action, which a Man, or Animal, can exert, we must determine the Velocity, which being given, this can be done conveniently; we must then seek the Intensity of the Action, so that it may be continued for a sufficient Time. Therefore this Action is determin'd every way, both with respect to the Intensity, and with respect to the Velocity; and the Application of these Actions to Machines must be examin'd also; so that we must consider three different Kinds of Actions †. 1856.
* 1850.
1857.
† 1852.

Let the Plane L M be to be driven forwards *, an Action being applied along N P, whose Intensity is not chang'd from the different Velocity, which is communicated to the Plane L M, upon which Velocity, other Things being given, the Quantity of Water which runs out depends. As long as the Intensity of the Pressure does not exceed the Weight mention'd in N. 1816. it is of no Use; what is superadded produces the Effect, which is so much the greater, as the Part superadded is greater †. 1858.
Pl. LVIII. Fig. 4.
* 1813.
† 1817.

But we may refer hither the reasoning in N. 495. whereby it will appear that there is a determinate Part of the Action, to be superadded to the first, that the whole Effect may be the greatest of all; that is, that the Quantity of Water, which is rais'd in a certain Time, may be the greatest of all, with respect to the Power applied. In the second Scholium we shall demonstrate that such is the Effect, if the Intensity of the Power be equal to double the Action often mention'd *. 1859.
* 1816.

If.

1860. If the Plane L M be circular, and the Diameter be one Foot, and the Height E F be ten Foot, and the Power applied along N P be equal to five hundred Pounds, the Water will be ſuſtain'd at the Height E F, and will not run out; but if the Power be equal to a thouſand Pounds, the Effect is the greateſt of all with relation to the Power. If this Power be diminiſh'd, the Effect is diminiſh'd more than the Power; if it be encreas'd, this is encreas'd leſs.

1861. We determine this greateſt Effect, the Height being given to which the Water is to be rais'd; but this is not the greateſt, which this Power can produce.

1862. For if, the Plane L M remaining, any one ſhou'd ſeek the greateſt Effect, which the given Power, for Example, the ſaid Power of a thouſand Pounds, can exert: The Height muſt then be determin'd, to which the Water is to be rais'd, that the Product of the Height of the Velocity of the Water may be the greateſt; for the Effect is chang'd, as the Height is varied, and that follows alſo the Ratio of the Velocity, with which the Water is thrown out of the Pipe.

1863. But this Product is the greateſt, as we alſo demonſtrate in the ſecond Scholium, when the Water is rais'd to an Height, which is equal to two Thirds of that, at which the Power can ſuſtain the Water in the Pipe. The ſaid Power of a thouſand Pounds, wou'd ſuſtain the Water in the Pipe B F, continued upwards, at the Height of twenty Feet; now if the Height E F be thirteen Feet and a Third, the Effect will be the greateſt.

1864. We ſaid that we muſt next examine the Actions of the Wheels,
*1852, 1851. which are mov'd by the Impetus of the Water *.

In what manner ſoever ſuch a Wheel be join'd to a Machine, we may always proceed ſo far in the Computation, as to determine the Power, which acts upon the Surface L M, as the Water preſſes the Boards of the Wheel by its Impetus; and the Velocity of this Power will be determin'd in the ſame manner as the Velocity of the Water, and the Intenſity will depend upon the Velocity. The ſame Wheel remaining, and the ſame Motion of the Water, the Velocity will indeed depend upon the Conſtruction of the Machine, as likewiſe the Intenſity of the Power, which acts along N P; but the Machine being given, when the Velocity is diſcover'd, which anſwers to the greateſt Velocity of the Wheel, that is, to the Velocity of the Water, which moves the Wheel, we conſider the Thing thus; as if the Water, being mov'd with the Velocity diſcover'd, ſhou'd run againſt L M, and ſhou'd there exert a Preſſure, whoſe

Intensity shou'd follow the Ratio of the Square of the respective Velocity of this Water and the Plane L M; and whose Intensity shou'd be determin'd for any given Velocity whatever, from the Intensity of the Action upon the Wheel, and from the Construction of the Machine; by deducing the Reasonings from those Things, which are demonstrated in the second Part of the first Book.

We have seen that the Action of the Water, mov'd with a determinate Velocity, against the Plane is the greatest, when the Velocity of the Plane is equal to a third of the Velocity of the Water *. Yet the greatest Quantity of Water is not always rais'd in this Case; it is only so, when the Water is thrown out thro' a determin'd Orifice. 1865. * 1778.

But if the Orifice be chang'd with the Plane, which immediately thrusts forwards the Water, and the Orifice be always equal to this Plane, the Velocity of the Plane must be diminish'd a little; that is, this must be less than a third of the Velocity of the Water. But the Difference is small, except in a greater Velocity of the Water running against the Plane, or in a less Height, to which the Water is to be rais'd; for if a Body can rise to a greater Height with that Velocity, a less Velocity of the Plane is requir'd; as will be explain'd in the third Scholium. 1866.

It remains that we speak of a Power every way determin'd *. In this Case the Effect is determin'd also, and nothing particular is to be observ'd; the general things demonstrated before, may be refer'd hither, and we must chiefly have regard to N. 1829. 1867. * 1857.

If the Construction of Machines, whereby such Powers are to be applied, it must be observ'd, that the Plane L M must not be too large; for the Intensity of the Action given shou'd exceed the Action mention'd in N. 1816. 1868.

Moreover we shou'd observe, that, when the Plane L M is determin'd, the Height *e g* is given; and therefore *f g*. Therefore the Velocity is given, with which the Water is driven out of the Orifice I F; but the Velocity is given with which L M is mov'd; therefore the Orifice must be so order'd, that these Velocities may answer to one another, that is, that the Velocity of the Plane may be to the Velocity of the Water going out, inversly as the Plane to the Orifice, thro' which the Water goes out; otherwise the Machine wou'd not produce the Effect requir'd: If the Orifice be given, the Magnitude of the Plane L M shou'd be so determin'd, that the said Ratio may take place. 1869.

1870. The Theory, deliver'd in this Chapter, is very universal, I will not deny that there may be difficulties in the Application, in peculiar Machines, and chiefly those, in which the Water is not rais'd thro' Pipes; but this has no relation to the Design of these Elements; the retarding Causes also, which we have overlook'd, encrease the Difficulty; but these can't be avoided, and regard must be had to them of necessity in the Computations.

SCHOLIUM I.

The Demonstration of N. 1826. *concerning Water running out.*

1871. Plate LX. Fig 1. LET *f* O P *q* be a Rectangle, part of which only is represented, whose Base is equal to the Circumference of the Orifice, thro' which the Water flows, and whose Height *f q* is equal to a Quarter of its Diameter; so that the Rectangle may be equal to the Orifice itself.

* 1817. We suppose the Water to ascend thro' the Orifice with a Velocity equal to that, which a Body acquires in falling the Height *g f**, and which the Line *f r* denotes; if all the Water were to go out laterally with the same Velocity, it ought to be rais'd to the Height *f q* above the Orifice; then all the Water wou'd run down sideways, thro' a Space equal to the Aperture itself. But as the Efflux of the Water is made every where, thro' the whole Circumference, in the same manner, it is sufficient to exhibit it in one Point as *f*. The Efflux from *f* to *q*, with the Velocity *f r*, is represented by the Rectangle *f r s q*.

1872. This is the Efflux which is requir'd, that all the Water may run out; with which we ought to compare that, which really runs out. This, in the same Point *f*, is represented by the Surface *f g r*, which is terminated by the Curve *g b r*, whose Ordinates, as *a b*, denote the Velocities in the Points to which they correspond, as *a*; and the Velocity at *a* is equal to
* 380. that, which a Body acquires in falling the Height *g a**; wherefore *g a* is to
† 381. *g f*, in the duplicate Ratio of *a b* to *f r*†. Whence it follows that the
|| *La Hire* Curve *g b r* is a Parabola ||.
Sect. con. B. 3. Prop. 1. As the Rectangle *f q s r* is to the Surface *f g b r*, so is the Quantity of Water, which ascends thro' the Pipe, to that which can flow down at the Sides.

1873. When *f g* does not exceed *f q* one third, the Surface of the Parabola is
* *La Hire* less than the Rectangle*, and all the Water cannot run out; that is, unless
Sect. con B. 5. Prop. 26. the Action be encreas'd, the Velocity of the Water in the Pipe will not be kept.

1874. Let us suppose the Action to be so encreas'd, that this Velocity may be kept; that is, that the Surface, which represents the Water, which really runs out, may be equal to the Rectangle *f q s r*. Let this Encrease be *g n*; the Water will now run out every where laterally with a Velocity, acquir'd in falling from *n*, and the Surface of the Parabola *f n t* will denote all that

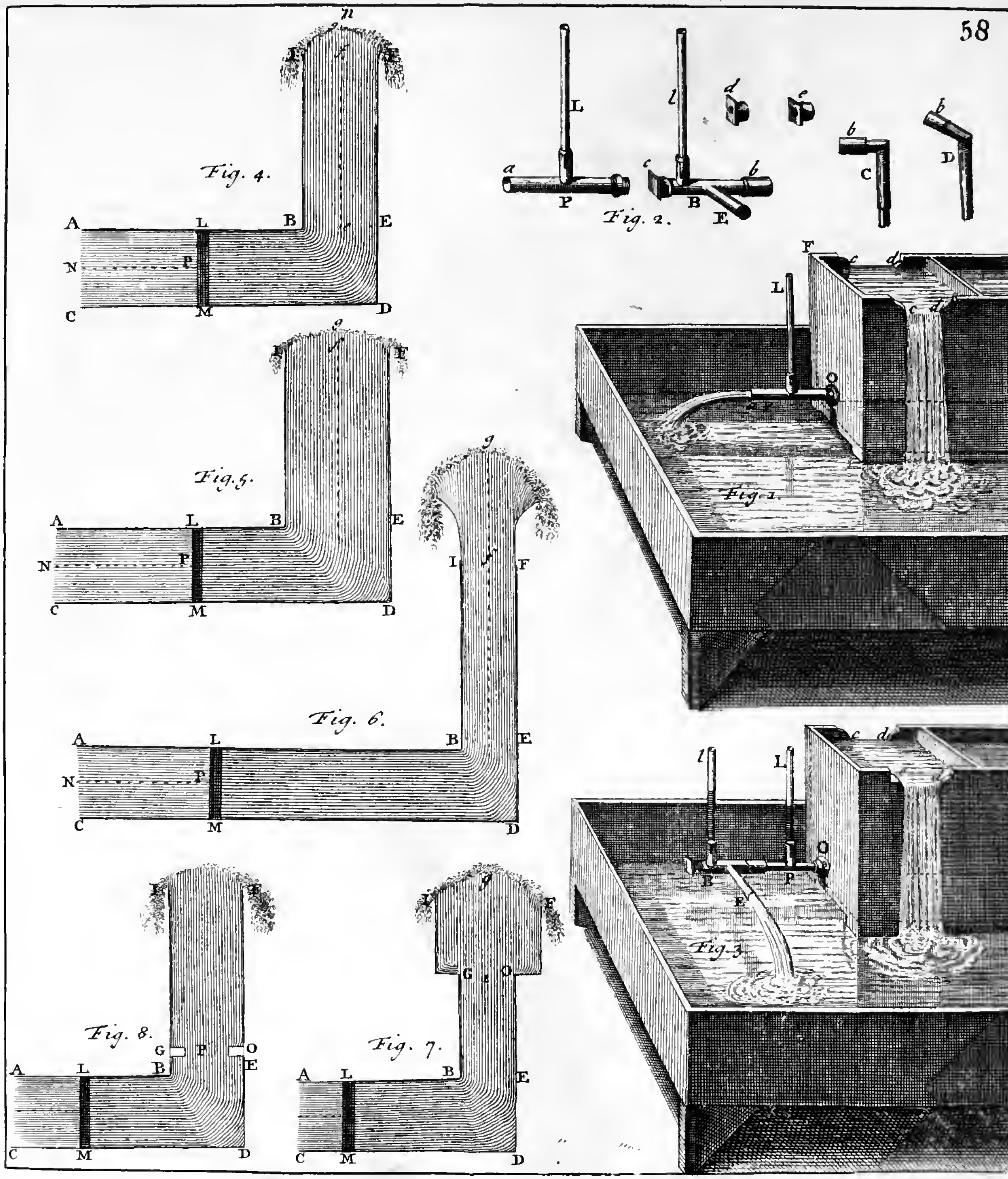


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.

that runs out, and this Surface will be equal to the said Rectangle $fqsr$. This Parabola agrees with the first, it is a greater Portion of the same Curve; for both denote the Velocities of the falling Body. The Rectangle made of ft and fn is to the Surface of the Parabola fnt, as 3 to 2 *; and it is equal to the Rectangle made of fr by three Eighths of the Diameter of the Orifice; for fq is equal to two such Eighths †. Thence it follows that the Square of the Line ft multiplied by the Square of the Height fn, is equal to the Product of the Square of the Line fr by the Square of three Eighths of the Diameter of the Orifice. But the Square of the Line ft is to the Square of the Line fr, as nf is to gf ||. Therefore the Product of the Line nf by the Square of the same, that is, the Cube of the Height sought nf, is equal to the Product of the Height gf by the Square of three Eighths of the Diameter of the Orifice; which we said in N. 1826. and which was now to be demonstrated.

* *La Hire Sect. con.* B. 5 Prop. 26.

† 1871.

|| *La Hire Sect. con.* B. 3 Prop. 1.

SCHOLIUM II.

The Demonstration of those Things, which are mention'd concerning the greatest Actions in N. 1859, *and* 1863.

WE speak in these Numbers 1859. and 1863. of the Action upon a determinate Surface, so that the Intensity of the Pressure may follow the Ratio of the Height at which the Water might be sustain'd in the Tube. 1875.

Let A B be the Height, to which the Water must be rais'd; let A C be the Height, at which it can be sustain'd in the Tube, by the Action of the Power; which Height is as the Intensity of this Power. The Water then goes out with a Velocity with which it cou'd ascend from B to C, which we suppose to be represented by C E. If the Intensity of the Power were as A D, the Water wou'd go out with a Velocity, with which it cou'd ascend along B D; and, D F being drawn, parallel to E C, which in this Case may represent the Velocity of the Water, E C, and F D will be Ordinates to the Diameter B D of the same Parabola B E F, as all this follows from what is demonstrated in the foregoing Scholium. 1876. Plate LX. Fig 2.

We seek the Case, in which the Effect is the greatest of all in respect of the Intensity of the Power; that is, in which the Ratio between the Lines E C and C A is the greatest. For the Quantity of Water, which is rais'd in a certain time, is as the Velocity, with which it runs out of the Pipe at B, which Velocity is as E C. 1877.

E A being drawn, E C will be the greatest of all in respect of C A, when the Angle E A C is the greatest; but this is the greatest, when the Line E A touches the Parabola; for it can't be encreas'd any more. Let this Tangent be A F, and A D will be the Intensity sought of the Power, which will be double A B *, as we said in N. 1859.

* *La Hire Sect. con.* B. 2. Prop. 20.

Let us now suppose the Intensity of the Pressure to be given; and that therefore the Height B A is given, at which the Water might be sustain'd in the Pipe. 1878.

Plate LX. Fig. 3. If the Length of the Pipe be B C; the Water will go out of it, with that Velocity, with which it can ascend along C A; which we suppose to be represented by the Ordinate C E, the Parabola A E D being drawn; as

* 1872. 1876. we have often seen already *.

* 1863. The Effect of which we are speaking *, follows the Ratio of the Height B C, to which the Water is rais'd, and of the Velocity C E, with which it goes out of the Pipe; that is, the Effect follows the Ratio of the

† 23. El. 6. Rectangle C B G E †.

1879. We seek therefore the Point C, that this Rectangle may be the greatest of all. Let us suppose this Point between C and *c*; and that these Points are so determin'd, that the Rectangles C B G E, *c* B *g e*, both which want but little of the greatest, may be equal; and therefore C *c* F E, G *g e* F are equal; whence it follows

* 14. El. 6. $$Fe : FE :: EC : FG^*, \text{ or } EG = CB.$$

But $Fe : FE :: EC : CL$; the Tangent E L being drawn.

* 9. El. 5. † *La Hire. Sect. con.* B. 2. Prop. 20. ‖ 1863. Therefore C B and C L are equal *; and C A is equal to the half of C B †, that is, it is equal to a third of the whole B A. Which was to be demonstrated ‖.

SCHOLIUM III.

The Demonstration of the greatest Action mention'd in N. 1865. 1866.

1880. THE Velocity of the Water being given, which acts against the Plane, we want to know with what Velocity the Plane must be mov'd, that the greatest Quantity of Water may be rais'd in a given Time.

* 918. † 1773. Let the given Velocity of the Water be call'd 1; the Velocity sought of the Plane x; the respective Velocity will be $1 - x$ *; and the Intensity of the Pressure against the Plane $\overline{1 - x}^2$ †.

Moreover we call the Height 1, to which the Water is to be rais'd; a denotes the Height to which a Body can ascend, with the Velocity 1; that is, 1 is to a, as the first Height to the second, both of which are given; lastly z is the Base of the Column of Water, which is rais'd; or rather, z denotes the Magnitude of the Surface mov'd forwards.

1881. Let us suppose the Water to be thrown out thro' a determinate Orifice, which we call 1. All the Water, which is driven forwards by the Plane z, with the Velocity x, passes thro' this Orifice, with a Velocity, which is to x, as the Plane itself is to the Orifice, that is, as z is to 1; therefore that Velocity is zx. With which Velocity a Body can rise to the Height $azzxx$: for

* 381. $$1 : a :: zzxx : azzxx^*.$$

This Height is to be added to that, to which the Water shou'd be rais'd; and the whole Height at which the Power shou'd sustain the Water is $1 + azzxx$. If we multiply this by the Base z, we shall have the whole Column,

Column, which is ſuſtain'd, and to which the Impetus of the Water againſt the Plane is proportional; therefore $\overline{1 - x}^2 = z + a z^3 x^2$. The Quantity of Water, which is rais'd, is that, which the Plane z thruſts forwards, with the Velocity x, in the Caſe, in which this Product is the greateſt of all.

The algebraical Rules *de maximis & minimis* muſt be call'd to our Aſſiſtance here; if a Computation be made, and z be taken away, we have this ſimple Equation $3x = 1$; which agrees with what is deliver'd in N. 1865. 1882.

But if the Orifice, which we ſuppos'd fix'd, be equal to z, the Water goes out of it with the Velocity x; and the Height to which it can riſe with this Velocity, is $a x x$, and we have $\overline{1 - x}^2 = z + a z x x$. Now if the Caſe be ſought, in which the Product $z.x$ is the greateſt, and z be taken away, we have $x^3 + x x + \frac{3x}{a} = \frac{1}{a}$. 1883.

If the Height, which we expreſs by Unity, to which the Water is to be rais'd, be ſmall, or the Velocity of the Water cauſing the Impetus be great, a may exeed Unity; and the greater a ſhall be, the leſs will x be. But if a be leſs than Unity, as it almoſt always happens, x wants but little of $\frac{1}{3}$; as all theſe Things follow from an Inſpection of the Equation, and were ſhewn in N. 1866. 1884.

BOOK. III.

Part IV. Of Bodies mov'd in Fluids.

CHAP. XV.

Of the Resistance which Bodies, mov'd thro' Fluids, undergo.

1885. EVERY *Body, that is mov'd in a Fluid, suffers a Resistance,* and indeed *from two Causes.*

Tho' the Parts of Fluids cohere together but a little, yet it
* 76. 77. is certain that they do cohere together with some Force *. But
1886. *whilst a Body in its Motion separates the Particles of Fluids, it must overcome the Cohesion* just mention'd; *and this is the first Cause of Resistance.*

1887. *This Action is like that, whereby the Parts of soft Bodies are separated, whilst a Cavity is made in them*; which we find is done by an Action, which follows the Proportion of the Cavity itself
* 841. that is made *; which Demonstration we may also apply to a Body mov'd in a Fluid; but in this Motion the Body makes a Cavity proportional to the Space pass'd thro'; tho' this Cavity be again fill'd every time, by the Afflux of the Fluid. Whence we infer,
1888. that *a Body suffers a Resistance from this first Cause,* proportional to this Space run thro'; *which* therefore *is encreas'd and diminish'd*
† 109. *with the Velocity* †.

1889. Whilst a Body makes a Cavity in a soft Body, the Parts are press'd into one another by the immediate Action of the Bodies only, which Actions ceasing, the Motion of the Particles ceases; for this reason, so much Force is consum'd, in making the Cavity, as is necessary to overcome the Cohesion of the Parts; and Bodies may lose their whole innate Forces, in making Cavities.

1890. But a Body, in its Motion along a Fluid, does not only move the Particles of it by an immediate Action, whilst it opens a Way to itself between them, in which immediate Motion of the Parts it overcomes the Cohesion; but it moreover communicates a Force to to the Particles themselves, whereby they are mov'd among one another after the Action of the Body ceases: But *the Re-action of the Particles, whilst this Motion is impress'd upon them, arising from their Inertia, is the second Cause of Resistance.* 1891.

To conceive more clearly what relates to these Resistances, we shou'd have regard to this, that *the mutual Action of a Body and a Fluid is the same, whether the Body be mov'd in a quiescent Fluid with a certain Velocity; or whether, the Body being at rest, the Fluid runs against it with the same Velocity.* For this Action depends upon the respective Motion, which is the same in these Cases. 1892.

Now if, setting aside the Cohesion of the Parts, we have regard to the Motion of the Fluid, and consider it, whilst it runs against a quiescent Body, we shall easily see, that *the Action of a Fluid is the Pressure*; and that the Particles don't impinge against the Body; but that they are mov'd along it, or along the Particles of the Fluid, which touch the Body, and that besides they press the Body, in the same manner as the Body presses a Plane upon which it moves; such are the Pressures, arising from Forces, which have been mention'd above *. 1893. *1003. 1004. 1005.

This Pressure, arising from the innate Force of the Particles, is as this Force, that is, as the Square of the Velocity *; which appears more plainly, if we consider the Analogy, which there is between this Action and the central Force, which *cæteris paribus*, is also as the Square of the Velocity †. The Pressure is also encreas'd, of which we speak here, as the Number of Particles impinging in a certain time, which Number follows the Proportion of the Velocity; lastly, this Pressure follows the Ratio of the Time, during which every Particle acts upon a certain part of the Surface; which Time is so much the less, as the Velocity is greater, and it follows the inverse Ratio of the Velocity: The two last Ratios mutually destroy one another, and there remains only *the Ratio of the Square of the Velocity*; which therefore *the Resistance from the second Cause follows.* 1894. * 753. † 616. 1895.

We easily perceive, by attending to what follows, how both Causes of Resistance act together, in the Case, in which the Body is at rest, and the Fluid is in motion, as we have suppos'd in this Demonstration. That the Particles, which act upon a Body, as A, flow down at the Sides B and D; and exert no Action there, if we set aside the Cohesion. But if we suppose Cohesion, these lateral Particles draw along with 'em the next to them, and separate them by their Actions; which Separation is requir'd, that the Fluid 1896. Pl. LX F. 4.

may

may flow down every way: But the Cohesion can by no means be separated, unless the Body resists, and there be an Action against
* 361. it *; which must be superadded to the Action, arising from the Inertia.

A Machine,

Whereby Experiments concerning the Resistance of Fluids are made.

1897. Plate LIX. Fig. 1. The wooden Trough A B is five Feet long, two Feet and an half broad, and eight or ten Inches high.

This is supported by four wooden Pillars, five Feet high, which stand upon a smaller Trough C D, which has four Feet, about ten Inches high: The Height of these Feet must not be less, because Water which runs out of the Cock E, is to be receiv'd into a Vessel of this Height.

The hollow Parallelopiped of Wood F is three Feet and an half long from *g* to *h*; the Base of its Cavity is five Inches square. We shew in the Figure how this Parallelopiped is fasten'd in a vertical Position between wooden Rulers. The Distance, between the upper Surface of it *l* and the Bottom of the Trough A B, is fifteen Inches.

In this Bottom, in the middle of its Length, there is a round Hole, of about four Inches and an half Diameter, which is at a less Distance from one Side than the other; that the Experiments may be more conveniently made.

To this Hole, answers a Hole, which differs little from the foregoing one, but is less, in the middle of the upper Surface *l* of the Parallelopiped F.

1898. In these Holes a leaden Pipe T, whose Diameter is equal to four Inches, is fasten'd vertically; whereby there is a Communication between the Trough A B and the Parallelopiped F. This Pipe is eighteen Inches long; its Cavity is cylindric, and its Inside is made very smooth.

The Pipe is well fasten'd, and the Water is hinder'd from running out between it and the Wood, by putting small Linnen Threads between.

1899. Pipes of a smaller Bore are also often used, in which Case their Ends are surrounded with wooden Rings, that they may be fasten'd in the same manner in the Holes mention'd.

1900. At the lower End of the Parallelopiped F there are four Cocks, I, L, M, N. Their Holes are in horizontal Plates, which are all

plac'd

plac'd in the ſame horizontal Plane: And theſe Holes are much leſs than the Capacities of the Cocks; that the Water may run out without any ſenſible Friction. The Holes of the two ſmaller Cocks I and L, which are equal, are equal; the Hole of the Cock M is double; and laſtly, that of the greateſt N is triple. The Diameter of the mean Hole is equal to half an Inch.

Although theſe Holes be meaſur'd ever ſo exactly, all Error 1901.
can't be avoided, which how it may be corrected, I ſhall preſently ſhew *. * 1905.

The Board P is put on the Edges of the Trough A B in the middle 1902.
of it, whoſe Length ſomewhat exceeds the Breadth of the Trough, and whoſe Breadth is ſixteen or eighteen Inches. This, leſt what is put upon it, ſhou'd happen to fall into the Water, is ſurrounded by a Border half an Inch high. This Board is faſten'd by four wooden Rulers, two of which are ſeen at *o* and *q*, join'd to it, and between which is the wooden Prominence *r*, join'd to the Trough.

The wooden Croſs S is put upon it, which goes under the 1903.
Table, that it may be faſten'd by means of a Screw. The Balance V is hung upon the Croſs, which we make uſe of in other Experiments, and of which we ſpoke before *. * 1480.

This is ſuſpended in ſuch manner, that, when it is in Equilibrio, the Feet of the Scales may be remov'd from the Board a little more than a Quarter of an Inch.

But the Hook of the Scale *k* anſwers to a Hole in the Board, whoſe Diameter is three Quarters of an Inch, and whoſe Center is in the Axis continued of the Pipe T.

In the Experiments, made with this Machine, we make uſe of 1904. Pl. LIX. Fig. 2. 3 4.
the Globes, Cylinders, and different Cones, which are every one ſuſpended by Horſe-hairs; in which Suſpenſion with reſpect to the Cylinders and Cones we muſt take care, that their Axis be vertical, and the Vertices of the Cones be directed upwards.

To try whether the Holes of the Cocks have that Ratio to one another, which I mention'd *, and to correct the Errors, I made * 1900.
uſe of a Method, which I will now explain.

Things being diſpos'd as I explain'd, and a Pipe being applied at 1905. Fig 1.
T *, whoſe Diameter was the Hypotenuſe of a Right-angled * 1689.
Iſoſceles Triangle, whoſe Sides are two Inches, I filled the Trough A B with Water in ſuch manner, that the Edges of the Trough were only two Inches diſtant from the Water; whereby F and T were fill'd alſo.

In

In the Pipe T, I hung by an Horſe-hair a Braſs Cylinder, whoſe Diameter was almoſt an Inch and a Quarter, and whoſe Height is an Inch and an half; the upper Surface is a little Convex, and it is hollow, that it may load the Balance leſs, and it is well clos'd, that the Water may not get into it: The Horſe-hair was join'd to the Hook of the Scale *k* of the Balance V, and, by a Weight put into the oppoſite Scale, there was made an Equilibrium.

I wanted to know what Weight muſt be added to make an Equilibrium, when one of the ſmaller Cocks was open'd; this Weight is diſcover'd by trial. You firſt put in any Weight that you pleaſe, and the Balance is kept in Equilibrio by the Hand, and, after the Cock is open'd, it is let go; if the Balance be mov'd, the Weight is encreas'd, or diminiſh'd, according to the different Motion; and the ſame Operation is repeated, till, when the Balance is left to itſelf, it remains in Equilibrio; then we have the Weight, which is equal to the Action, which the Water, whilſt it moves along the Tube, exerts againſt a Body.

By this Method I diſcover'd, the ſmaller Cocks being open'd ſucceſſively, that their Actions differ a little; and that therefore the Quantities of Water, running thro' each of 'em, were not exactly equal; which Error was eaſily amended, by making one of the Holes ſomewhat larger.

* 1900. Both of the Cocks I, L *, being then open'd together, that twice the Quantity of Water might run out, I wanted to know the Action of the Water againſt the Cylinder; and I took care that the Action ſhou'd be the ſame, when the ſingle Cock M was open'd.

Laſtly, by the ſame Method, I ſo order'd the Cock N, that there might flow ſuch a Quantity of Water out of it, as was equal to that which run out of the Cock M, and one of the Cocks I, or L together, in the ſame time.

In all this I took care to keep the Water in the Trough at the ſame Height, which muſt be obſerv'd in all the Experiments, that are made with this Machine; wherefore, when the Surface of it ſinks one Inch, more Water muſt be pour'd in.

1906. The Machine is now fit to make the Experiments. The Cock I, or L being open'd, a certain Quantity of Water runs out, and the Water in the Pipe T is mov'd with a determinate Velocity, which Velocity is uniform throughout the Pipe; for a Quantity of Water, equal to that, which runs out of the Cock, continually enters into the Pipe, and goes out of it in the ſame Time. The Velocity of the Water in the Pipe is double, if a double Quantity of of Water runs out, that is, if both the Cocks I and L, or only M be open'd. It is triple if you open M and I or L together, or N alone. The Velocity is quadruple if you open the three Cocks I, L, and M; or N, and one of the Cocks I and L. It is quintuple if you open M and N at the same time. It is sextuple if N, M, and one of the Cocks I and L be open'd. Lastly, it is septuple if you open them all together.

In all these Motions there can be no Accelaration of the Water 1907.
in the Pipe T arising from the Cohesion, such as we mention'd in another Place *; which if it shou'd obtain, this Conclusion cou'd * 1652.
not be drawn, *viz.* That an equal Quantity of Water runs out of a Cock in a certain time, whether it be open'd alone, or together with others, which is here past all doubt; because a Velocity, which very much exceeds the greatest, which the Water has here in the Pipe, can arise only from the Pressure of the Water which is above the Orifice of the Pipe.

EXPERIMENT I.

Things being dispos'd, as has been explain'd in the Description of 1908.
the Machine, and the Pipe T, above mention'd *, being applied, Pl. LIX. Fig. 1. 2.
whose Diameter is the Hypotenuse of a Right-angled Isosceles Triangle, * 1905.
whose Sides are two Inches each, the Brass Globe G, whose Diameter is half an Inch, is suspended at any Depth, of six, eight, or ten Inches in the Pipe. In whose Axis the Globe is; because the Horse-hair to which it is fasten'd, is join'd to the Hook of the Scale *k*.

By the Method deliver'd in N. 1905. I sought the Actions of the Water upon the Globe, whilst the Water pass'd thro' the Pipe with different Velocities successively; which Actions were equal to the Resistances of a Body, if, the Water being at rest, it had been mov'd in it, with the same Velocities.

The smallest Weights, which I made use of in determining these Actions, were quarters of a Grain; the Actions discover'd were as follow.

Velocities.	*Resistances.*
1.	Gr. $\frac{3}{4}$.
2.	Gr. 1 $\frac{1}{2}$.
3.	Gr. 3.
4.	Gr. 4 $\frac{3}{4}$.
5.	Gr. 7 $\frac{3}{4}$.
6.	Gr. 10 $\frac{1}{2}$.
7.	Gr. 14.

1909. In the three firſt Velocities the Actions wanted a little of the Weights ſet down.

Theſe Experiments were made with a very nice Balance, and great Care was taken in the making of them; but I cannot affirm that there was no Error at all, howſoever ſmall, in them.

I muſt confeſs that ſome ſmall Errors, leſs than a quarter of a Grain, cou'd not be avoided; and I do not think that we recede from the Experiment in ſupplying ſo ſmall a Matter, when a regular Series requires it.

That there is ſuch an Error in the firſt Action, here determin'd, which wants but little of $\frac{3}{4}$ of a Grain, and which is ſenſible with reſpect to this Weight, is not only ſhewn by the regular Series, to be deduc'd from the reſt of the Experiments, but is alſo confirm'd by
* 1913. the following Experiment *.

The Experiments are deliver'd here, as they were made by me, before any computation was made.

A Grain being now divided into an hundred Parts, it appears in the following Series, that the Reſiſtance in part follows the Ratio of the Velocity, and in part the Ratio of the Square of the Velocity.

1911.

Velocities.	*Reſiſtances from the firſt Cauſe.*	*Reſiſtances from the ſecond Cauſe.*	*The Sums of both.*	*Reſiſtances in the Exp.*
1.	1. × 20 = 20.	1 × 26 = 26.	46.	75.
2.	2. × 20 = 40.	4 × 26 = 104.	144.	150.
3.	3. × 20 = 60.	9 × 26 = 234.	294.	300.
4.	4. × 20 = 80.	16 × 26 = 416.	496.	475.
5.	5. × 20 = 100.	25 × 26 = 650.	750.	775.
6.	6. × 20 = 120.	36 × 26 = 936.	1056.	1050.
7.	7. × 20 = 140.	49 × 26 = 1274.	1414.	1400.

1912. *When ſimilar Bodies are mov'd in like manner, and with equal Velocities, along the ſame Fluid,* it is deduc'd from what is demon-
* 1886. 1893. ſtrated before *, that both Reſiſtances are encreas'd, and diminiſh'd, as the Number of the Particles of the Fluid mov'd out of their Place at the ſame time, is encreas'd and diminiſh'd; that is, *the whole Reſiſtance follows the Ratio of the Squares of the homologous*
† 20. El. 6. *Sides* †; *and in Globes, Cylinders, or Cones, it follows the Ratio of*
‡ 2. El. 12. *the Squares of the Diameters* ‡.

Experiment 2.

1913. Pl. LX. Fig. 1, 2. This differs from the laſt Experiment only with reſpect to the largeneſs of the Globe, which is ſuſpended in the Pipe T. In this we

we use the Globe H, whose Diameter is the Hypotenuse of a
Right-angled Isosceles Triangle, whose Sides are half an Inch each,
namely equal to the Diameter of the Globe G, made use of in the
first Experiment; wherefore the Squares of the Diameters are as
one to two *, in which Ratio also were the Resistances discover'd, * 47. El. 1.
as appears from the following Table, in which + denotes Excess,
and — Defect.

Velocities.	*Resistances* of the Globe H.	*Resistances* of the Globe G. in *Exp.* 1.
1.	$\frac{3}{4}$ +	$\frac{3}{4}$ —
2.	2 $\frac{3}{4}$	1 $\frac{1}{2}$ —
3.	6 —	3 —
4.	9 $\frac{3}{4}$ +	4 $\frac{3}{4}$
5.	15 $\frac{1}{4}$	7 $\frac{3}{4}$
6.	21	10 $\frac{1}{2}$
7.	28	14.

The Resistances in the less Velocity only do not agree with the 1915.
Proposition; but we have already seen in the foregoing Experiment, that that must be corrected, which was discover'd in that Experiment, when the Velocity was the smallest of all; but the Resistance there plac'd in the regular Series, is half of that, which was determin'd, in the same Velocity, in this last Experiment.

The Resistance from the first Cause is not chang'd according to the 1916.
different Figure of the Body, if the Cavity made in the Motion be but
the same *; wherefore in a Cone and Cylinder, mov'd in the Di- * 1887. 841.
rection of their Axis, as also in the Globe, if the Diameters of these Bodies be equal, and the same Fluid, and Velocity be given, the Resistance is the same.

But *the Resistance from the second Cause differs according to the* 1917.
different Figure of the Body; for tho' a Fluid at rest presses every way with the same Force, it is manifest that this shou'd not be refer'd to Pressure arising from Motion; this acts in one Direction only, and is not wholly sustain'd but by a Plane perpendicular to this Direction.

We demonstrate in the following Scholium that *the Resistance of* 1918.
a Cylinder is to the Resistance of a Cone, if they are both right ones, and mov'd with the same Velocity, in the Directions of their Axes, in the same Fluid, as a Line drawn on the Surface of the Cone from the Vertex to any Part of the Base, is to the Semidiameter of the Base.

1919. We demonſtrate in the ſame Scholium, that *the Reſiſtances of a right Cylinder and a Globe are to one another as three to two, if the Diameters are equal, and the former be mov'd in the Direction of its Axis.*

1920. Whence it follows that *the Reſiſtance of a Globe is to the Reſiſtance of a right Cone, mov'd in the Direction of its Axis, and the Diameter of whoſe Baſe is equal to the Diameter of the Globe, as two thirds of a Line, drawn on the Surface of the Cone to a Point of the Baſe, are to the Semidiameter of the Baſe.*

It is to be obſerv'd that the Vertex of the Cone muſt move foremoſt; for if the Baſe ſhou'd undergo a Reſiſtance, it wou'd be manifeſt that this wou'd not differ from the Reſiſtance of a Cylinder of the ſame Diameter.

Experiment 3.

1921. Pl. LIX. Fig. 1. 3. This Experiment is made in the ſame manner as the foregoing, only the Body is different upon which the Water acts. I made uſe of the Cone repreſented at O, the Diameter of the Baſe is half an Inch, the Height half an Inch from the Vertex *v* to the Center of the Circle, which terminates the conical Figure, below which conical Figure the Body was cylindric, and the Height of the cylindric Part was about an Eighth of an Inch. But this lower Part of the Body is not to be regarded, becauſe the Water, which moves in the Direction of the Axis of the Body, cannot run againſt it.

The Actions of the Water againſt the Body are contain'd in the following Table.

1922.

Velocities.	*Reſiſtances.*
1.	Gr. $\frac{1}{2}$ —
2.	Gr. $1\frac{1}{4}$ +
3.	Gr. $2\frac{1}{2}$ —
4.	Gr. 4.
5.	Gr. 6. —
6.	Gr. $8\frac{1}{2}$
7.	Gr 11.

A Grain being divided into an hundred Parts, we ſeparate the Reſiſtances from each Cauſe in the following Table.

Velocities.

Velocities.	*Resistances from the first Cause.*	*Resistances from the second Cause.*	*The Sums of both.*	*Resistances in Exp.*
1.	1. × 20 = 20.	1 × 20 = 20	40	50—
2.	2. × 20 = 40.	4 × 20 = 80	120	125+
3.	3. × 20 = 60.	9 × 20 = 180	240	250—
4.	4. × 20 = 80.	16 × 20 = 320	400	400
5.	5. × 20 = 100.	25 × 20 = 500	600	600—
6.	6. × 20 = 120.	36. × 20 = 720	840	850
7.	7. × 20 = 140.	49 × 20 = 980	1120	1100

Whosoever shall examine this Table, will easily see, that hardly any thing more accurate can be expected in such Experiments. 1924.

By comparing this Experiment with the first *, N. 1916. is confirm'd. *1907. 1911.

It is also plain, as to the Resistance from the second Cause, that it is, in this Case, to the Resistance of a Globe of the same Diameter, as 20 to 26 *. *1910. 1923.

Now to make a Computation of these Resistances; let them be to one another as $\frac{1}{3}$ $v\,b$ to the Semidiameter of the Base * : If this Semidiameter be call'd 1, the Height of the Cone will be 2; and $v\,b$ will be equal to the Square Root of the Number 5 †; therefore the Resistances are as $\frac{1}{3}\sqrt{5}$. to 1. But the Conical Figure is not kept in the upper Part, that the Cone may be suspended by its Vertex; wherefore the Resistance must be encreas'd. 1925. * 1920. † 47. El. 1.

At the Sides of the Hole thro' which the Thread goes, there are two small plane Surfaces, which together are equal to about $\frac{1}{25}$ of the Surface of a Circle, whose Diameter is $b\,d$; wherefore a twenty-fifth Part of the Resistance of the Cone must be encreased in the Ratio of the Resistance of this Cone to the Resistance of the Cylinder; that is, in the Ratio of 1 to $\sqrt{5}$ *. Therefore the Resistances sought are as $\frac{25\times 2}{3}\sqrt{5}$. to $24+\sqrt{5}$; which Ratio differs little from the Ratio of 26. to 19. In which Computation we have neglected the Consideration of the Figure of the Vertex, to which the Thread was tied. * 1918.

This Resistance from the Computation differs from the Resistance in the Experiment a twentieth Part, howsoever the Velocity be altered; whence it appears, that this Difference is to be attributed to the Figure. 1926.

But

But as this Difference is not very great, and as a certain Part of a Figure cannot eaſily be reduced to Calculation, it is manifeſt that the Propoſition N. 1920. is confirm'd by this Experiment.

1927. I did not make Experiments with Cylinders, to confirm the Demonſtrations; the Difficulty of theſe Experiments was the Reaſon why I did not: for a Cylinder cannot well be ſuſpended without moving, whilſt the Water paſſes along by it; whence the Series of Reſiſtances is irregular, and in greater Velocities very uncertain.

I alſo found that the Reſiſtances of Cylinders, whoſe Diameters were equal, but Heights different, were different; which is a certain Sign of ſome Agitation, as it is paſt all doubt, that the Reſiſtance of a Cylinder, which moves in the Direction of its Axis, does not depend upon its Height. But as Spheres and Cones are eaſily ſuſpended in ſuch manner, that no Agitation is to be fear'd, I thought it beſt to make uſe of theſe Bodies.

Nevertheleſs I ſhall add ſome Things concerning Experiments made with Cylinders.

1928. Among the four Cylinders, with which I made the Experiments, there is one, whoſe Diameter is half an Inch, and Height $\frac{2}{3}$ of an Inch, whoſe Reſiſtances give a Series nearly regular; which being reduced to a Regularity, exactly agrees with what is before demonſtrated; the greateſt Correction anſwers to the Velocity ſix, in which the Reſiſtance wants $1\frac{1}{4}$ Gr. that is, about a twelfth Part of that, which is requir'd in the Series; which is certainly a remarkable Difference.

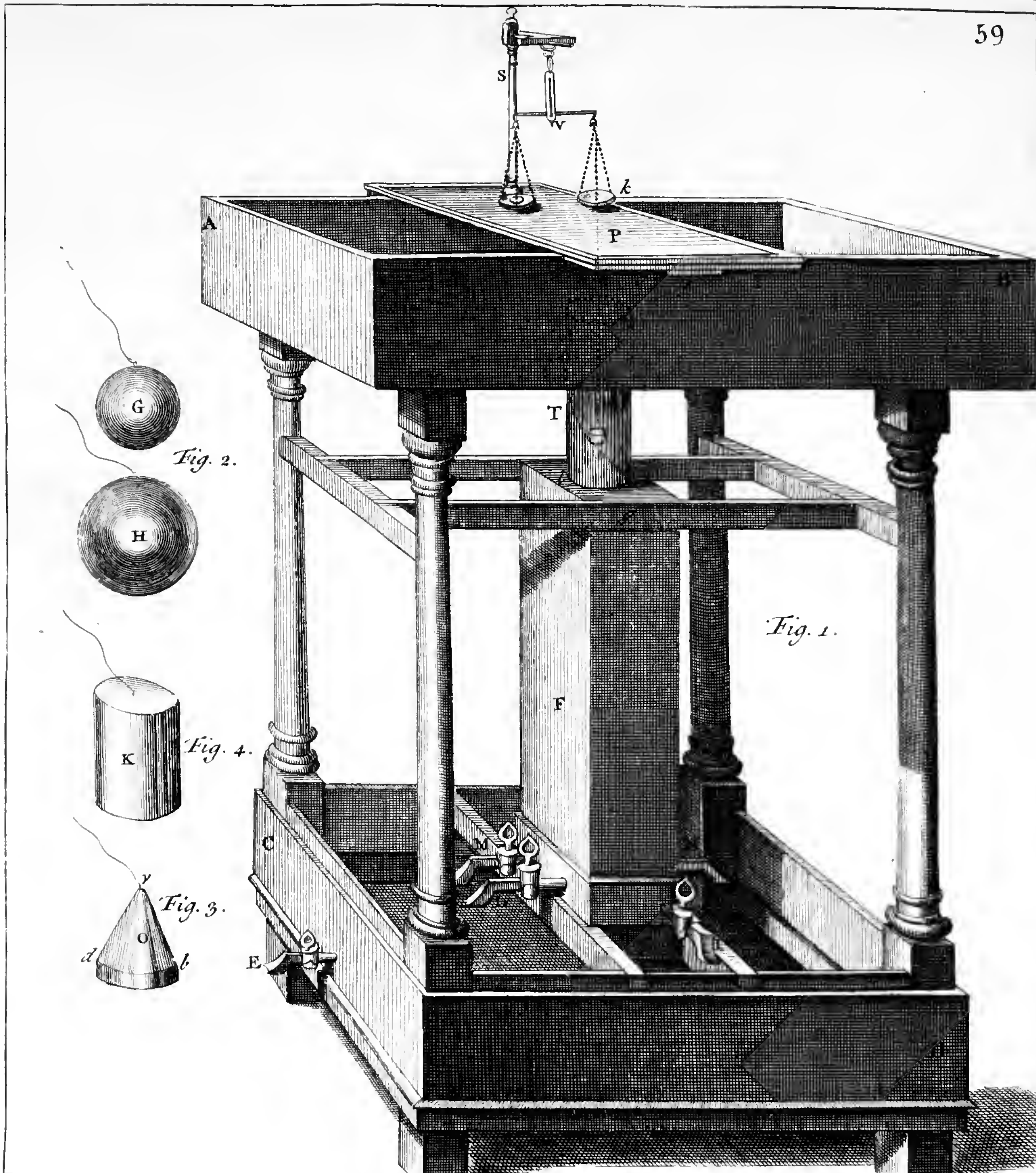
Experiment 4.

1929. Pl. LX. Fig. 1. 4. This was made in the ſame manner as the foregoing Experiments, by ſuſpending the Cylinder K, whoſe Diameter was half an Inch: The Water mov'd in the Direction of the Axis of the Cylinder.

Velocities.	*Reſiſtances.*
1.	Gr. $\frac{3}{4}$.
2.	Gr. 2.
3.	Gr. 4.
4.	Gr. 7. $\frac{1}{2}$.
5.	Gr. 11.
6.	Gr. 14.
7.	Gr. 20. $\frac{1}{2}$.

A Grain being divided into an hundred Parts, the Reſiſtances from the two Cauſes are ſeparated.

Velocities.


S
V
k
A
P
T
Fig. 2.
G
H
Fig. 1.
F
K
Fig. 4.
C
M
γ
Fig. 3.
O
d
b
E

Velocities.	*Resistances from the first Cause.*	*Resistances from the second Cause.*	*The Sums of both.*	*Resistances in the Exp.*	1930.
1.	1 × 20 = 20.	1 × 39 = 39.	59.	75.	
2.	2 × 20 = 40.	4 × 39 = 156.	196.	200.	
3.	3 × 20 = 60.	9 × 39 = 351.	411.	400.	
4.	4 × 20 = 80.	16 × 39 = 624.	704.	750.	
5.	5 × 20 = 100.	25 × 39 = 975.	1075.	1100.	
6.	6 × 20 = 120.	36 × 39 = 1404.	1524.	1400.	
7.	7 × 20 = 140.	49 × 39 = 1911.	2051.	2050.	

Whence it appears that the Reſiſtance from the firſt Cauſe, in this Caſe, is that, which was obſerv'd, in the Experiments made with the Globe, and Cone, of the ſame Diameter with this Cylinder; according to what is demonſtrated in N. 1916. It is alſo manifeſt that the Reſiſtance from the ſecond Cauſe, in this Experiment, is to the Reſiſtance of the Globe, as 39 to 26. *; that is, as 3 to 2; as I ſaid in N. 1919. 1931. *1930. 1911.

The Reſiſtance from the firſt Cauſe is different in different Fluids; and it is manifeſt that this Difference can only be determin'd by Experiments. 1932.

In ſwifter Motions, if we except glutinous Fluids, *the Reſiſtance from the Coheſion of Parts is ſmall, if compar'd with the Reſiſtance from the ſecond Cauſe*; which follows from the different Ratio's, according to which they are encreas'd. For the Velocity, in which theſe Reſiſtances are equal, being encreas'd, for example, an hundred times, the firſt will be to the ſecond, as one to an hundred. 1933.

But *the Reſiſtance from the ſecond Cauſe, in different Fluids*, follows the Ratio of the Particles mov'd out of their Place; for it depends upon the Inertia of Matter, which follows the Ratio of the Quantity of Matter *: Therefore this Reſiſtance *is, cæteris paribus, as the Denſity of the Fluid.* 1934. * 19.

A Computation may be made of the Reſiſtance from the ſecond Cauſe, without making any Experiment, by determining the Weight, which is equal to this Reſiſtance. 1935.

Let there be a Body, whoſe Surface A B ſuffers a Reſiſtance, whilſt the Direction of the Motion is perpendicular to this Surface; but we ſuppoſe, as above, that the Body is at reſt, whilſt the Fluid moves, whereby the Action of the Fluid againſt the Body is not chang'd *. 1936. Plate LX. Fig. 5.

* 1892.

Let

Let the Surface C D at the Bottom of the Veſſel be equal to the Surface A B, which Veſſel contains a ſimilar Fluid to the Height D F; moreover, let us ſuppoſe the Preſſure, which the Part C D of the Bottom undergoes, to be equal to the Action, which AB undergoes, ſetting aſide the Coheſion of the Parts.

Theſe two equal Planes hinder the Motion of the Fluid, and are preſſed for this Reaſon: Therefore, as the Actions are equal, they hinder equal Motions. Therefore, if the Planes are taken away, the Fluid moves with the ſame Velocity, in the Places where the Planes acted; that is, the Fluid, which acts upon the Surface A B, is mov'd with a Velocity, with which the Fluid can go out thro' a Hole at C D; which Velocity is equal to that, which a Body acquires in falling the Height E C *in vacuo* *; for we ſet aſide the Coheſion of the Parts; and all Friction. Therefore the Action, which the Surface A B undergoes, whilſt the Fluid acts upon it, is equal to the Weight of a Column of the Fluid, whoſe Baſe is C D, or A B, and Height E F; for this is the Preſſure which C D undergoes *.

* 1583.

* 1531.

1937. Whence it appears, that *the Reſiſtance of a Right Priſm, mov'd in a Fluid, in a Direction perpendicular to its Baſe, is equal to the Weight of a Column of the ſame Fluid, whoſe Baſe is equal to the Baſe of the Priſm, and whoſe Height is that, from which a Body, falling* in vacuo, *acquires the Velocity with which the Priſm is mov'd in the Fluid.*

1938. This Demonſtration only obtains, when the Surface, which ſuffers the Reſiſtance, is perpendicular to the Direction of the Motion *; in other Surfaces, we muſt have regard to what is demonſtrated concerning theſe *.

* 1917.

* 1918. 1919.

1939. Wherefore in a Globe, the Reſiſtance will be equal to two thirds of the Weight of a Cylinder of the Fluid, whoſe Diameter is equal to the Diameter of the Globe, and whoſe Height is that, from which a Body falling *in vacuo*, acquires the Velocity with which it is mov'd in the Fluid *.

* 1937. 1919.

1940. *The Height, from which a Body in falling acquires a Velocity, with which if it be carried in the Fluid, the Reſiſtance from the ſecond Cauſe is equal to the Weight of the Body, is eaſily diſcover'd* from theſe Things. *In a Priſm, the Denſity of the Fluid will be to the Denſity of the Priſm, as the Height of this is to the Height ſought* *.

* 1937. 1464.

1941. *In a Globe the Denſity of the Fluid will be to the Denſity of the Globe* as the Height of a Cylinder, of the ſame Weight with the Globe, and that has its Diameter equal to that of the Globe, which Height is equal

equal to two thirds of the Diameter, is to two thirds of the Height ſought *, that is, *as the Diameter is to the Height ſought.* * 1937. 1919. 1464.

The Weight which is equal to the Reſiſtance, and therefore *the Reſiſtance* itſelf *from the ſecond Cauſe, follows the Ratio of the Baſe of the Priſm, the Denſity of the Fluid, and the Square of the Velocity of the Body* *. Which agrees with what is demonſtrated before †. 1942. * 1937. 374.

What is ſaid of the Weight, which is equal to the Reſiſtance ‡, *agrees with the Experiments* alſo, as will appear, if a Computation be made of the Weight, which is equal to the Reſiſtance, any Velocity being given of thoſe, which the Water had in the Experiments. † 1912. 1934. 1895. 1943. ‡ 1937.

We ſaid that the Velocity of the Water was 2, when the Cock was open'd, whoſe Hole was a Circle of half an Inch Diameter, the Water being five Feet above this Hole; ſo that a Cylindric Foot of Water might run out in 46,66 Seconds †. A Cylindric Foot would take up 18 Feet in the Pipe continued, in which the Experiments were made ||. Therefore the Water paſſed thro' the Pipe with a Velocity, with which 18 Feet are run thro', in 46,66 Seconds; and when the Velocity in the Experiment was 6, the ſame Space of 18 Feet might be run thro' in 15,55 Seconds. By the Computation, made from what is before demonſtrated *, we diſcover this to be the Velocity, which a Body acquires, in falling *in vacuo* from an Height of 0,257 Inch; which is but little more than a Quarter of an Inch. 1944. † 1637. || 1905. 47 El. I.] * 883. 374.

A Cubic Foot of Water weighs 487360 Grains *; and a Cylindric Foot weighs 382772 Grains; and a Cylindric Inch $221\frac{1}{2}$ Grains. * 1551.

The Reſiſtance of a Cylinder whoſe Diameter is half an Inch, and that Velocity, which is call'd 6 in the Experiments, is equal to the Weight of a Cylinder of Water, whoſe Diameter is half an Inch, and whoſe Height is equal to 0,257. Inch * therefore it is equal to 14,23 Gr. * 1937.

Now ſuppoſing this Reſiſtance to be in a duplicate Ratio of the Velocities *, and the Reſiſtance of the Globe equal to two thirds of the Reſiſtance of the Cylinder †, we made the following Table; in which Parts, leſs than the hundredth Part of a Grain, are overlook'd. * 1912. † 1919.

1945. 1930. † 1911.

Velocities.	*Resistances from the second Cause.*			
	Of the Cylinder. Comp.	*Exp.**	*Of the Globe. Comp.*	*Exp.†*
1.	39.	39.	26.	26.
2.	158.	156.	105.	104.
3.	356.	351.	237.	234.
4.	632.	624.	421.	416.
5.	988.	975.	659.	650.
6.	1423.	1404.	949.	936.
7.	1937.	1911.	1291.	1274.

1946. It is no wonder that there is a ſmall Difference between the Reſiſtances, diſcover'd by the Computation, and thoſe, which are found by the Experiments; as this Compariſon depends, 1. upon the Meaſure of the Water that runs out in a certain Time; 2. upon the Meaſure of the Space run thro' by a falling Body in a certain Time; 3. upon the Meaſure of the Weight of a Cubic Foot of Water; and 4. laſtly, upon the Meaſure of the Reſiſtances. In every one of theſe four Meaſures ſmall Errors can't be avoided; yet they are not ſuch as to cauſe any Doubt concerning the Experiments.

1947. In the ſecond Chapter of Book 2. I ſaid *, that I would give a
* 756. Demonſtration in this Chapter, different from that, which is deliver'd there, concerning the Meaſure of Forces, which we affirm'd
* 753. to be proportional to the Squares of the Velocities in the ſame Body *. The Demonſtration is this.

1948. There is no doubt that the Velocity of a Fluid ariſing from the Preſſure of the ſuperincumbent Fluid, follows the ſubduplicate
* 1586. Ratio of the Height of the Fluid *; we have demonſtrated in this
† 1937. Chapter †, that the Reſiſtance from the ſecond Cauſe follows the Ratio of this Height; and therefore the duplicate Ratio of the Velocity: But we have alſo ſeen that the ſame Reſiſtance is in the
* 1894. Ratio of the Quantity of Force of all the Particles of the Fluid *; therefore that Force is alſo as the Square of the Velocity. *Q. D. E.*

SCHOLIUM.

SCHOLIUM.

The Demonſtrations of N. 1914. *and* 1915. *concerning the Reſiſtance of a Cone, and a Globe.*

LET ABCD, EFG, be Sections of a Cylinder and Cone thro' their Axes, the Diameters of whoſe Baſes are equal; let the Fluid move in the Directions of their Axes. The Plane AB ſuſtains the whole Action of the Fluid, whilſt it continually runs down from every Part along this Surface. But the Surface FE ſuſtains a leſs Preſſure, and ſo much the leſs, as its Obliquity to the Direction of the Motion is greater *: And the Preſſure, againſt any Point M, is reduced to a Preſſure perpendicular to the Surface, if, IM being put in the Direction of the Motion, equal to FE, ML be perpendicular to FE at M, and IL be drawn parallel to it. Then the Preſſure, ariſing from the Motion, is to the Preſſure which the Surface undergoes, as IM to ML *; and the Surface FE ſuffers a like Preſſure againſt every Part of it; for the Fluid, which touches every Point of the Surface, undergoes ſuch an Action, from the Fluid which continually comes towards it. This would not be the Caſe in the Motion of ſeparate Bodies; for then the Number of Bodies, running againſt the Surface EF, would be equal to the Number of Bodies, which, taking away the Surface EF, could act upon the Surface EH; but Fluids act always againſt all the Points of the Surfaces, which they preſs. 1949. Pl. LX. Fig. 6.

* 1917. * 319.

If the Preſſure along LM be reſolv'd into two, LN being drawn perpendicular to IM, NM will denote the Action, with which the Body is driven on, in the Direction of the Motion of the Fluid.

Now the whole Action upon the Cone is to the Action upon the Cylinder, as the convex Surface of the Cone to the Baſe of the Cylinder; for ſuch are the Surfaces, upon which the Preſſures act; that is, as EF is to EH: And as the Action, which acts againſt every Part of the Cone, in the Direction of the Motion of the Fluid, is to the Action, which drives the Cylinder on in every Point, that is, as NM is to IM; the Ratio compounded of theſe is the Ratio of the Product EF by NM to the Product of EH by IM.

Which Products, becauſe EF, IM, are equal, are as NM to EH, or ML; for theſe Lines are equal, by reaſon of the equal and ſimilar Triangles IML, EFH, The Triangles LMN, LMI, are alſo ſimilar *; wherefore MN is to ML, as ML is to MI, or as EH is to EF. Therefore the Reſiſtance of a Right Cone is to the Reſiſtance of a Right Cylinder, ſuppoſing them to have equal Baſes, and to be carried with equal Velocities, in the Directions of their Axes, in the ſame Fluid, as the Semidiameter of the Baſe is to a Right Line drawn on the Surface of the Cone from the Vertex to any Point of the Baſe; as was ſaid in N. 1918.

* 8. El. 6.

Let us now ſuppoſe the Cylinder and Sphere, having equal Diameters, to be mov'd with the ſame Velocity, in the ſame Fluid, and that the Cylinder is carried in the Direction of its Axis. 1950.

Pl. LXI. Fig. 1. Let this be ABLM, whilst the Sphere is represented by DFEG; and C is the Center. The Resistance which the infinitely small Part I*i* of the Base of the Cylinder undergoes, is to the Resistance, which the corresponding Part F*f* of the Surface of the Sphere undergoes, IH, *ih*, being drawn parallel to the Axis of the Cylinder, and therefore to the Direction of its Motion, as F*f* is to F*g*, which is drawn parallel to AB; which appears by applying here the Demonstration delivered in the foregoing Number. The Triangles F*fg*, FHC, which are both rectangular, and have the Angles *f*F*g*, CFH, equal, whose Complement to a Right Angle is the Angle *g*FC, are similar: Therefore

$$Ff : Fg :: FC, \text{ or } IH : FH.$$

Therefore if IH represents the Resistance which the Part I*i* of the Surface undergoes, FH will represent that, which the correspondent Part F*f* of the Surface of the Globe undergoes. And, as this Demonstration may be apply'd to every one of the Points of the Surface of the Hemisphere DFE, it follows, that the Cylinder ADEB, circumscrib'd by the Hemisphere, is to the Hemisphere, as the whole Resistance of the Cylinder to the whole Resistance of the Sphere; which Resistances therefore are as three to two, as was said in N. 1918.

1951. From the same Principles is deduced, what relates to the Resistances of any Bodies whatsoever. For Example, it is easily prov'd from hence, that
1952. *the Resistance of a Right Cylinder, whose Height is equal to its Diameter, from the second Cause is the same, if the Velocity be the same, in what Direction soever it is carried.*

CHAP. XVI.

Of the Retardation of Bodies mov'd in Fluids.

1953. † 1885. ‖ 708. WE have seen above that *a Body mov'd in a Fluid* undergoes a Resistance †, and that there is a Pressure contrary to its Motion, whereby it is manifest that the Body *is retarded* ‖.

As there is a double Resistance, the Body also loses Part of its Motion *from two Causes.*

1954. As the Nature of *the* two *Resistances* is different, they *generate different Retardations, in those very Cases, in which the Pressures, which they exert upon the Body, are equal:* Which we deduce from a peculiar Examination of both Resistances.

1955. *In the Case in which the Body is at rest, whilst the Fluid is in motion,* the Causes, which would retard the Body, if it should be in motion, now communicate Motion to it; and *the Velocity acquir'd is equal to the Retardation, which the Body undergoes, when, the Fluid being at rest, the Body is mov'd with that Velocity, which the Fluid*
* 1892. *had in the first Case* *.

But

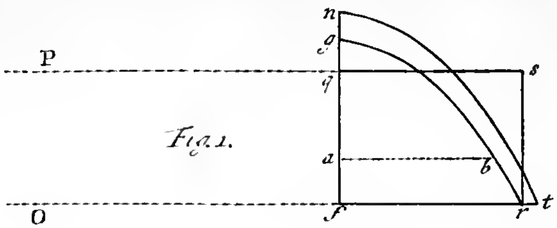


Fig. 1.

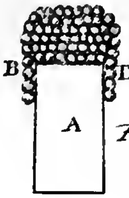


Fig. 4.

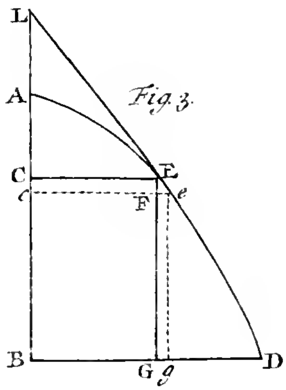


Fig. 3.

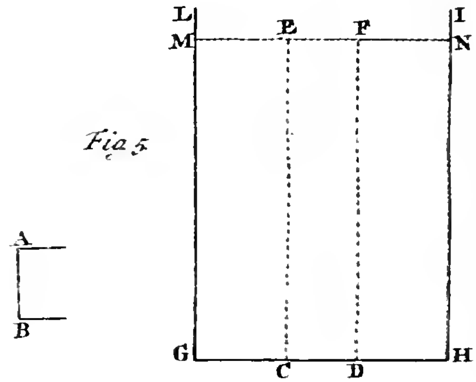


Fig. 5.

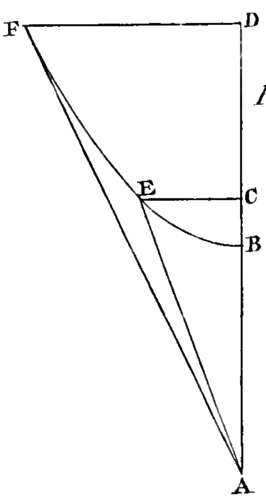


Fig. 2.

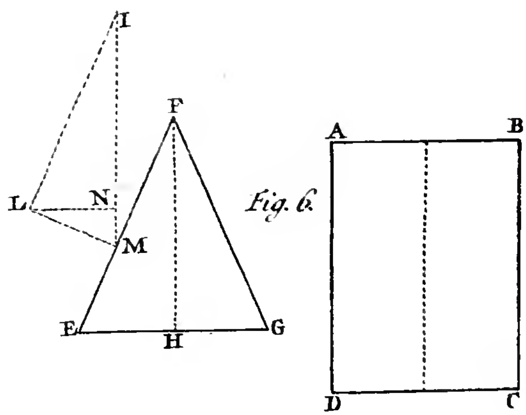


Fig. 6.

But *in the Case* here mention'd, *in which the Fluid is in motion, the Cohesion of the Parts can never communicate Motion to the Body immediately*, but only by means of the Motion of other Particles, as has been explain'd †; which cannot be apply'd in like manner to the second Cause of Resistance, which communicates Motion to the Body immediately: Wherefore *what relates to the Retardations, arising from the different Resistances, is to be deduced from quite different Principles.* 1956. † 1896. 1957.

When the Body is at rest, and the Fluid in motion, the Particles, which run down at the Sides, overcome the Cohesion, and lose something of their Force; which Action must be consider'd, to determine the Velocity from hence communicated to the Body, but it is difficult to determine this Celerity; which neverthelesss shall be explain'd in the last Scholium of this Chapter, where I shall remove some Scruples about it. 1958.

I shall here determine the Retardation, which a Body suffers in the Case, in which it is in motion, and the Fluid at rest.

We have seen that the Resistance from the first Cause is of the same kind as the Resistance of soft Bodies, whilst a Cavity is made in them *. 1959. * 1887.

We have also seen that this Cavity follows the Proportion of the Force lost in making it †; but the Cavity, which a Body makes in a Fluid, whilst it moves along it, is proportional *to the Space run thro'*: Therefore *the Force, lost from the Resistance from the first Cause, is proportional* to this Space also. † 841. 1960.

A Body, which is projected upwards vertically in a Vacuum, in its Ascent continually loses a Force proportional to the Space run thro' †; therefore the Retardation, in this Ascent, follows the same Ratio, which *the Retardation* of a Body, *arising from the Resistance, of which we are speaking*, follows; but the Retardation of a Body ascending *is equable* †; therefore such is the Retardation also, which we are examining. † 754. 380. 1961. † 377.

Therefore, *as long as the same Body is mov'd in the same Fluid, in the same manner, with what Velocity soever it is carried, setting aside the Resistance from the second Cause, in equal Times, it loses equal Degrees of Velocity*; and it will lose its whole Motion, in running thro' a certain Space †, which will in the beginning be proportional to the Square of the Velocity *, in a Time, proportional to this Velocity †. 1962. † 1960. * 1961. 377. 381. † 1961. 377. 378. 374.

From hence we find that *Bodies mov'd in Fluids will at last be at rest*, which, admitting the common Opinion, *viz.* that the Forces are 1963.

are proportional to the Velocities, cannot be explain'd without a great deal of difficulty, if at all; for the whole Velocity could not be confum'd, except in an infinite Time.

The Retardation from the fecond Caufe is determin'd, by fuppofing the Body at reft, and the Fluid running againft it; becaufe it is more eafy to find out the Velocity, which is communicated to a Body at reft by a Fluid, than the Retardation which the Body fuffers; therefore it will be better to confider this Velocity, which does not differ from the Retardation of a Body mov'd along a Fluid
* 1955. at reft *.

1964. The Preffure, which the Fluid produces upon the Body at reft, can move the Body immediately; whence it follows, that there is communicated an infinitely fmall Velocity, in an infinitely fmall, conftant Moment, proportional to the Space, thro' which this Body at reft is mov'd, by the immediate Action of the Fluid; which
* 133. Space is proportional to the Preffure *; which follows the Ratio of
† 1895. the Square of the Velocity †.

1965. Therefore *the Diminutions of the Velocity, which a Body mov'd in a Fluid, fuffers, in Moments, infinitely fmall, and equal, from the Refiftance from the fecond Caufe, are as the Squares of the Velocities of the Body.*

1966. From which Demonftration it follows, that *a Body can never lofe its whole Velocity, from the Refiftance from the fecond Caufe only.*

1967. It appears alfo in every Cafe that *the Retardation, from this Refiftance, follows the fame Ratio as this does, as long as the Body in motion contains the fame Quantity of Matter*; but when this is dif-
1968. ferent, *the Retardation is,* cæteris paribus, *inverfly as the Quantity of*
* 138. *Matter* *. From whence we eafily fee, how, the Demonftrations in the foregoing Chapter being laid down, the Retardations for different Bodies, and different Fluids, may be compar'd together.

1969. *In Spheres, Cylinders, or fimilar Cones,* for Example, *fuppofing the Cylinders and Cones, to be mov'd in the Directions of their Axes, the Retardations from the fecond Caufe will be directly* as the Squares of
† 1967. 1912. the Diameters †, *as the Squares of the Velocities* *, *as the Denfities*
* 1965. *of the Fluids* ‖; *and inverfly as the Denfities of the Bodies* ‡, and the
‖ 1967. 1934. Cubes of the Diameters *: but the direct Ratio of the Squares,
‡ 1968. and the inverfe Ratio of the Cubes of the Diameters, is reduced to
* 1698. the inverfe Ratio of the Diameters; therefore, the laft and firft Ratio's being join'd, the Retardations are inverfly *as the Diameters.*

Numbers

Numbers are found out, in the Ratio compounded of thoſe Ratio's, by multiplying, for each Body, the Denſity of the Fluid by the Square of the Velocity of the Body, and dividing the Product by the Diameter multiply'd by the Denſity of the Body, and the Quotients of the Diviſions will expreſs the Relations of the Retardations. 1970.

These are alſo diſcover'd, if, for each Body, the Weight, which is equal to the Reſiſtance *, be divided by the Weight of the Body; for the Quotients are as the Retardations †. 1971. * 1957. † 1967. 1968. 156.

Whilſt a Body in a Fluid is retarded, the Retardation is changed every Moment, as the Velocity is changed; whence many Things are deduced concerning the Continuation of Motion of a Body in a Fluid, ſome of which are demonſtrated in the Scholiums, annex'd to this Chapter; a few of them I ſhall mention here. 1972.

Setting aſide, as in the laſt Propoſitions, *the Reſiſtance from the Coheſion of Parts, let a Body be mov'd in a Fluid, it will run thro' equal Spaces, in unequal Times, which will be in geometrical Progreſſion; in which Progreſſion, but inverſe, are the Velocities at the Beginnings of theſe Moments.* 1973.

If a Globe, or right Cylinder, be mov'd in a Fluid in the Direction of their Axis, the Length of the Cylinder, or Diameter of the Globe, will be to the Spaces reſpectively moved thro' by theſe Bodies, ſo as to loſe half their Velocity, in a Ratio compounded of the Denſity of the Fluid to the Denſity of the Body, and that of the Number 10000 *to* 13863. 1974.

But *the Retardation of a Body, that is mov'd in a Fluid,* depends upon both Cauſes of Reſiſtance, and *is partly equable* ‡, *partly as the Square of the Velocity* *. 1975. ‡ 1961. * 1965.

Which may be applied to aſcending and deſcending Bodies alſo.

A Body ſpecifically heavier than a Fluid, which aſcends, or ſpecifically lighter than a Fluid, which deſcends, beſides the Retardation ariſing from the Inactivity of the Fluid †, *ſuffers another equable Retardation,* not only from the Coheſion *, but moreover, fom the reſpective Gravity ‡, in the firſt Caſe, and from the Force, with which it is driven upwards in the Fluid *, in the ſecond Caſe. 1976. † 1965. * 1962. ‡ 1491. 376. * 1476.

On the contrary, *if a Body, ſpecficially heavier than the Fluid, in which it is immers'd, deſcends, or ſpecifically lighter than the Fluid aſcends, it is* continually *accelerated* with a Force, which is equal to the Difference of the ſpecifick Gravities of the Body and Fluid ‖, which Acceleration, ariſing from Gravity, is equable *: This is diminiſh'd by the Retardation ariſing from the Coheſion, 1977. ‖ 1478. 1512. * 370.

but.

† 1962. but equably †, and the Acceleration is hitherto equable. But as
1978. the Retardation from the ſecond Cauſe encreaſes with the Velocity, *the Acceleration is continually diminiſh'd; and the Body comes nearer and nearer to a certain determinate Velocity which is the greateſt, to which nevertheleſs it can never arrive.*

1979. But *that is the greateſt Velocity, in which the Retardation is equal to the Acceleration*; for if the Body ſhould come to this, it would continue its Motion equably, the oppoſite Preſſures mutually deſtroying one another.

1980. *A Cylindric Body acquires this greateſt Velocity, by falling* in vacuo *from an Height, which is to the Length of the Cylinder, if it deſcends in a Fluid in the Direction of its Axis*, or in a Globe, to its Diameter, *as the Difference of the Denſity of the Body, mov'd in the*
† 1940. 1941. *Fluid, from the Denſity of the Fluid to this Denſity of the Fluid* †; namely, *if we ſet aſide the Retardation ariſing from the Coheſion of Parts:* but this being ſuppos'd, the Height will be leſs from which, the Body falling *in vacuo*, acquires the greateſt Velocity, of which we are ſpeaking.

Having now done with the Motions in Right Lines, I ſhall add ſomething concerning the Motion of Pendulums.

1981. Let ABD be the Arc of a Cycloid, in which a Pendulum vi-
Plate LXI. brates; B the loweſt Point. The Acceleration ariſing from Gravity
Fig. 2. in any Point whatſoever as E, is as EB †; but this is diminiſh'd
† 414.
‖ 1962. equably by the Coheſion ‖: let this Diminution be as BF, the Acceleration will now be as EF, and at A it will be as AF. In the Aſcent of the Body, the Retardation at G, ariſing from Gravity, will be as GB, that from the Coheſion will be as BF, and from theſe Cauſes jointly it is as GF; and in the whole Vibration, ſetting aſide the other Reſiſtance, the Body is mov'd in reſpect of the Point F, as it would be mov'd *in vacuo* in reſpect of B.

Therefore we will call the Motion of the Pendulum to F its Deſcent, and the Motion beyond this Point its Aſcent; for I ſhall ſpeak of Pendulums deſcending from the Part A.

1982. But to demonſtrate, what obtains, when the Pendulum is retarded by the Reſiſtance from the ſecond Cauſe alſo, I will ſuppoſe a Reſiſtance, which generates a Retardation in a Ratio of the Velocity; and demonſtrate ſome Propoſitions, upon this Suppoſition; which being deliver'd, it will more eaſily appear, what takes place, when the Retardation is as the Square of the Velocity.

1983. Now *ſuppoſing the Retardation to be in the Ratio of the Velocity, let two Pendulums, entirely ſimilar, which vibrate in a Cycloid, perform*

form unequal Vibrations, and begin to fall the same Moment; they begin to move with Velocities, which are as the Arcs describ'd in the Descent †; if these Impressions of the first Moment only should † 414.
be consider'd, after a given Time, the Celerities will be in the same Ratio as in the Beginning; for the Retardations, which are as the Velocities themselves, cannot change their Proportions; for the Ratio between Quantities is not chang'd, by the Addition, or Substraction, of Quantities in the same Ratio *. Therefore in equal * 16. 17. 18. El. V.
Times, in what manner soever the Celerity of the Body be alter'd from the Resistance during the Motion, Spaces are run thro', which are as the Velocities at the Beginning †; that is, as the Arcs to be † 110.
describ'd in the Descent: Therefore, after any Time whatsoever, the Bodies are in corresponding Points of these Arcs. But the Accelerations in these Points are in the same Ratio as in the Beginning *; * 414.
and the Ratio between the Celerities, which is not alter'd from the Resistance, undergoes no Alteration from the Acceleration. In the Ascent the Motion of the Bodies is retarded, but, in corresponding Points, the Retardations are in the same Ratio, in which the Accelerations are in the Descent. Therefore the Celerities are every where, in corresponding Points, in the same Ratio. But as the Bodies are in those corresponding Points, in the same Moments, it follows that the Motion of both is destroy'd at the same time; that is, that *their Vibrations are perform'd in the same Time.* The Spaces, run thro' in the whole Vibrations, as they are passed through in equal Times, and as the Velocities are in the same Ratio to one
another, in each Moment, are also in this Ratio; that is, *the Arcs,* 1984.
of the whole Vibrations, are as the Arcs describ'd in the Descent, the
Doubles of which are the Arcs to be describ'd *in Vacuo.* Therefore 1985.
the Deficiencies of the Arcs, describ'd in a Fluid, from the Arcs, to be describ'd in Vacuo, are the Differences of Quantities in the same Ratio, and *are as the Arcs describ'd by the Descent* *. * 16. 17. El. V.

Now *let the Retardation encrease in a duplicate Ratio of the Velocity, and let a Pendulum perform unequal Vibrations, the greater* 1986.
will be of longer Duration, because the Resistance encreases more than in the Case N. 1982.

Neverthelefs *the Celerities, supposing the Arcs not very unequal,* 1987.
in corresponding Points of the Arcs describ'd, are every where nearly in the same Ratio, and indeed in *the Ratio of the Arcs describ'd*
in the Descent. If the Retardation should be in the Ratio of the 1988.
Celerity, this Proportion would obtain; but is now disturb'd, by reason of the greater Resistance in the greater Vibration, whereby

the Motion in this is diminish'd more: but is accelerated more from a double Cause. 1. This greater Vibration lasts longer *; and the
* 1986. Body continues longer in a certain Space, than in a corresponding Space when the Vibration is smaller; therefore it is accelerated for a longer time. 2. The Defect of the Arc describ'd, from the Arc to be describ'd *in vacuo*, is greater, the Proportion being kept, in the greater Vibration; because in this the Retardation differs more from the Retardation in the smaller Vibration, than in N. 1984. Therefore the corresponding Points, keeping the Proportion, are at a greater Distance from the Point F, in a greater Arc than in a less, as long as the Body descends in it; therefore, the Proportion being kept, there is a greater Acceleration in that; because this is as the Distance of the Body from the Point F. Therefore there is a Compensation, and the Proportion mention'd is restor'd. In the Ascent of the Body, the Duration of the Retardation concurs with the Retardation itself to disturb this Proportion; but now the corresponding Points in the greater Arc, the Proportion being kept, are at a less Distance from the Point F, than in a less Arc, and, keeping the Proportion, the Retardation from Gravity is less; and thus, the Proportion being kept, the Difference of the Distance of the corresponding Points from the lowest Points has encreas'd, so that a Compensation is easily had from this alone.

The Retardations, which are as the Squares of the Celerities, are therefore every where, in corresponding Points, nearly as the Squares of the Arcs describ'd in the Descent; and, the Sums of all the Re-
* 12 El. V. tardations will be in the same Ratio also *, which are *the Differences*
1989. *between the Arcs describ'd in the Descent and the next Ascent.* These Differences therefore, *if the Vibrations have not been very unequal, are nearly as the Squares of the Arcs describ'd in the Descent.* This agrees with the Experiments also pretty exactly.

A Machine,

Whereby Experiments, concerning the Retardations of Pendulums are made.

1990. Plate LXI. Fig. 3. The Trough AB three Feet long, and one Foot broad and one Foot high, is fill'd with Water; the Pendulum *gp* is suspended by the Plate *i*, which is fix'd, and answers to the Middle of the Trough. This Pendulum consists of a Brass Wire *gh*, seven or eight Feet long, and of a leaden Ball *p*, of an Inch and a half Diameter. When the Pendulum is at rest, the Ball is at the Distance

of

of three Inches from the Bottom of the Trough. At P there is a larger Ball of Lead, of three Inches Diameter, join'd to the Wire mention'd; that the Ball *p* may be leſs retarded in the Water.

The Plate *i* above-mention'd is repreſented by itſelf at I; it is faſten'd by two Screws which go into the Wood, and to it are join'd two ſmaller Plates L and M, which have Holes in them to receive the Axis, upon which the Pendulum moves. This Axis is ſharp underneath, like the Axis of a Balance Beam, and is faſten'd to the ſolid Piece of Braſs O, which is join'd to the Wire of the Pendulum. The Axis is put into the Holes, by removing the Plate M, which, being again applied, is faſten'd by means of the Screw *n*. Fig. 4.

Along the Trough, upon the Edges of it, may be mov'd a ſmall Board about five Inches high, to which are applied the divided Braſs Rulers, CD, CD, and the Indices F, F, to meaſure the Angles, deſcrib'd by the Pendulum in its Deſcent and Aſcent, by the Method deliver'd in N. 737. Fig. 3.

EXPERIMENT.

Place the Rulers CD, CD, in ſuch manner, that the Ends D, D may anſwer to the Pendulum, when it is at reſt, and that the Diſtance between thoſe Ends may be equal to the Diameter of the Braſs Wire to which the Bodies P, *p* are join'd. Let go the Pendulum from different Heights ſucceſſively, which in every Caſe are mark'd by the Index: The Heights are diſcover'd to which the Pendulum aſcends, if it be let go ſeveral times from the ſame Height, and one of the Indices be alter'd, till the Pendulum comes to it in its Aſcent; but does not reach the Index, if it be remov'd a little. 1991.

The Differences of the Arcs, deſcrib'd in the Aſcent and Deſcent, will be nearly to one another as the Squares of the Arcs deſcrib'd in the Deſcent, if we have regard to this, that each of the Vibrations muſt be diminiſh'd equally, becauſe of the Reſiſtance from the Coheſion of the Parts.

But it muſt be obſerv'd, that the Pendulum is not to be let down, except when the Surface of the Water is at reſt.

SCHOLIUM I.

Of the Logarithmic Line.

WHAT is demonſtrated in the following Scholiums, concerning the Retardations of Bodies, mov'd in Fluids, is founded upon the Properties of the logarithmic Line. Therefore I ſhall explain the Formation of this Curve, and the Properties of it, which we ſhall have occaſion for in what follows.

1992. Plate LXII. Fig. 1. Let A B be a Right Line, and let A D, D F, F H, &c. be infinitely ſmall Parts in it, which are equal to one another. Moreover, let A C, D E, F G, H I, &c. be perpendicular to A B, the Difference between which is infinitely ſmall, and let them be in geometrical Progreſſion continued. Now if a Curve paſſes thro' the Ends C, E, G, I, &c. this will be a logarithmic Line, whoſe Aſymptote A B will be, to which the Curve continually approaches, but can never reach it.

1993. *There is the ſame Ratio between any two Ordinates whatſoever, if there is the ſame Diſtance between them.*

A C is to H I, as L M is to R S, if the Diſtance A H be equal to the Diſtance L R. For the Ratio, which is given between A C and H I, is compounded of the Ratio's of A C to D E, of D E to F G, and F G to H I; the Ratio of L M to R S, is compounded of the Ratio's of L M to N O, of N O to P Q, and P Q to R S: The compounding Ratio's are equal one to the other *; and the Number of the compounding Ratio's, in each Caſe, is the ſame; by reaſon of the equal Diſtances A H, L R: Therefore the compound Ratio's are equal alſo. *Q. E. D.*

* 1992.

DEFINITION I.

1994. *The correſpondent Abſciſs of any Ordinate is call'd the Logarithm of that Ordinate, whereſoever the Beginning of the Abſciſſes be ſuppos'd.*

DEFINITION 2.

1995. *The Diſtance between two Ordinates is call'd the Logarithm of the Ratio, which is given between them.* And it is the Difference of the Logarithms of the Ordinates themſelves.

A H and L R being again put equal, we have

* 1993. A C : H I :: L M : R S *; and by Diviſion

† 17 El. V. A C — H I = T C : A C :: L M — R S = V M : L M †. Wherefore

‖ 16 El. V. T C : V M :: A C : L M ‖.

1996. That is, *the Ordinates are to one another, as the Differences of every one of them from the other Ordinates, which are equally diſtant from them.*

In

In any Point whatsoever C, of the Logarithmic C M, the Tangent C T being drawn, which cuts the Asymptote at T, *the Subtangent* A T is had: and this *is constant in all Points of the Curve*; and the Tangent M V being drawn at M, A T, L V, will be equal. To make this appear, let A D, L N, be infinitely small, and equal; and the Ordinates D E, N O, being drawn, let E *c*, O *m*, be parallel to A B. The Triangles C *c* E, C A T, are similar; as also M *m* O and M L V; therefore Pl. LXII. Fig. 2. 1997. 1998.

C *c* : *c* E :: C A : A T, by Altern. C *c* : C A :: *c* E : A T; also,

M *m* : *m* O :: M L : L V, by Altern. M *m* : M L :: *m* O : L V.

But C *c* : M *m* :: C A : M L *, and by Altern. C *c* : C A :: M *m* : M L; therefore also *c* E : A T :: *m* O : L V. But *c* E, *m* O, are equal; therefore also A T, L V. Which was to be demonstrated. * 1996.

If the Ordinates A C, D E, F G, H I, &c. being kept, and the Equality of the Distances A D, D F, F H, &c. being kept also, these Distances be encreas'd or diminish'd, it is manifest that the Logarithmic is alter'd, and that the Subtangent is alter'd also in the same Ratio in which these Distances are alter'd; for in the Triangle C *c* E, the Side C *c* being kept, if *c* E be alter'd in the similar Triangle C A T, whose Side C A is kept, A T will be alter'd in the same Ratio with *c* E. 1999. Pl. LXII. Fig. 1, 2.

In the same Ratio also, in which all the smaller Distances are chang'd, the Sums of any Distances whatsoever are chang'd; that is, as A D is chang'd, so is A H chang'd also, the Logarithm of the Ratio of A C to H I; whence it follows, that *in different Logarithmics the Subtangents are to one another, as the Logarithms of the equal Ratio's are.* Fig. 1. 2000.

In the Tables of Logarithms, which are publish'd, the Logarithm of the Ratio of one to ten is Unity itself; and the Logarithms of the intermediate Ratio's are expressed by decimal Fractions, and the Subtangent of the logarithmic Line is 0,43429.44819. 2001.

SCHOLIUM II.

Of Retardation in general.

Retardation and Acceleration is measur'd, supposing Moments infinitely small and equal; the Retardation, which depends upon the first Cause, is said to be equable, because the Diminutions of the Velocity, in equal Times, are equal *. The Retardation from the second Cause is said to be as the Square of the Velocity, because the Diminutions, in Moments infinitely small and equal, are as these Squares †. 2002. * 1962. † 1965.

But *in every one of the Moments infinitely small, the Retardations and Accelerations, during the Moment, are equable*; for in such a Moment the Change in the respective Action may be look'd upon as nothing; therefore, if the Moments differ, *the Retardations and Accelerations* will be as the Moments themselves; that is, these *are in Moments infinitely small and unequal, in a Ratio* 2003. 2004.

Ratio compounded of the Ratio of the Retardations and Accelerations, ſuppoſing the Moments equal *, *and of the Ratio of the unequal Moments* †.

* 2002.
† 2003. When the infinitely ſmall Spaces are equal, the Moments in which all the ſmall Spaces are paſſed thro', are inverſly as the Velocities ‖; therefore *the*
‖ 120.
2005. *Retardations and Accelerations which a Body undergoes, in running thro'* every one of thoſe *ſmall and equal Spaces, are directly as the Retardations, ſuppoſing the Moments equal, and inverſely as the Velocities* *.

* 2004.
2006. Therefore *in the Retardation from the firſt Cauſe, if the infinitely ſmall Spaces are equal, the Diminutions of the Velocity are inverſely as the Velocities* *.

* 1962.
2007. *In the Retardation from the ſecond Cauſe the Diminutions of the Velocity are, in equal Spaces,* directly as the Squares of the Velocities, and inverſly as the Velocities themſelves *; that is, *directly as the Velocities.*

* 1965. 2004.

SCHOLIUM III.

Of Retardation from the firſt Cauſe.

2008. Plate LXI. Fig. 5. LET A C be the Space, in which a Body loſes its whole Velocity, when it is retarded from the firſt Cauſe alone, whilſt the Velocity at the Beginning is repreſented by the Line A D.

Whilſt this Space A C is run thro' by the Body, it undergoes the ſame Changes to which an aſcending Body is ſubject, which ſhould be retarded by Gravity alone, and which, in aſcending to the Height A C, ſhould loſe its whole Velocity *. Therefore the Square of the Velocity at A is to the Square of the Velocity at any other Point whatſoever B, as A C to B C †. Therefore if A D be to B E, in the ſubduplicate Ratio of A C to B C, B E will repreſent the Velocity at B. But there is given this Ratio between the Ordinates of the Parabola, which paſſes thro' C and D, C being ſuppos'd the End of the Diameter A C ‖.

* 377. 1962.
† 380. 374.
‖ *La Hire Sect. con.* B. 3. Prop. 1.

2009. Therefore, *if the Diameter of the Parabola repreſents the Space run thro', the Ordinates to the Diameter will repreſent the Velocities, in any Points whatſoever, if the Body be retarded from the firſt Cauſe only, or undergoes any other equable Retardation whatſoever.*

* 2006. If the infinitely ſmall Spaces A *a* and B *b* be equal, the Diminutions of the Velocities D F, G E, will be inverſly as the Velocities A D, B E *; if A *a* or B *b* be changed, D F or G E is changed in the ſame Ratio; there-

2010. fore *in the Parabola, the infinitely ſmall Differences of the neighbouring Ordinates are directly as the Differences of the correſponding Abſciſſes, and inverſly as the Ordinates themſelves.* Which alſo might have been deduced from the Conſideration of the Parabola alone.

2011. *If there be two Bodies given, carried with equal Velocities, which ſuffer different Reſiſtances from the firſt Cauſe, or in general different, equable Reſiſtances, the Spaces, in paſſing thro' which the whole Velocities are conſum'd, are inverſely as the Retardations in equal Moments*; as is eaſily deduced from the Demonſtrations concerning Aſcent upon inclin'd Planes. For with equal

equal Velocities Bodies ascend to the same Height on different Planes *; * 399.
that is, the Spaces, in running thro' which they lose their Velocities, are as the Length of the Planes, supposing the Heights equal; but in this Case the Pressures, whereby the Bodies endeavour to descend along these Planes, which are as the Velocities communicated in the same time, or taken away, * 341.
are in the inverse Ratio of the Lengths *. *Q. E. D.*

SCHOLIUM IV.

Of Retardation from the second Cause.

2012. IF A B, *the Asymptote of the logarithmic Curve, represents the Space run thro' by a Body in a Fluid, the Velocities, in every one of the Points, may* Pl. LXII. Fig. 1.
be represented by the Ordinates; for the Decrements of the Velocities, in the infinitely small and equal Spaces, A D, D F, F H, *&c.* are as the Velocities themselves *, and the Decrements of the Ordinates A C, D E, F G, * 2007.
&c. as the Ordinates themselves †. † 1996.

2013. Whence it follows that, if the Spaces are equal, as A L, L X, X B, the Velocities in the Points A, L, X, B, which are denoted by the Ordinates A C, L M, X Z, B K, are in geometrical Progression *; as we observ'd * 1993.
in N. 1973.

2014. Let A T be the Asymptote of the logarithmic Curve; B Y the logarithmic Curve; B M its Continuation, being in a contrary Position. Plate LXII. Fig. 3.

Now if we take any Ordinate whatsoever, as T Y M, the Logarithm of the Ratio of T M to A B is A T *, which also is the Logarithm of the Ratio * 1995.
of A B to T Y; therefore T M, A B, T Y, are in continued Proportion †: † 1993.
and the Square of A B is equal to T M × T Y ‖; and all the Rectangles as ‖ 17 El. 6.
T Y × T M, S X × S L; P E × P G, *&c.* are equal to the same Square of A B, and therefore to one another.

2015. Therefore the Ordinates encrease, which are terminated by the Curve B M, as the corresponding ones are diminish'd, which are terminated by the Curve B Y; and the first are inversely as the second.

2016. All the infinitely small Spaces are run thro' with an equable Velocity; therefore the Moments, in which small and equal Spaces A C, C P, P Q, *&c.* are run thro', are inversely as the Velocities, in which they are run thro' †; that is, inversely as A B, C D, P E, *&c.* ‖; or directly as A B, † 120.
C F, P G, *&c.* ‡; which are as the Differences, B *b*, F *f*, G *g*, *&c.* * ‖ 2012.

Therefore the whole Time, in which a Line as A Q is run thro' is represented by all these Differences jointly, that is, by the Line N H; in the ‡ 2015. * 1996.
same manner O M represents the time, in which Q T is run thro': but if the Spaces A Q, Q T, be equal, N H will be to O M, as Q H to T M *; * 1996.
that is, inversely Q K to T Y †, or A B to Q K ‖. † 2015.

‖ 1993.

2017. Therefore the Times, in which the equal Spaces are run thro' successively, are inversely as the Velocities at the End, or inversely as the Velocities at the Beginnings of the Spaces; as was said in N. 1973.

Let

2018. Pl. LXII. Fig. 2. * 2012. Let us again ſuppoſe a Body, which is mov'd in the Line A B, and retarded from the ſecond Cauſe only; let A C be the Velocity at A, and C M the logarithmic Curve, which determines the Velocity in the other Points *; that we may make uſe of this Curve and the Tables, in the Computations, it is neceſſary, that we determine the Magnitude of the Subtangent of the logarithmic Curve, which may be of uſe in any Caſe whatſoever that is propos'd; or, which is the ſame thing, we ſhould determine, what Space is repreſented by the Subtangent in any given Figure whatſoever.

Let us ſuppoſe A C to be the Velocity, with which if a Body be carried in a Fluid, the Reſiſtance from the ſecond Cauſe will be equal to the Weight of the Body.

2019. Therefore the Weight of the Body, that is, *the Preſſure from Gravity, which* retards the aſcending Body, *is equal to the Preſſure*, which the Body, of which we are ſpeaking, ſuffers *from the Reſiſtance from the ſecond Cauſe.* By both theſe Preſſures the Body is immediately mov'd, when they act upon it; therefore *the ſame Motion of the ſame Body* can be *equally* chang'd by them; and the Retardation, which the Body ſuffers in the Fluid in the firſt Motion, is equal to the Velocity, which in an equal Moment an aſcending Body, and which *is retarded* by Gravity, loſes.

2020. Now let C *c* be the Retardation which the Body ſuffers in running thro' A D, C *c* will be the Velocity, which the Body loſes, in aſcending to the Height A D, when it is retarded by Gravity. Let us now ſuppoſe a Parabola deſcrib'd, whoſe Axis let A B be, and which paſſes thro' the Points C and E, that is, let it have the ſame Tangent C T as the logarithmic Curve, which paſſes thro' C and E, and whoſe Aſymptote is A B.

The Ordinates of this logarithmic Curve will denote the Velocities of the
* 2012. Body mov'd in the Fluid, whoſe Velocity at A is A C *: and A X the Axis of the Parabola, whoſe Vertex is X, will ſhew the Height, to which the Body, projected upwards with the Velocity A C, and retarded by Gra-
† 2009. vity alone, can aſcend †; therefore X A, *the half of the Subtangent* A T ‖,
2021. *denotes the Height from which a Body falling* in vacuo *acquires a Velocity, with*
‖ *La Hire Sect. con.* B. 2. Prop. 20. *which if the Body be mov'd along the Fluid, it ſuffers a Reſiſtance equal to the Weight of the Body itſelf*, which Height is given *.
* 1940. Theſe things being ſuppos'd, the reſt is a natural Conſequence. Let A L be the Space run thro' by the Body.

2022. *As* A X, *the Height from which a Body, falling* in vacuo, *acquires a Velocity, which gives a Reſiſtance equal to the Weight of the Body, is to* A L, *the Space run thro' by the Body in the Fluid, ſo is the half of the Subtangent of the Tables*; that is, A X, expreſſed in Numbers of the Tables, to A L denoted
* 2001. by the ſame Numbers, that is, as 0,21714.72409. * *to the Logarithm of the*
† 2012. *Ratio between the Velocities at the Beginning and at the End of the Space* †.

2023. Any Numbers whatſoever in the Tables, the Difference of whoſe Logarithms is the Logarithm found of the Ratio, are to one another as theſe
* 1995.1993. Velocities *.

2024. By the ſame Rule, the Ratio between the Velocities at the Beginning and End of the Space run thro' being given, this Space is diſcover'd.

The

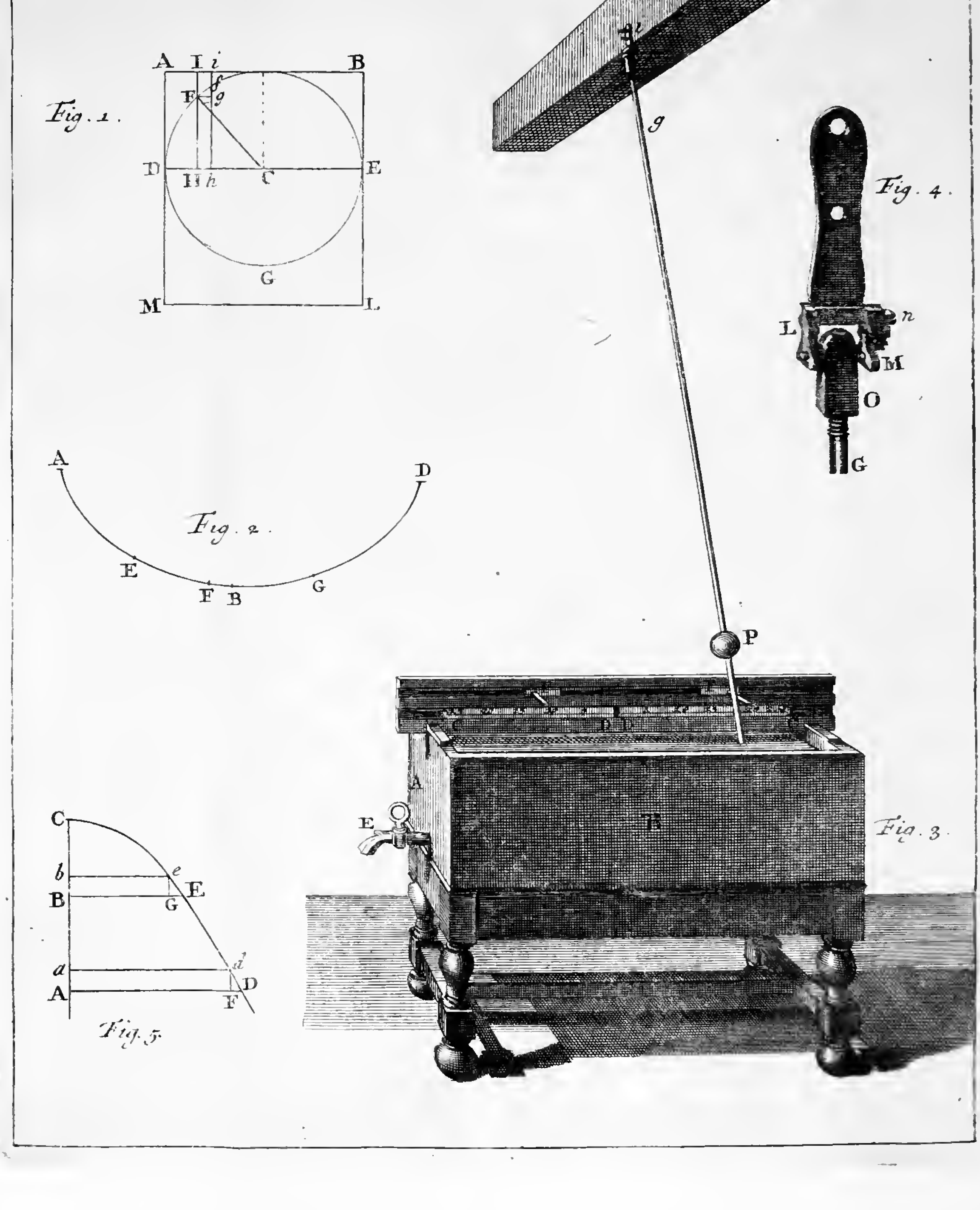


Fig. 1.
A I i B
F f g
D H h C E
G
M L
Fig. 2.
A D
E F B G
Fig. 3.
P
A
B
E
Fig. 4.
L n
M
O
G
Fig. 5.
C
b e
B G E
a d
A F D

The Logarithm of the Ratio of 2. to 1. is had, by substracting from the Logarithm of the Number two 0,30102.99957. the Log. 0. of Unity; therefore as 0,21714.72409, is to 0,30102.99957, that is, as 10000000000. is to 13862945972, so is the Height from which a Body, falling *in vacuo*, acquires a Velocity, which gives a Resistance equal to the Weight, to the Space, in which the Body loses half its Velocity *. This agrees with what is deliver'd in N. 1974. 2025.

* 2021.

If in any Point whatsoever, *the Retardation from the second Cause becomes equable, the Space in which the whole Velocity is destroy'd* is represented by the half Sub-tangent, as follows from the Demonstration N. 2019, which may be apply'd here also; but as the Subtangent is constant *, it follows also that in an homogeneous Fluid, such as we suppose here, that Space is not changed, howsoever the Velocity be alter'd; and that this *is equal to the Height from which a Body, falling* in vacuo, *acquires a Velocity, which being given, the Resistance is equal to the Weight* †. 2026.

* 1997.

† 2020.

SCHOLIUM V.

Of both Retardations jointly.

LET A M be the Line, which the Body runs thro' in the Fluid; let this be the Asymptote of the logarithmic Curve I S P, of which A I is an Ordinate; moreover, let G F B be a Parabola, whose Axis is I B, and Vertex B, its Ordinate G I parallel to A M, its Parameter B I: If A B be to B I, as the Retardation from the first Cause is to the Retardation from the second in the Point A, the Velocity in any Point whatsoever, as C, may be determin'd. For if in this Point there be given C D, perpendicular to A M, an Ordinate of the logarithmic Curve, and thro' D, D F be drawn parallel to I G and A M, G I and F E will be as the Velocities in the Points A and C, if the logarithmic Curve be rightly determin'd; of determining which I shall presently speak. 2027.

Pl. LXII. Fig. 4.

To demonstrate this we suppose A *a* and C *c* infinitely small and equal; if the Velocities, in the Points *a* and *c*, as in the Point C be determin'd, these will be K H and *e f*; therefore the Decrements of the Velocities, whilst the equal Spaces A *a*, C *c* are run thro', are G *g* and F L; we must demonstrate, if G *g* be resolv'd into two Parts which are as A B to B I, that F L may be resolv'd into two, in such manner, that the first Parts of each Decrement may be inversely as G I to F E *, and the second directly in the same Ratio of G I, or B I †, (because this is the Parameter of the Parabola) to F E ‖; that is, we must prove that G *g* is to F L, as $\frac{AB}{GI}+\frac{BI}{GI}$ is to $\frac{AB}{FE}+\frac{FE}{GI}$. 2028.

* 2006.

† *La Hire Sect. con.* B. 3. Prop. 2.

‖ 2007.

* 2010. But this is the Demonſtration; $Gg : FL :: \frac{IK}{GI} : \frac{Ee}{FE}$ *; but IK :

† 1996. $Ee :: AI : AE$ †; therefore $Gg : FL :: \frac{AI}{GI} = \frac{AB}{GI} + \frac{BI}{GI} : \frac{AE}{FE} = \frac{AB}{FE} + \frac{BE}{FE}$.

* *La Hire Sect. con.* B.3. Prop. 2. But $\frac{BE}{FE} = \frac{BE \times FE}{FE \times FE} = \frac{BE \times FE}{BE \times BI}$ * $= \frac{FE}{BI} = \frac{FE}{GI}$ by reaſon that BI, GI are equal: Therefore $Gg : FL :: \frac{AB}{GI} + \frac{BI}{GI} : \frac{AB}{FE} + \frac{FE}{GI}$.

Which was to be demonſtrated.

The Space in which the Body loſes its whole Velocity is BP, or AQ,
* 2027. for the Velocity in the Point Q is none *.

2329. Now to determine the logarithmic Curve, let this Figure ſerve for the Computation alſo, the Space, repreſented by the given Line, is to be determin'd; as alſo the Ratio which is given between IB and BA; which we cannot arrive at, without Experiments made concerning the Retardations themſelves.

2030. Therefore we ſuppoſe *the Space* AQ to have been diſcover'd by Experiment, *in which the Body loſes its whole Velocity*; which Space *being given*, the Ratio between AB and BI, which is *the Ratio of the Retardations* in the Point A, namely *at the beginning*, *may be diſcover'd*, in the following manner.

2031. The Velocity in A is repreſented by the Line GI, or BI equal to it; and the Retardation, whilſt the Space Aa is run thro', is Gg, as we have ſeen; this (by reaſon of the Subtangent of the Parabola being double the Abſciſs
‖ *La Hire Sect. con.* B.2. Prop. 20. BI ‖, and therefore double GI) is half of gH, or ik.

The Line IkO touches the logarithmic Curve ISP; taking AM double of AO, and drawing IM, which cuts ki in m, ki will be double mi, which therefore is equal to Gg, and repreſents the Retardation.

Let MT be parallel to AI; which BP produced cuts at N; ſo that AB, MN, and BI, NT alſo, may be equal; therefore drawing IN, which cuts mi in n AB, will be to BI, that is, the firſt Retardation to the ſecond in the Point A, as mn is to ni; therefore theſe repreſent ſeparately each Retardation; for the Sum denotes the Retardations jointly.

Now ni is the Retardation, which the Body ſuffers from the ſecond Cauſe alone, whilſt BI, which is equal to GI, expreſſes the Velocity at A. Therefore if we ſuppoſe the logarithmic Curve IR, whoſe Aſymptote is BN, and which paſſes thro' I and n, BR will denote the Velocity which the Body, if it ſhould be retarded from the ſecond Cauſe only, would have
* 2012. left, in running thro' the Space AQ, or BP * diſcover'd by the Experi-
† 2022. ment; and the Ratio between BI and PR may be diſcover'd †.

The Subtangent of the logarithmic Curve IR is BN, or AM the double of AO, which is the Subtangent of the logarithmic Curve IP.

Therefore

Therefore if A Q, equal to B P, the Logarithm of the Ratio of B I to P R, be divided into two equal Parts at V, and V S be given perpendicular to A Q, B I will be to P R, as A I to V S *. But A I, V S, Q P are in continued Proportion †; therefore AI^q is to VS^q, that is, BI^q is to PR^q, as A I is to Q P, or A B; and by Division * 2000. † 1993.

$$BI^q - PR^q : PR^q :: AI - AB = BI : AB.$$

Which may be thus express'd: *As the Square of the Velocity of the Body in the beginning minus the Square of the Velocity, which, if the Body should be retarded from the second Cause alone, it would have remaining, after a Space run thro', in which, whilst it is retarded from both Causes, it loses its whole Motion, is to this last Square; so is the Retardation from the second Cause to the Retardation from the first, in the first Moment of the Motion.* 2032.

These things being premis'd, we discover by Computation the Velocity in any given Point whatsoever of the Line A Q, as C. 2033.

We seek in the Numbers of the Tables the Logarithm of the Ratio of B I to P R *, which is the Logarithm of the Ratio of A I to V S; if this be doubled, we have the Number that represents A Q, if we suppose I S P to be the logarithmic Curve of the Tables: for the Demonstrations may be applied to any logarithmic Curve whatsoever; let this Number be call'd L. * 2022.

As the Space A Q, in which the Body loses its whole Motion, is to the given Space A C, that is, as A Q is to A C, so is L to the Logarithm of the Ratio of A I to C D or A I to A E; which therefore is given, and may be denoted by the Letter M.

Now taking a Number at pleasure, which denotes A I, the Logarithm A I — M will be the Logarithm of the Number which denotes C D *, or A E. The Logarithm A I — L is the Logarithm of the Number, that denotes Q P, or A B; which Numbers we determine: therefore there are three Numbers given, which are to one another as A I, A E, A B; wherefore the last being substracted from the two first, there remain Numbers, which are as B I to B E, that is, as the Squares of the Velocities at A and C †, at the beginning and given Point. * 1995. † 2027. *La Hire Sect. con.* B. 3. Prop. 1.

By a contrary Operation, the Velocities G I and F E being given, and the Space A Q also, in which the Body loses its whole Velocity, the Point C is discover'd. For A Q being given, the Ratio between B I and B A * is discover'd; and taking a Number, which expresses the Velocity G I, equal to B I, B A is given; but as GI^q is to FE^q, so is B I to B E, therefore the Number is given that expresses this Line; and therefore we determine Numbers, which are to one another as A B, A E, A I. But it appears from the Demonstrations *, that as the Difference of the Logarithm A I, A B, is to the Difference of the Logarithm A I, A E, so is A Q to A C, the Space run thro', which is therefore discover'd. 2034. * 2032. * 2028.

C Q, the Space in which the Body loses its whole Motion, is determin'd also, a different Velocity F E being given in the beginning, by substracting A C from A Q. 2035.

2036. We suppos'd the Space A Q to have been discover'd by Experiment, in
which the Body loses its whole Velocity, when at A; that is, in the begin-
ning, it has the Velocity with which the Experiment was made. But if
the Experiment had been made, any other Velocity being given; such, for
* 1940. example, as should be to the Velocity in the beginning, as F E to G I *,
we might make the same Computations.

2037. The Space discover'd by the Experiment is C Q. If we suppose I R
to be the logarithmic Curve of the Tables, as B I is equal to G I the Velocity
* 2022. at A, we discover the Number of the Tables that expresses X P *, or C Q;
this Number expresses the half of C Q, if we consider the logarithmic Curve
† 2000. I D P †; therefore, the Number being doubled, we have C Q, the Lo-
garithm of the Ratio between C D and Q P, that is, A E and A B. But
as the Ratio between F E and G I is given, the Ratio between B E, B I, is
given also, which is the Duplicate of that; from which we deduce the
Numbers which are as B I to A B, as in N. 2031.

Now if we suppose, the Velocity G I being given, the Retardation from
the second Cause to take place only, and that this becomes equable, there is
* 2026. given the Space in which the whole Velocity is destroy'd *; but this Space
is to *the Space, in which the whole Velocity is destroy'd from the Resistance from*
2038. *the first Cause only*, as A B is to B I *, which Ratio as it is given †, this
* 2011. † 2032. 2037. Space *is determin'd* also. But *these Spaces, in different Fluids, are inversely as*
2039. *the Cohesion of the Parts* ||.
|| 2011.

SCHOLIUM VI.

Of Bodies projected upwards.

2040. A *Body, specifically heavier than a Fluid, which is projected upwards in
it, is retarded from three Causes*, from Gravity, and the two Causes
explain'd in this Chapter. The Retardation from Gravity, and from the
* 377. 1961. first Cause, are both equable *, and being join'd, only cause an equable
2041. Retardation; wherefore *what is demonstrated in the Scholium above, may be
applied here* also.

2042. Therefore *if it appears by one Experiment only, to what Height a Body
ascends in a Fluid, with a given Velocity*, the following Problems are solv'd.
1. *The Height is discover'd, to which a Body can ascend, with any other
* 2035. given Velocity whatsoever* *.
2. *The Velocity at the beginning being given, the Velocity in the given*
2043. *Point is found* †.
† 2033. 3. *The Velocity being given, the Space is discover'd, in which*, setting aside
2044. the Resistance of the second Cause, *the Body would lose its Motion, from the
* 2038. Retardation only from the respective Gravity and Cohesion jointly* *.
2045. 4. *The Space is discover'd in which the Body*, mov'd with a given Velo-
city, *would lose its Motion from the Cohesion alone.*

2046. For as the Velocity is given, the Height is given, to which the Body can
ascend *in vacuo*; this is to the Height to which the Body ascends in the
Fluid,

Fluid, whilst it is retarded by the respective Gravity alone, as this respective
Gravity is to the whole Weight *. * 2011.

But this last Height is to the Height, to which the Body ascends, whilst it is retarded by the respective Gravity and Cohesion, which Height is also
given †, as the Retardation from those two Causes is to the Retardation † 2044.
from the respective Gravity alone ‖. ‖ 2011.

Therefore by Division *as the Difference of these Heights is to the last* 2047.
Height, so is the Retardation from the Cohesion to the Retardation from the respective Gravity; and in the same Ratio is the Height, whilst the respective Gravity alone retards, to the Space, in which the Motion is lost by the Co-
hesion alone *. * 2011.

5. Lastly, *the Velocity being given, we discover the Space, in which the* 2048.
Body would lose its Motion, in an horizontal Motion, whilst it is retarded by the Cohesion and Inertia. Which must be farther explain'd.

In the foregoing Computation there is given the Ratio between the Re- 2049.
tardation from the Cohesion and Retardation from the respective Gravity *. * 2047.
Therefore the Ratio between the first of these and the Sum of them is given also. The Height also is given, to which a Body ascends, with a given Velocity, whilst it loses its whole Motion, when it is retarded from
these two Causes, and the Inertia also † ; whence we deduce the Ratio, † 2042.
which is given between the Retardation from the Cohesion and from the respective Gravity jointly, that is, between the said Sum, and the Retar-
dation from the Inertia *. The Ratio which is compounded of these two * 2032.
Ratio's, is that, which is given between the Retardation from the Cohesion and the Retardation from the Inertia. If we refer this to the Figure of the
foregoing Scholium, the Ratio between A B and B I is given, G I being Plate LXII.
suppos'd the Velocity, of which we are speaking; and A Q is sought; F E Fig. 4.
might indeed be taken for the Velocity propos'd, in which Case, the Parameter B I of the Parabola would be to be discover'd, from the Ratio between BE and F E; but it is needless to confine one's self to a determin'd Figure.

From the known Ratio between A B and B I may be deduced the Ratio
of B I to P R *; which being given B P †, or A Q is discover'd. * 2031.
† 2032.

A Body specifically lighter than a Fluid, is carried upwards in it in the 2050.
same manner, as one heavier than a Fluid goes towards the Bottom of it; wherefore the Demonstrations in this Scholium may be applied to Bodies specifically lighter than Fluids, which descend in them with an impressed Motion.

SCHOLIUM VII.

Of Bodies falling in Fluids.

A Body, which falls in a Fluid spontaneously, is continually accelerated 2051.
equably *; *but in the mean time it suffers a Resistance, which is as the* * 370. 1961.
Square of the Velocity †. † 1965.

What

What relates to this Motion, is alſo repreſented by a Parabola, and logarithmic Curve.

2052. Pl. LXII. Fig. 5. Let Q A R be the Aſymptote of the logarithmic Curve B D H, A B an Ordinate of this Curve perpendicular to the Aſymptote; which alſo is the Axis of the Parabola B F Q, whoſe Parameter we ſuppoſe A to be, and its Vertex at B.

If A R repreſents the Space run thro' in falling, the Point from which the Body is let down being ſuppos'd at A, the Velocity in any Point whatſoever as C, is determin'd by drawing C D parallel to A B, and thro' D, D E F parallel to R A Q; E F the Ordinate of the Parabola will denote the Velocity ſought, whilſt A Q expreſſes the greateſt Velocity, to which the Body does not arrive, till after the Space A R, produced *in infinitum*, is run thro'.

Theſe things will appear if, the infinitely ſmall and equal Spaces C c, G g being taken at pleaſure, we demonſtrate the Increaſes of the Velocities, which are here expreſſed by the Lines f L and k M, to be to one another inverſely as the Line F E and K I, which we ſay expreſs the Velocities, Parts being taken away, which are as theſe Lines F E and K I †.

† 2006. 2007. 2051.
* 2010.
† 1996.

$$fL : kM :: \frac{Ee}{FE} : \frac{Ii}{KI} * :: \frac{CD}{FE} = \frac{BA}{FE} - \frac{BE}{FE} : \frac{GH}{KI} = \frac{BA}{KI} - \frac{BI}{KI} \dagger;$$

|| La Hire Sect. con. B. 3. Prop. 2.

But B E x B A = F E x F E ||; therefore $\frac{BE}{FE} = \frac{FE}{BA}$. In the ſame manner $\frac{BI}{KI} = \frac{KI}{BA}$. Therefore

$$fL : kM :: \frac{BA}{FE} - \frac{FE}{BA} : \frac{BA}{KI} - \frac{KI}{BA}.$$

Which was to be demonſtrated.

2053. That we may make uſe of this Figure in the Computation, the greateſt Velocity, to which the Body can arrive, and which is repreſented by Q A, muſt be determin'd:

Therefore we ſeek the Velocity, which being given, the Retardation from the ſecond Cauſe is equal to the Acceleration from the reſpective Weight, taking away the Retardation from the firſt Cauſe; for this is the uniform Acceleration, which is to be deſtroy'd by the Retardation from the ſecond Cauſe, that the Acceleration may ceaſe *.

* 1979.

2054. Here we ſtand in need of an Experiment again; therefore let the Height be given, to which the Body aſcends in the Fluid, with any given Velocity whatſoever; from this being known, we infer the Ratio between the Acceleration from the reſpective Weight and the Retardation from Coheſion *; and therefore the Ratio of this Acceleration to this, taking away the Retardation from Coheſion: And this is the Ratio, which is given between the Height, from which a Body falling *in vacuo* acquires a Velocity, which gives

* 2047.

gives a Resistance equal to the respective Weight, which Height is given *, * 1937. 1938.
and the Height from which a Body, falling *in vacuo*, acquires the Velocity sought QA †. † 374. 1965.

But this Height being discover'd, we discover another also, namely, 2055.
from which the Body falling in a Fluid, setting aside the Resistance from the second Cause, would acquire the same Velocity QA; for the Height *in vacuo* is to the Height in the Fluid, as the Retardation from the respective Weight, taking away the Retardation from the Cohesion of the Parts, to the Retardation from the whole Weight *. Let us conceive this * 2011.
Height to be represented by the Line B A, *b* O will denote the Velocity acquir'd, in falling in the same manner from the Height B *b* †. † 2009.

Moreover, we ought to determine the Space, denoted by a certain known 2056.
Portion of the right Line A R; which will be done, if we have regard to this; that in the beginning of the Fall the Body is accelerated by the respective Weight, taking away the Retardation from the first Cause, because this Acceleration is equable: but that it is not retarded from the second Cause, because the Velocity is none; and that therefore the Velocity *b* O, in the first infinitely small Moment, in falling from the Height which is represented by A *a*, is acquir'd as in the Motion mention'd in falling thro' B *b*; and therefore B *b* and A *a*, in different Lines, represent equal Spaces: but B *b* is to A *a*, or *b* N, as B A is to A P, the Subtangent of the logarithmic Curve; therefore B A and A P also denote equal Spaces; and the Space, represented by the Subtangent, is *the Height, from which the Body* 2057.
falling in a Fluid, setting aside the Resistance from the Inertia, can acquire the greatest Velocity.

Now when the Tables are to be made use of, it appears, that this Height *is to any given Height whatsoever,* AG, *as the Subtangent of the Tables* 0,43429,44819 * *is to the Number in the Tables, which expresses the given* * 2001.
Height. This Number is the Logarithm of the Ratio B A and B G, which 2058.
Ratio is therefore given; wherefore the Ratio A B and B I is given also, which is the Ratio *of the Squares* of the Velocities A Q and I K ‖; that is, ‖ *La Hire Sect. con.* B. 3. Prop 3.
of the greatest Velocity and of the Velocity, which a Body really acquires in a Fluid, in falling from the given Height A G †. † 2052.

SCHOLIUM VIII.

An Illustration of some Things relating to Retardation.

MANY things relating to Retardations must be illustrated, which, whilst they follow from what is demonstrated before, yet do not seem to agree well together, or with what is demonstrated before, at least at first sight: to remove which Scruples, and by removing which, to confirm the Theories of Forces and Retardations the more, I thought proper to add this Scholium to the rest.

The first Scruple relates to what is said in N. 2003, that the Retardation 2059.
and Acceleration, in every one of the infinitely small Moments, during the Moment,

Moment, are equable; but there is a Difficulty with reſpect to the Acceleration, and it relates to the Agreement of this Propoſition with what is demonſtrated concerning innate Forces.

2060. Let us ſuppoſe a Body at reſt in a Fluid that is in motion; this communicates to it an infinitely ſmall Velocity, in the firſt, infinitely ſmall Moment: Let the Moment be divided into two equal Parts, in each of the equal Parts there is communicated an equal Velocity, by reaſon of the equable Acceleration; that is, in the firſt Part only one, infinitely ſmall Degree of Force is communicated to the Body, and in the ſecond three
* 753. ſimilar Degrees *; tho' the reſpective Action be not encreas'd, which ſeems impoſſible.

2061. To remove this Scruple, I ſay that we muſt diſtinguiſh between abſolute and reſpective Actions. Whilſt we conſider theſe, in the Caſe of which we are ſpeaking, the Degrees of Velocity are equal, which are communicated, in equal Parts of an infinitely ſmall Moment, becauſe the reſpective Action is not ſenſibly changed; moreover, if we have regard to the reſpective Motions, the Force that is impreſſed upon the Body, is not greater in the ſecond than in the firſt Part of the Moment: The Body, to which one Degree of Velocity is ſuperadded, acquires only one Degree of Force in the Ship, in which the Body was at reſt, with what Velocity ſoever the Ship be carried.

2062. But in the Examination of the abſolute Actions, not only the reſpective Actions, but the abſolute alſo, muſt be conſider'd; as we before demon-
* 995. ſtrated, when we treated of Colliſions *. The Body A mov'd with the Velocity *a*, running againſt the Body B, communicates a greater Force to it,
† 996. 997. 998. if B be carried towards the ſame Part with A, than if it were at reſt †, tho' the reſpective Velocity in that Caſe be leſs, if the Velocity of the Body B does not exceed certain Limits. The Action upon the Body is different according to the different Force which it now has; and if it ſeems impoſſible that the ſame Body, mov'd in the ſame manner, and running againſt the ſame Body, ſhould communicate a greater Force to it in a certain Caſe, in which the reſpective Velocity is leſs, it muſt be referred to the Theory of Forces, which is not well underſtood; for it is manifeſt, that what is immediately prov'd by Experiments, cannot be accounted impoſſible; but we
* 1002. have ſufficiently illuſtrated the Matter *.

2063. When we undertake to determine Effects from Cauſes, we ought to have regard to the whole Effects. In the reſpective Action, it will be proportional to the whole reſpective Effect: If the Action be abſolute, we ought to conſider every Effect whatſoever; and the Cauſe anſwers to all the Effects join'd.

It is manifeſt that theſe things ought to be referred to the Actions of Fluids; and as the Changes of the reſpective and abſolute Actions do not follow the ſame Ratio, it plainly appears that the ſame Ratio can't take place in the reſpective and abſolute Effects alſo.

2064. The ſecond Scruple, to be remov'd in this Scholium, relates to the Re-
* 1961. tardation from the firſt Cauſe, which we demonſtrated to be equable *; whence

whence it follows from the Action, ariſing from the Coheſion of Parts, which we explain'd above †, that an equal Velocity is communicated to a † 1896.
Body at reſt, in an equal Time, with what Velocity ſoever the Fluid runs againſt it ‖. ‖ 1955. 1962.

But this does not ſeem to agree with what is before demonſtrated; for we have ſeen that a Body ſuffers a Preſſure from the Action, ariſing from the Coheſion of Parts, when it is kept in its Place, which is encreas'd with the Velocity *; and we have demonſtrated with regard to Preſſure in ge- * 1888. 1911.
neral, that it communicates a Velocity, to a Body at reſt, in a determin'd and infinitely ſmall Moment, which follows the Ratio of the Preſſure itſelf ‖. ‖ 133. 355.

2065. We have given above * the Foundation of the reaſoning, whereby we think this Difficulty is remov'd; we ſhall now more clearly explain the * 1956.
Reaſoning itſelf.

We ſaid that we muſt diſtinguiſh between the Preſſure, which immediately moves a Body, and the Preſſure which does not move a Body immediately. The firſt is treated of in N. 133. and its Demonſtration cannot be applied to the Caſe, in which the Preſſure, which ſeparates the Particles, acts in ſuch manner, as it ſhould to move the Obſtacle at the ſame time alſo.

This exerts Actions entirely different, as it acts upon immoveable, or moveable, greater or leſs Obſtacles. But to determine what relates to this kind of Preſſure, what follows muſt be conſider'd.

The Action of the Fluid upon the Body, ariſing from the Coheſion of Parts, is analogous and ſimilar to the Action, which Bodies as A, B, join'd by a Thread, exert upon the Body C, whilſt they paſs along by the Sides of it, and break the Thread by their Action upon C *. Plate LXII. Fig. 6. * 1896.

The Bodies A and B, as long as the Parts of the Thread cohere, preſs the Point C; the Thread being broke, the Preſſure ceaſes: but if immediately two other ſimilar Bodies D and F, and after theſe G and H, *&c.* preſs in the ſame manner, there will be given a Preſſure, which does not differ from the Preſſure of a Fluid, ariſing from Coheſion. Therefore it will be ſufficient to demonſtrate, theſe Bodies being in motion, that in equal Times an equal Velocity is communicated to the Body C, with what Velocity ſoever the Bodies A, B, D, F, G, H, *&c.* be carried, which we ſuppoſe equal, and mov'd with equal Velocity; but that theſe Bodies exert a Preſſure upon an immoveable Obſtacle, which follows the Ratio of the Velocity, with which they are carried.

2066. *A Body*, which is at reſt, or whoſe Velocity is given, *reſiſts the more the faſter it is accelerated*; for whilſt it acquires a determinate Degree of Velocity, a determin'd Degree of Force is communicated to it; and whilſt it acquires a determin'd Degree of Force, it exerts a determin'd Reſiſtance *: * 361.
This therefore is the ſame, whether this Degree of Force be communicated ſlower or faſter, namely, if we conſider the whole Reſiſtance. For the ſame Reaſon, the inſtantaneous Reſiſtance is the greater, the faſter the Body is accelerated; for the whole Reſiſtance follows the Proportion of the inſtantaneous Reſiſtance, and of the Time, it laſted. Therefore if this

be diminished, that will be to be increas'd, that the whole Resistance may be kept: but the Time is diminish'd in the Ratio, in which the Acceleration

2067. is encreas'd, *and the instantaneous Resistance encreases with the Acceleration, if the whole Resistance be determin'd.*

When the Acceleration is equable, the Body resists in the Ratio of the

* 751. Velocity which it has *.

2068. Therefore in general *the instantaneous Resistance of a Body, which is accelerated, is in a Ratio compounded of the Velocity, which it has, and the Acceleration.*

2069. Therefore *if the instantaneous Resistance be constant, the Velocity of the Body is inversely as the Acceleration.*

This Proposition is now to be applied to the Case of which we are speaking.

2070. The Bodies A and B act upon the Body C, till the instantaneous Resistance of this last, which alone can act contrary to the Pressure, is equal to the Pressure with which the Parts of the Thread cohere. The Acceleration continues so long; but when this Equality is given, the Action ceases, and the Thread is broke; and, whether the Bodies A and B be mov'd faster or slower, the instantaneous Resistance of the Body C is requir'd to be constant, which must be equal to the Cohesion of the Parts of the Thread, that the Thread may be broke. But the faster A and B are mov'd, the greater is the Acceleration, whilst they draw the Body C; therefore the less is the

* 2069. Velocity communicated to C, whilst the Thread is broke *. If, for example, the Velocity of the Bodies A and B in one Case be triple of that in another, whilst C is at rest in each Case; because the Acceleration in the first Case is triple, a third Part of the Velocity only is communicated to the Body C, whilst the Action of the Bodies upon C continues. If this Degree of Velocity be small, that the respective Action of the following Bodies D and F may not differ sensibly, these will communicate an equal Degree of Velocity; and not till three Threads are broke will C have the Velocity, which it has, whilst one Thread only is broke in the second Case. But in the Time in which, in the second Case, the Bodies A and B only pass along by C, in the first Case A, B, and D, F, as G, H also pass by, that is, three Threads are broke in the first Case, whilst only one is broke in the second, and in equal Times the equal Velocities mention'd are communicated. Which was to be demonstrated. Something like this we have

* 1034. 1035. seen before, where we treated of the Collision of Bodies *.

2071. It is a thing commonly known, that the less Velocity is communicated to a Body drawn by a Thread, whilst the Thread is broke, the faster it is drawn; for this reason, if the Body be accelerated slowly, a great Velocity may be communicated to it, tho' it be drawn by a small Thread.

2072. When the Bodies A and B, in breaking the Thread, communicate Force to the Body C, they lose so much of their Forces, as they communicate, and as is requir'd to break the Thread; therefore they lose the less Force, the faster they are mov'd.

If

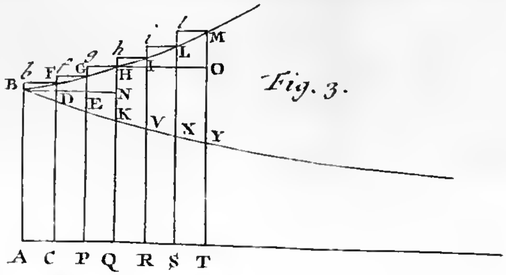


Fig. 3.

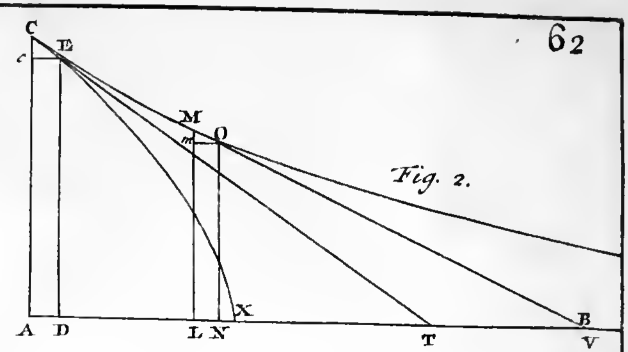
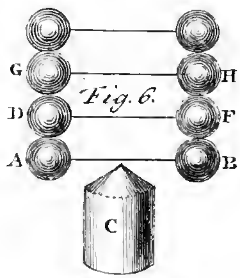


Fig. 2.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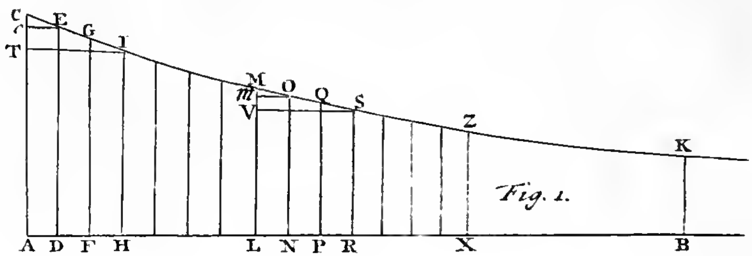
Fig. 1.

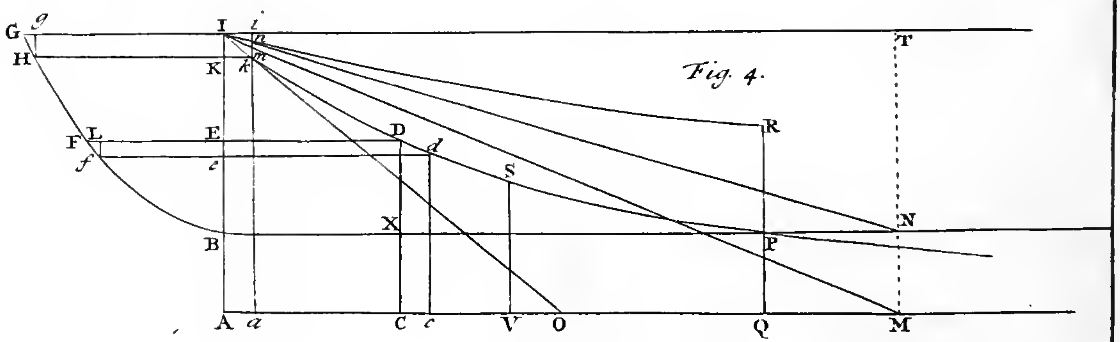
Fig. 4.

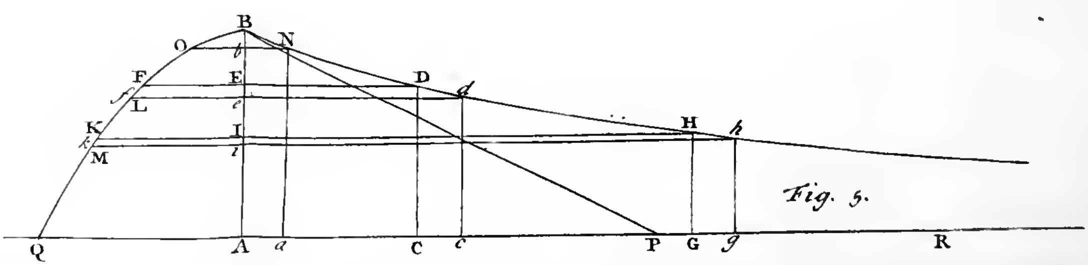
Fig. 5.

If the Obſtacle C cannot quit its Place, there is only one Effect of the Action of the Bodies A and B, and they loſe ſo much of their Force as is requir'd to break the Thread; and the Action, which that ſuffers which keeps C in its Place, is the ſame for each of the Threads which are broke. In the foregoing Caſe, the ſlower the Bodies A and B are mov'd, the longer they act before C reſiſts ſo much as is requir'd, that the Thread may be broke; but in this Caſe, at the very Moment when the Thread comes to the Body C, this Reſiſtance is given: Wherefore in this Caſe the Action, which C ſuffers, follows the Ratio of the Threads, broke in a determin'd Time, that is, of the Velocity of the Bodies. Which was to be demonſtrated alſo. 2073.

The End of the Firſt Volume.

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